



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/04

Paper 4 Further Probability & Statistics

For examination from 2020

SPECIMEN PAPER

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.

- 1 (a) State briefly the circumstances under which a non-parametric test of significance should be used rather than a parametric test. [1]

. . . .

The level of pollution in a river was measured at 17 different locations. The results, in mg/l, are shown below, with the highest 7 values representing greater pollution

8 3 6 8 0 3 8 4 6 8 0 9

- (b) Use a Wilcoxon signed rank test to test whether the average pollution level in the river is more than 10 mg/l. Use a 5% significance level. [5]

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6

- 4 The number, x , of a certain type of sea shells was counted at 6 randomly chosen sites, each 1 metre square, along the coastline in country A. The number, y , of the same type of sea shells was counted at 6 randomly chosen sites, each 1 metre square, along the coastline in country B. The results are summarised as follows, where \bar{x} and \bar{y} denote the sample means of x and y respectively.

$$\bar{x} = 2 \quad \Sigma(x - \bar{x})^2 = 3 \quad \bar{y} = 2 \quad \Sigma(y - \bar{y})^2 = 0$$

Find a 95% confidence interval for the difference between the mean number of sea shells, per square metre, of the coastlines in country A and in country B. [7]

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5 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 0 & x < 0, \\ \frac{6}{5}x & 0 \leq x \leq 1, \\ \frac{6}{5}x^{-4} & x > 1. \end{cases}$$

(a) Find $P(X > 1)$. [1]

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(b) Find the mean value of X . [2]

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(c) Given that $E(X) = \mu$ and σ^2 is the variance of X . [3]

(d) Find $E(\sqrt{X})$. [2]

6 Aisha has a bag containing 3 red balls and 3 white balls. She selects a ball at random, notes its colour and returns it to the bag. The same process is repeated twice more. The number of red balls selected by Aisha is denoted by X .

(a) Find the probability distribution of $X(t)$ for X . [2]

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Basim also has a bag containing 3 red balls and 3 white balls. He selects three balls at random, with replacement, from his bag. The number of red balls selected by Basim is denoted by Y .

(b) Find the probability distribution of $Y(t)$ for Y . [3]

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The random variable Z is the total number of red balls selected by Aisha and Brian.

(c) Find the probability distribution of Z , expressing your answer as a probability table. [3]

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(d) Use the probability distribution of Z to find $E(Z)$ and $\text{Var}(Z)$. [5]

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Additional page

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