UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/21 Paper 21, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Find ω or ω^2 from $2\pi/T$:	$\omega = 2\pi / 0{\cdot}5$ [$= 4\pi = 12{\cdot}57$]	B1		
	Relate F to acceleration:	$F = 0{\cdot}2\ \mathrm{d}^2x/\mathrm{d}t^2$	M1		
	Relate acceleration to ω and x:	$\mathrm{d}^2x/\mathrm{d}t^2 = [-]\omega^2 x$	M1		
	State or use value of x giving max of F [or $\mathrm{d}^2x/\mathrm{d}t^2$]:	Maximum when $x = [\pm]\ 0{\cdot}3$	M1		
	Evaluate maximum F_{max} of F:	$0{\cdot}2\ (4\pi)^2\ 0{\cdot}3 = 0{\cdot}96\pi^2$ *or* $9{\cdot}47$	A1	5	**[5]**
2	Take moments for rod about A (A.E.F.):	$2P \cos 60° = W \sin 60°$	M1 A1		
	Find P:	$P = W\sqrt{3}/2$ *or* $0{\cdot}866W$	A1	3	
	Resolve vertically for friction F at A (√ on P):	$F = W - P \cos 30° = W/4$	M1 A1√		
	Resolve horizontally for reaction R at A (√ on P):	$R = P \sin 30° = W\sqrt{3}/4$ *or* $0433W$	B1√		
	Use $F \le \mu R$ to find values of μ:	$\mu \ge 1/\sqrt{3}$ *or* $\mu \ge 0{\cdot}577$	B1	4	**[7]**
3	Use conservation of momentum:	$3mv_A + mv_B = 3mu$	B1		
	Use Newton's law of restitution:	$v_A - v_B = -u$	B1		
	Solve for v_A *and* v_B:	$v_A = \frac{1}{2}u$ *and* $v_B = 3u/2$	M1 A1		
	Find rebound speed of B after collision with barrier:	$w_B = \frac{1}{2}v_B$ [$= 3u/4$]	M1	5	
	EITHER: Find time for B to colln. at d from barrier:	$t_1 = a / v_B$ [$= 2a/3u$] *and* $t_2 = d / w_B$ [$= 8a/15u$]	B1		
	Find time for A to same collision:	$t_1 + t_2 = (a - d) / v_A$	B1		
	Equate times and solve for d:	$2(a - d) = 2a/3 + 4d/3$, $d = 2a/5$	M1 A1		
	OR: Find dist. A moves in time t_1:	$s_A = v_A \times (a/v_B)$ [$= a/3$]	(B1)		
	Find t_2 from both A and B:	$t_2 = (a - s_A - d) / v_A$, $t_2 = \mathrm{d} / w_B$	(B1)		
	Equate times and solve for d:	$2(2a/3 - d)/u = 4d/3u$, $d = 2a/5$	(M1 A1)	4	
	MR: Taking $v_A - v_B = -\frac{1}{2}u$ gives:	$v_A = 5u/8$, $v_B = 9u/8$, $w_B = 9u/16$ $t_1 = 8a/9u$, $t_2 = 64a/171u$, $d = 4a/19$			
	or taking $v_A - v_B = -e\,u$ gives:	$v_A = (3 - e)u/4$, $v_B = 3(e + 1)u/4$ $w_B = 3(e + 1)u/8$, $t_1 = 4a/3(e + 1)u$ $t_2 = 32ea/3(e + 1)(e + 9)u$ $d = 4ea/(e + 9)$	(max 8)		**[9]**
4 (i)	Equate tangential speeds to find ω_B:	$0{\cdot}5\omega_A = 0{\cdot}3\omega_B$, $\omega_B = 5/3$ [rad s^{-1}]	M1 A1	2	
(ii)	Find tangential acceleration, $r\ \mathrm{d}^2\theta/\mathrm{d}t^2$:	$0{\cdot}5 \times \frac{1}{2} = 0{\cdot}25$	M1 A1		
	Find radial acceleration, $r\ (\mathrm{d}\theta/\mathrm{d}t)^2$:	$0{\cdot}5 \times 1^2 = 0{\cdot}5$	B1		
	Combine to give mag. of acceln:	$\sqrt{(0{\cdot}25^2 + 0{\cdot}5^2)}$	M1		
		$= \sqrt{5}/4$ *or* $0{\cdot}559$ [m s^{-2}]	A1		
	Find angle made with PA (A.E.F.):	$\tan^{-1}(0{\cdot}25/0{\cdot}5)$	M1		
		$= 0{\cdot}464$ rad or $26{\cdot}6°$	A1	7	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
5	Find extension and apply Hooke's Law:	$T = \frac{1}{2}mg\ (a / \sin\beta) / a$	M1		
		$= mg / 2\sin\beta$	A1		
	Resolve vertically for particle:	$mg = 2T\cos\beta$ $[T = mg / 2\cos\beta]$	B1		
