

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
GCE Advanced Level

**MARK SCHEME for the May/June 2012 question paper  
for the guidance of teachers**

**9231 FURTHER MATHEMATICS**

**9231/23**

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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<b>Page 2</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2012</b>	<b>9231</b>	<b>23</b>

### Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

<b>Page 3</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2012</b>	<b>9231</b>	<b>23</b>

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{\phantom{x}}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2012	9231	23

Question Number	Mark Scheme Details	Part Mark	Total
1	<p>Use conservation of momentum, e.g.: <math>3mv_A + 6mv_B = 3mu</math> M1</p> <p>Use Newton's law of restitution (A1 if both correct): <math>v_A - v_B = -eu</math> M1 A1</p> <p>Solve for <math>v_A</math> and <math>v_B</math> (A.E.F.; A0 if any dirn. error): <math>v_A = \frac{1}{3}(1 - 2e)u</math>, <math>v_B = \frac{1}{3}(1 + e)u</math> M1 A1</p> <p>Find loss of kinetic energy by substituting for <math>v_A, v_B</math>: <math>\frac{1}{2}m(3u^2 - 3v_A^2 - 6v_B^2)</math>  <math>= mu^2\{3/2 - (1 - 4e + 4e^2)/6 - (1 + 2e + e^2)/3\}</math>  <math>= mu^2\{3/2 - \frac{1}{2}(1 + 2e^2)\}</math>  <math>= mu^2(1 - e^2)</math> <b>A.G.</b> M1 A1</p> <p>(some working needed to earn A1; ignore any earlier dirn. error)</p>	7	[7]
2	<p>Find <math>AP</math> (or <math>BP</math>) in terms of <math>x</math> and <math>a</math>: <math>AP = \sqrt{4a^2 + x^2}</math> B1</p> <p>Find <math>T</math> by Hooke's Law for one string [or both]: <math>T = 2mg(AP - a) / a</math> B1</p> <p>Find eqn of motion at general point: <math>md^2x/dt^2 = -2Tx/AP</math> M1</p> <p>Combine to find <math>d^2x/dt^2</math>: <math>d^2x/dt^2 = -(4gx/a)(1 - a/AP)</math> M1  <math>= -(4gx/a)\{1 - \frac{1}{2}(1 + x^2/4a^2)^{-1/2}\}</math> A1</p> <p><b>A.G.</b></p> <p><b>M.R.</b> Vertical motion loses only this A1  Motion along <math>AB</math> loses all marks</p> <p>Neglect <math>(x/a)^2</math>: <math>d^2x/dt^2 = -(4gx/a)^{1/2} = -2gx/a</math> B1</p> <p>State period of motion using <math>2\pi/\omega</math> (A.E.F.): <math>2\pi\sqrt{a/2g}</math> or <math>\pi\sqrt{2a/g}</math> B1</p>	5 2	[7]
3	<p>Apply <math>v^2 = \omega^2(A^2 - x^2)</math> at both points: <math>4^2 = \omega^2(A^2 - 6^2)</math>, <math>3^2 = \omega^2(A^2 - 8^2)</math> M1</p> <p>Combine to find <math>A</math>: <math>7A^2 = 32^2 - 18^2</math>, <math>A = 10</math> <b>A.G.</b> M1 A1</p> <p>Find period <math>T</math> using <math>\omega</math>: <math>\omega = \frac{1}{2}</math>, <math>T = 2\pi/\omega = 4\pi</math> or 12.6 [s] B1</p> <p><i>EITHER:</i> Use <math>x = A \sin \omega t</math> to find time from <math>O</math> to <math>A</math>: <math>t_{OA} = \omega^{-1} \sin^{-1}(6/10)</math> [= 1.287] M1</p> <p>Use <math>x = A \sin \omega t</math> to find time from <math>O</math> to <math>B</math>: <math>t_{OB} = \omega^{-1} \sin^{-1}(8/10)</math> [= 1.855] M1</p> <p>Combine to find time from <math>A</math> to <math>B</math>: <math>t_{OA} + t_{OB} = 3.14</math> or <math>\pi</math> [s] M1 A1</p> <p><i>OR:</i> Use <math>x = A \cos \omega t</math> to find time to <math>A</math>: <math>t_A = \omega^{-1} \cos^{-1}(6/10)</math> [= 1.855] (M1)</p> <p>Use <math>x = A \cos \omega t</math> to find time to <math>B</math>: <math>t_B = \omega^{-1} \cos^{-1}(-8/10)</math> [= 4.996] (M1)</p> <p>Combine to find time from <math>A</math> to <math>B</math>: <math>t_B - t_A = 3.14</math> or <math>\pi</math> [s] (M1 A1)</p>	4	[8]
4	<p>Use conservation of energy (B0 for <math>v^2 = \dots</math>): <math>\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos \theta)</math> B1</p> <p>Equate radial forces: <math>R = mg \cos \theta - mv^2/a</math> B1</p> <p>Eliminate <math>v</math> to find <math>R</math>: <math>R = mg(3 \cos \theta - 2) - mu^2/a</math> <b>A.G.</b> M1 A1</p> <p>(i) Find <math>\cos \alpha</math> when <math>R = 0</math>: <math>\cos \alpha = \frac{1}{3}(\frac{1}{4} + 2) = \frac{3}{4}</math> M1 A1</p> <p>(ii) Find <math>v^2</math> at this point: <math>v_1^2 = \frac{1}{4}ga + \frac{1}{2}ga = \frac{3}{4}ga</math> B1</p> <p>Find vertical comp. of <math>v_1</math> at plane: <math>v_2^2 = v_1^2 \sin^2 \alpha + 2ga(1 + \cos \alpha)</math> M1 A1  <math>= (21/64 + 7/2)ga</math> A1</p> <p>(using <math>u</math> in place of <math>v_1</math> can earn M1 A0) (A.E.F.) <math>v_2 = \sqrt{(245ga/64)}</math> or <math>(7/8)\sqrt{(5ga)}</math> or <math>1.96\sqrt{(ga)}</math> or <math>6.19\sqrt{a}</math> A1</p>	4 2 5	[11]

