



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1

**October/November 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.



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**3**

- 1** The coefficient of  $x^3$  in the expansion of  $(1 + kx)(1 - 2x)^5$  is 20.

Find the value of the constant  $k$ .

[4]

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- 2 The first, second and third terms of a geometric progression are  $2p + 6$ ,  $-2p$  and  $p + 2$  respectively, where  $p$  is positive.

Find the sum to infinity of the progression. [5]

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4 The sum,  $S_n$ , of the first  $n$  terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The  $k$ th term in the progression is greater than 200.

Find the smallest possible value of  $k$ .

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5 Functions  $f$  and  $g$  are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x + 1}, \quad \text{for } x \in \mathbb{R}, x \neq -1.$$

(a) Find the value of  $fg(7)$ . [1]

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(b) Find the values of  $x$  for which  $f^{-1}(x) = g^{-1}(x)$ . [5]

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6 (a) Prove the identity  $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$ . [4]

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(b) Hence solve the equation  $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2 \tan^2 x$  for  $0^\circ \leq x \leq 180^\circ$ . [2]

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7 The point (4, 7) lies on the curve  $y = f(x)$  and it is given that  $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$ .

- (a) A point moves along the curve in such a way that the  $x$ -coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the  $y$ -coordinate when  $x = 4$ . [3]

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- (b) Find the equation of the curve. [4]

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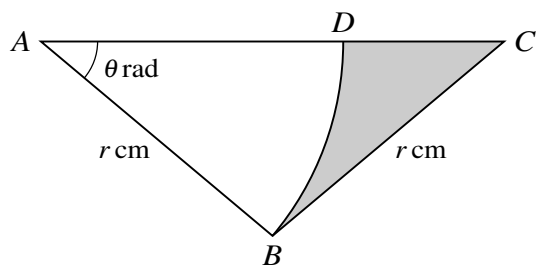
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In the diagram,  $ABC$  is an isosceles triangle with  $AB = BC = r$  cm and angle  $BAC = \theta$  radians. The point  $D$  lies on  $AC$  and  $ABD$  is a sector of a circle with centre  $A$ .

(a) Express the area of the shaded region in terms of  $r$  and  $\theta$ . [3]

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9 A circle has centre at the point  $B(5, 1)$ . The point  $A(-1, -2)$  lies on the circle.

(a) Find the equation of the circle. [3]

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Point  $C$  is such that  $AC$  is a diameter of the circle. Point  $D$  has coordinates  $(5, 16)$ .

(b) Show that  $DC$  is a tangent to the circle. [4]

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The other tangent from  $D$  to the circle touches the circle at  $E$ .

(c) Find the coordinates of  $E$ . [2]

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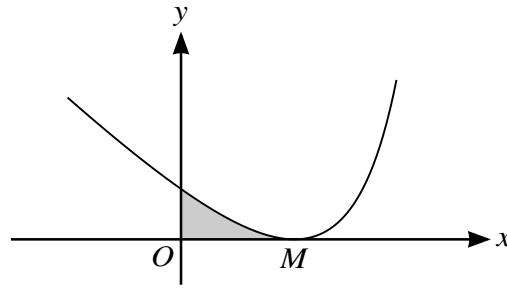
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The diagram shows part of the curve  $y = \frac{2}{(3 - 2x)^2} - x$  and its minimum point  $M$ , which lies on the  $x$ -axis.

- (a) Find expressions for  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  and  $\int y dx$ . [6]

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- (b) Find, by calculation, the  $x$ -coordinate of  $M$ . [2]

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- (c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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**11** A curve has equation  $y = 3 \cos 2x + 2$  for  $0 \leq x \leq \pi$ .

**(a)** State the greatest and least values of  $y$ . [2]

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**(b)** Sketch the graph of  $y = 3 \cos 2x + 2$  for  $0 \leq x \leq \pi$ . [2]

**(c)** By considering the straight line  $y = kx$ , where  $k$  is a constant, state the number of solutions of the equation  $3 \cos 2x + 2 = kx$  for  $0 \leq x \leq \pi$  in each of the following cases.

**(i)**  $k = -3$  [1]

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**(ii)**  $k = 1$  [1]

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**(iii)**  $k = 3$  [1]

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Functions  $f$ ,  $g$  and  $h$  are defined for  $x \in \mathbb{R}$  by

$$f(x) = 3 \cos 2x + 2,$$

$$g(x) = f(2x) + 4,$$

$$h(x) = 2f\left(x + \frac{1}{2}\pi\right).$$

- (d) Describe fully a sequence of transformations that maps the graph of  $y = f(x)$  on to  $y = g(x)$ . [2]

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- (e) Describe fully a sequence of transformations that maps the graph of  $y = f(x)$  on to  $y = h(x)$ . [2]

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**Additional Page**

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