	Equate two expressions for T:	$\sin\beta = \cos\beta,\ \beta = \frac{1}{4}\pi$ **A.G.**	B1	4	
	Use Newton's Law for vertical motion (A.E.F.):	$m\,d^2x/dt^2 = mg - 2T'\cos\beta'$			
		$= mg - (mg / \sin\beta')\cos\beta'$	M1 A1		
	Simplify:	$d^2x/dt^2 = g - g\ (\frac{1}{2}a + x) / \frac{1}{2}a$			
		$= -2gx/a$ **A.G.**	A1	3	
	Use $v = A\omega$ to find amplitude A of motion:	$A = \sqrt{(ag)}\ \sqrt{(2g/a)} = a/\sqrt{2}$	M1 A1		
	Hence show particle reaches pins:	$a/\sqrt{2} > a/2$	A1		
	Use $x = A\sin\omega t$ to find time t:	$t = (\sin^{-1}(-\frac{1}{2}a/A))/\omega$			
		or $\frac{1}{2}T + (\sin^{-1}(\frac{1}{2}a/A))/\omega$	M1		
		$= (\sin^{-1}(-1/\sqrt{2})) / \sqrt{(2g/a)}$			
		or $(\pi + \sin^{-1}(1/\sqrt{2})) / \sqrt{(2g/a)}$	A1		
	Simplify (A.E.F.):	$= (5\pi/4)\sqrt{(a/2g)}$ *or* $0{\cdot}878\sqrt{a}$	A1	6	**[13]**
6	Find relation for median M:	$\log_{10} M = 1{\cdot}5$	M1 A1		
	Evaluate M:	$M = 10^{1\cdot5} = 31{\cdot}6$	A1	3	
	Relate $P(X \geq 50)$ to Normal distribution:	$P(X \geq 50) = P(\log X \geq \log 50)$			
		$= 1 - \Phi((\log 50 - 1{\cdot}5) / 0{\cdot}2)$	M1		
	[log 50 = 1·699]	$= 1 - \Phi(0{\cdot}995) = 0{\cdot}160$	A1	2	**[5]**
7	Relate $P(X \leq 2 \leq 4X)$ to $F(x)$:	$= P(\frac{1}{2} \leq X \leq 2) = F(2) - F(\frac{1}{2})$	M1 A1		
	Evaluate:	$= (1 - e^{-1}) - (1 - e^{-\frac{1}{4}})$			
		$= 0{\cdot}632 - 0{\cdot}221 = 0{\cdot}411$	A1	3	
	EITHER: State $E(X)$ or find using $f(x)$ for $x > 0$:	$f(x) = \frac{1}{2}e^{-\frac{1}{2}x}$, $E(X) = 2$	M1 A1		
	Find width of interval $(X, 4X)$:	$E(3X) = 3 \times E(X) = 6$	M1 A1		
	OR: Find f(y) for $Y = 4X - X$:	$F(y) = P(X < y/3) = 1 - e^{-y/6}$			
		$f(y) = e^{-y/6}/6$	(M1 A1)		
	Find width of interval $(X, 4X)$:	$E(Y) = \int y(e^{-y/6}/6)\,dy = 6$	(M1 A1)	4	**[7]**
8	State assumptions (A.E.F.):	Equal variances	B1		
		Normal populations	B1		
	Find difference in sample means to 2 dp, e.g.:	$\overline{x}_A - \overline{x}_B = 21{\cdot}417 - 25{\cdot}75 = -4{\cdot}33$	B1		
	Estimate common population variance:	$s^2 = (5629 - 257^2/12$			
		$+ 5359 - 206^2/8) / 18$			
		$= (124{\cdot}9 + 54{\cdot}5)/18$			
		$= 9{\cdot}968$ *or* $3{\cdot}157^2$ (3 sf)	M1 A1		
	Use of correct tabular t value:	$t_{18,\ 0.975} = 2{\cdot}101$ (to 3 sf)	B1		
	Find confidence interval for e.g. $\overline{\mu}_A - \overline{\mu}_B$:	$x_A - \overline{x}_B \pm ts\ \sqrt{(1/12 + 1/8)}$	M1		
	Evaluate:	$-4{\cdot}33 \pm 3{\cdot}03$ *or* $[-7{\cdot}36, -1{\cdot}31]$	A1*	8	
	State reason and conclusion (A.E.F.):	Interval does not include zero			
	(dep *A1 apart from rounding)	so statement not supported	B1	1	**[9]**

Question Number	Mark Scheme Details	Part Mark	Total
9 **(i)**	Valid comment on scatter diagram (A.E.F.): Approx str. line with neg. gradient B1	1	
(ii)	*EITHER:* Find gradient b directly using r: $b = r\sqrt{(S_{yy} / S_{xx})}$ M2 $= r\sqrt{\{(8245 - 240^2/20) / (2125 - 200^2/20)\}}$ A1 $= -0.992\sqrt{(5365 / 125)}$ *OR:* Find S_{xy} (to 3 sf): $S_{xy} = r\sqrt{(S_{xx}\ S_{yy})} = -812.37$ (M1 A1) Find b: $b = S_{xy} / S_{xx}$ [$= -812.37/125$] (M1) Evaluate b: [$= -6.499$] $= -6.50$ A1 Find equation of regression line: $y = b(x - 10) + 12$ M1 $= 77.0 - 6.50x$ A1	6	