<b>Page 5</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2012</b>	<b>9231</b>	<b>23</b>

Question Number	Mark Scheme Details	Part Mark	Total	
<b>5</b>	Find MI of rod about $A$ : Find MI of circular lamina about $A$ : Find MI of rod plus lamina about $A$ : (i) Find new MI including particle about $A$ : Use eqn of circular motion to find $d^2\theta/dt^2$ : (using $I$ can earn B0 M1 A1 A0) (ii) Use energy when $AB$ vertical to find $\omega$ : (or by integration) (using $I$ can earn M1 A1 A0)	$I_{rod} = (4/3)m(3a)^2 = 12ma^2$ B1 $I_{lamina} = 2ma^2 + 64ma^2 = 66ma^2$ M1 A1 $I = I_{rod} + I_{lamina} = 78ma^2$ A.G. A1 $I' = I + (2m)(6a)^2 = 150ma^2$ B1 $I'd^2\theta/dt^2 = [-] mg(3 + 8 + 12)a \sin \theta$ M1 A1 $d^2\theta/dt^2 = -(23g/150a) \sin \theta$ A1 $\frac{1}{2}I'\omega^2 = \frac{1}{2}I'\omega_0^2 + (3 + 8 + 12)mga$ M1 A1 $\omega^2 = 9g/25a + 46mga/150ma^2$ $\omega = \sqrt{(2g/3a)}$ or $0.816\sqrt{(g/a)}$ or $2.58/\sqrt{a}$ (A.E.F.) A1	4 4 3	[11]
<b>6</b>	Estimate population variance: (allow biased here: $0.0912$ or $0.3020^2$ ) State hypotheses (B0 for $\bar{x}$ ...): Calculate value of $t$ (to 3 sf): State or use correct tabular $t$ value: (or can compare $\bar{x}$ with $8.05 + 0.184[5] = 8.23$ ) Valid method for reaching conclusion: Conclusion (A.E.F., needs correct values):	$s^2 = 0.912 / 9$ $= 0.1013$ or $38/375$ or $0.3183^2$ M1 $H_0: \mu = 8.05, H_1: \mu > 8.05$ B1 $t = (\bar{x} - 8.05)/(s/\sqrt{10}) = 2.03$ M1 A1 $t_{9, 0.95} = 1.83[3]$ B1 Reject $H_0$ if $t >$ tabular value M1 $2.03 > 1.83$ so mean is greater A1	7	[7]
<b>7</b>	(i) State or find $\lambda$ : (ii) Find $F(10) - F(5)$ : (iii) Formulate eqn for median $m$ of $T$ : Find value of $m$ :	$\lambda = 1/(\text{standard deviation}) = 1/8$ M1 A1 $e^{-5\lambda} - e^{-10\lambda} = e^{-0.625} - e^{-1.25} = 0.249$ M1 A1 $1 - e^{-\lambda m} = 1/2$ M1 $m = -8 \ln 1/2 = 5.55$ (or $5.54$ ) M1 A1	2 2 3	[7]
<b>8</b>	Find expected values (to 1 d.p.): (lose A1 if rounded to integers) State (at least) null hypothesis (A.E.F.): Calculate value of $\chi^2$ to 1 d.p.: Compare with consistent tabular value (to 1 d.p.): Conclusion consistent with values (A.E.F.): State town with max. contribn. ( $\checkmark^h$ on exp. values): State valid comment ( $\checkmark^h$ on exp. values), e.g.:	$24.48 \quad 30.72 \quad 16.8$ $48.96 \quad 61.44 \quad 33.6$ $28.56 \quad 35.84 \quad 19.6$ M1 A1 $H_0$ : Grade independent of town B1 $\chi^2 = 0.009 + 0.350 + 0.467$ $+ 1.669 + 0.034 + 1.719$ $+ 2.567 + 0.094 + 5.518$ $= 12.4[3]$ M1 A1 $\chi_{4, 0.975}^2 = 11.1[4]$ B1 Grade not independent of town A1 $\checkmark^h$ Town C B1 $\checkmark^h$ Signal poorer than expected in C B1 $\checkmark^h$	7 2	[9]
<b>9</b>	Integrate $f(x)$ to find $F(x)$ for $-a \leq x \leq a$ (A.E.F.): (Finding $F(x) = x/2a$ earns M1 A0) State $F(x)$ for other intervals: Relate dist. fn. $G(y)$ of $Y$ to $X$ for central interval: (working may be omitted; ignore other intervals) Find $k$ from $P(Y \geq k) = 1/4$ with $a = 4$ : (Using $G(k) = 1/4$ can earn M1)	$F(x) = x/2a + c = (x + a)/2a$ M1 A1 $F(x) = 0$ ( $x < -a$ ), $F(x) = 1$ ( $x > a$ ) B1 $G(y) = P(Y < y) = P(e^X < y)$ $= P(X < \ln y) = F(\ln y)$ $= (\ln y + a)/2a; (e^{-a} \leq y \leq e^a)$ M1 A1; A1 $1 - G(k) = 1/4, (\ln k)/2a + 1/2 = 3/4$ M1 A1 $\ln k = 2, k = e^2$ or $7.39$ A1	3 3 3	[9]