(iii)	Find b' using $r^2 = bb'$: [*or* S_{xy} / S_{yy}] $b' = -0.992^2/6.499 = -0.151$ M1 A1	2	**[9]**
10	Tabulate observed data with totals: *A* *B* *C* *Flu* 30 29 16 75 *No flu* 148 120 157 425 178 149 173 M1 Find expected values: *A* *B* *C* (lose A1 if 1 or 2 errors; *Flu* 26·7 22·35 25·95 lose A1 if rounded to integers) *No flu* 151·3 126·65 147·05 M1 A2 State (at least) null hypothesis (A.E.F.): H_0: Catching flu indep of vaccine B1 Calculate value of χ^2 (to 2 dp): $\chi^2 = 7.30$ M1 A1 **S.R.** If rounded to integers above allow (to 2 dp): $\chi^2 = 7.53$ (earns max 8/10) (B1) Compare with consistent tabular value (to 2 dp): $\chi_{2,\,0.95}{}^2 = 5.991$ B1 Valid method for reaching conclusion: Reject H_0 if $\chi^2 >$ tabular value M1 Correct conclusion (A.E.F., requires correct values): Catching flu depends on vaccine A1 Find proportions (or complements) for *A*,*B*,*C*: 0·169, 0·195, 0·092 (to 2 dp) M1 A1 Correct conclusion (A.E.F., requires correct values): *C* appears most effective A1	10 3	**[13]**
11 EITHER	Find MI of disc about *O* [or *A*]: $I_{disc} = \frac{1}{2}\,4ma^2$ [$= 2ma^2$ *or* $18ma^2$] B1 Find MI of ring about *O* [or *A*]: $I_{ring} = m(2a)^2$ [$= 4ma^2$ *or* $8ma^2$] B1 Find MI of *AO* about *O* [or *BO* about *A*]: $I_{rod} = (4/3)ma^2$ [*or* $22ma^2/3$] B1 Find MI of wheel about *A*: $I_{wheel} = 10ma^2 + 8m(2a)^2$ M1 $= 42ma^2$ **A.G.** A1 Find angular speed ω using energy: $\frac{1}{2} I_{wheel}\,\omega^2 = 8mg \times 2a \sin 30°$ M1 A1 $\omega^2 = 8mga^2/21ma^2$ $\omega = \sqrt{(8g/21a)}$ *or* $1.95/\sqrt{a}$ (A.E.F.) A1 Find new MI about *A*: $I_{new} = 8ma^2 + 4m(2a)^2 = 24ma^2$ M1 A1 Find reqd. angle θ using energy: $\frac{1}{2} I_{new}\,\omega^2 = M_{new}g \times 2a \sin\theta$ M1 Find and use new mass: $M_{new} = m + 3m = 4m$ A1 Substitute for I_{new}, M_{new}, ω^2: $(32/7)mga = 8mga \sin\theta$ (A.E.F.) A1 Solve for θ: $\theta = \sin^{-1}(4/7) = 0.608$ rad *or* 34·8° A1	5 3 6	**[14]**

Question Number	Mark Scheme Details			Part Mark	Total
11 OR (i)	Integrate to find F(t) for $t \ge 2$ [c needed]:	$F(t) = c - (t-1)^{-2}$	M1		
	Use F(2) = 0 to find c:	$F(t) = 1 - (t-1)^{-2}$	A1		
	Find $p = P(T > 5)$:	$p = 1 - F(5) = 1 - (1 - 4^{-2}) = 1/16$	B1	3	
(ii)	State or imply distribution:	$P(N > n) = p\,(1-p)^{n-1}$ *or* geometric distn. with par. p	M1		
	Find $P(N > E(N))$:	$(1-p)^{1/p} = (15/16)^{16} = 0{\cdot}356$	M1 A1	3	
(iii)	Relate dist. fn. G(y) of Y to T:	$G(y) = P(Y < y) = P(1/(T-1) < y)$	M1		
	Rearrange :	$= P(T > 1 + 1/y)$	A1		
	Relate to dist. fn. F:	$= 1 - F(1 + 1/y)$	M1		
	Substitute expression for F:	$= 1 - \{1 - (1 + 1/y - 1)^{-2}\}$	A1		
	Simplify:	$= y^2$	A1		
	Differentiate to find prob. density fn:	$g(y) = 2y$	M1 A1		
	Give complete statement of g(y):	$g(y) = 2y\ (0 \le y \le 1)$, 0 otherwise	A1	8	**[14]**