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2012	9231	23

Question Number	Mark Scheme Details	Part Mark	Total
10	<p>Estimate population variance using <math>P</math>'s sample: <math>s_P^2 = (1224 - 240^2/50) / 49</math>  (allow use of biased: <math>\sigma_{P,50}^2 = 1.44</math> or <math>1.2^2</math>) <math>= 1.469</math> or <math>72/49</math> or <math>1.212^2</math> M1 A1</p> <p>Estimate population variance using <math>Q</math>'s sample: <math>s_Q^2 = (754 - 168^2/40) / 39</math>  (allow use of biased: <math>\sigma_{Q,40}^2 = 1.21</math> or <math>1.1^2</math>) <math>= 1.241</math> or <math>242/195</math> or <math>1.114^2</math> A1</p> <p>Estimate population variance for combined sample: <math>s^2 = s_P^2/50 + s_Q^2/40</math>  (allow use of <math>\sigma_{P,50}^2, \sigma_{Q,40}^2</math>: <math>0.05905</math> or <math>0.243^2</math>) <math>= 0.0604</math> or <math>0.246^2</math> M1</p> <p>Find confidence interval: <math>(240/50 - 168/40) \pm zs</math> M1</p> <p>Use <math>z = 1.96</math> and evaluate correct to 3 d.p.: <math>0.6 \pm 0.482</math> or <math>[0.118, 1.082]</math> A1</p> <p>(allow use of <math>0.05905</math> instead of <math>0.0604</math>): <math>0.6 \pm 0.476</math> or <math>[0.124, 1.076]</math></p> <p>State hypotheses: <math>H_0: \mu_P = \mu_Q, H_1: \mu_P &gt; \mu_Q</math> B1</p> <p>Calculate value of <math>z</math> (to 2 d.p.): <math>z = (240/50 - 168/40) / s = 2.44</math> M1 *A1  (allow <math>0.6/0.243 = 2.47</math>)</p> <p>State or use correct tabular <math>z</math> value: <math>z_{0.99} = 2.326</math> [or <math>2.33</math>] *B1  (or can compare <math>0.6</math> with <math>0.572</math> or <math>0.56[5]</math>)</p> <p>Valid method for reaching conclusion: Reject <math>H_0</math> if <math>z &gt;</math> tabular value M1</p> <p>Correct conclusion (AEF, dep *A1, *B1): <math>\mu_P</math> is greater than <math>\mu_Q</math> A1</p> <p><b>S.R.</b> Allow (implicit) assumption of equal variances, but deduct A1 if not explicit:</p> <p>Pooled estimate of common variance <math>s^2</math>: <math>(50\sigma_{P,50}^2 + 40\sigma_{Q,40}^2)/88 = 1.368</math></p> <p>Confidence interval: <math>0.6 \pm 1.96s\sqrt{(1/50+1/40)}</math>  <math>= 0.6 \pm 0.486</math> or <math>[0.114, 1.086]</math></p> <p>Value of <math>z</math> (to 2 dp): <math>0.6/s\sqrt{(1/50+1/40)} = 2.42</math></p>	6	[12]
11 (a)	<p><b>EITHER:</b></p> <p>Take moments about <math>A</math> for rod to find <math>R_C</math>: <math>R_C 3a = kW 4a \cos \theta + W 2a \cos \theta</math> M1 A1  <math>R_C = (2k + 1) (2W/5)</math> A1</p> <p>Resolve horizontal forces on rod to find <math>F_A</math>: <math>F_A = R_C \sin \theta = (2k + 1) (8W/25)</math> M1 A1</p> <p>Resolve vertical forces on rod to find <math>R_A</math>: <math>R_A = W + kW - R_C \cos \theta</math>  <math>= (13k + 19) (W/25)</math> M1 A1</p> <p><b>OR:</b> Take moments about <math>C</math> for rod:  <math>F_A 3a \sin \theta - R_A 3a \cos \theta = kWa \cos \theta - Wa \cos \theta</math> (M1 A1)  <math>4F_A - 3R_A = (k - 1)W</math> (A1)</p> <p>Resolve forces parallel to rod:  <math>F_A \cos \theta + R_A \sin \theta = kW \sin \theta + W \sin \theta</math> (M1)  <math>3F_A + 4R_A = 4(k + 1)W</math> (A1)</p> <p>Combine to find <math>F_A, R_A</math>:  <math>F_A = (2k + 1) (8W/25)</math> and  <math>R_A = (13k + 19) (W/25)</math> (M1 A1)</p> <p>Use <math>F_A \leq \mu R_A</math> to find <math>\mu_{\min}</math>:  (No use of inequality loses A1)  <math>\mu_{\min} = 8(2k + 1)/(13k + 19)</math> <b>A.G.</b> M1 A1</p> <p>Find bound on <math>k</math> (allow use of equality):  <math>160k + 80 \leq</math> (or <math>=</math>) <math>117k + 171</math> M1  Bound = <math>91/43</math> or <math>2.12</math> A1</p> <p>Correct use of inequality  <math>k \leq 91/43</math> or <math>2.12</math> A1</p>	9	[12]

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2012	9231	23

Question Number	Mark Scheme Details	Part Mark	Total
11 (b)	Find correlation coefficient $r$ : $r^2 = (-0.5)(-1.2) = 0.6$ or $0.775^2$ M1 A1 $r = -0.775$ *A1	3	
	State both hypotheses (B0 for $r \dots$ ): $H_0: \rho = 0, H_1: \rho \neq 0$ B1 State or use correct tabular one-tail $r$ value: $r_{5, 2.5\%} = 0.878$ *B1 Valid method for reaching conclusion: Accept $H_0$ if $ r  <$ tabular value M1 Correct conclusion (AEF, dep *A1, *B1): Coeff. does not differ from zero A1	4	
	<i>EITHER</i> : Find two indep. eqns for $p, q$ , e.g.: (M1 for either) $14 + q = (-0.5)(13 + p) + 5 \times 5$ or $(14 + q)/5 = (-0.5)(13 + p)/5 + 5$ M1 A1 $13 + p = (-1.2)(14 + q) + 5 \times 7.6$ or $(13 + p)/5 = (-1.2)(14 + q)/5 + 7.6$ A1 Combine to find $p, q$ : $p = 7, q = 1$ A1, A1		
	<i>OR</i> : Combine regression lines to find $\bar{x}, \bar{y}$ : $\bar{x} = 4; \bar{y} = 3$ (M1 A1; A1) Use sample data to find $p, q$ : $p = 7, q = 1$ (A1, A1)	5	[12]