UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

7188280838

FURTHER MATHEMATICS **9231/13**

Paper 1 **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Express $\dfrac{1}{r(r+1)(r-1)}$ in partial fractions. [1]

Find

$$\sum_{r=2}^{n} \frac{1}{r(r+1)(r-1)}.$$ [4]

State the value of

$$\sum_{r=2}^{\infty} \frac{1}{r(r+1)(r-1)}.$$ [1]

2 Show that the matrix $\begin{pmatrix} 1 & 4 & 2 \\ 3 & 0 & -2 \\ 3 & -3 & -4 \end{pmatrix}$ has no inverse. [2]

Solve the system of equations

$$\begin{aligned} x + 4y + 2z &= 0, \\ 3x \qquad - 2z &= 4, \\ 3x - 3y - 4z &= 5. \end{aligned}$$ [4]

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 4x^2 + 8.$$ [7]

4 A curve has parametric equations

$$x = 2\theta - \sin 2\theta, \quad y = 1 - \cos 2\theta, \quad \text{for } -3\pi \leqslant \theta \leqslant 3\pi.$$

Show that

$$\frac{dy}{dx} = \cot\theta,$$

except for certain values of θ, which should be stated. [4]

Find the value of $\dfrac{d^2y}{dx^2}$ when $\theta = \frac{1}{4}\pi$. [3]

5 The equation

$$8x^3 + 36x^2 + kx - 21 = 0,$$

where k is a constant, has roots $a - d$, a, $a + d$. Find the numerical values of the roots and determine the value of k. [8]

6 [In this question you may use, without proof, the formula $\int \sec x \, dx = \ln(\sec x + \tan x) + \text{const.}$]

(a) Let $y = \sec x$. Find the mean value of y with respect to x over the interval $\frac{1}{6}\pi \leqslant x \leqslant \frac{1}{3}\pi$. [4]

(b) The curve C has equation $y = -\ln(\cos x)$, for $0 \leqslant x \leqslant \frac{1}{3}\pi$. Find the arc length of C. [4]

7 The curve C has equation

$$y = \frac{2x^2 + 5x - 1}{x + 2}.$$

Find the equations of the asymptotes of C. [3]

Show that $\frac{dy}{dx} > 2$ at all points on C. [3]

Sketch C. [3]

8 The points A, B, C have position vectors

$$4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}, \qquad 5\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}, \qquad 2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin O. Find a cartesian equation of the plane ABC. [4]

The point D has position vector $6\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$. Find the coordinates of E, the point of intersection of the line OD with the plane ABC. [4]

Find the acute angle between the line ED and the plane ABC. [3]

9 Prove by mathematical induction that, for every positive integer n,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta. \quad [5]$$

Express $\sin^5\theta$ in the form $p\sin 5\theta + q\sin 3\theta + r\sin\theta$, where p, q and r are rational numbers to be determined. [6]

10 The curve C has polar equation $r = 2\sin\theta(1 - \cos\theta)$, for $0 \leqslant \theta \leqslant \pi$. Find $\frac{dr}{d\theta}$ and hence find the polar coordinates of the point of C that is furthest from the pole. [5]

Sketch C. [2]

Find the exact area of the sector from $\theta = 0$ to $\theta = \frac{1}{4}\pi$. [6]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

Let $I_n = \int_0^1 (1 + x^2)^n \, dx$. Show that, for all integers n,

$$(2n + 1)I_n = 2nI_{n-1} + 2^n. \qquad [5]$$

Evaluate I_0 and hence find I_3. [4]

Given that $I_{-1} = \frac{1}{4}\pi$, find I_{-3}. [5]

OR

The vector **e** is an eigenvector of each of the 3×3 matrices **A** and **B**, with corresponding eigenvalues λ and μ respectively. Justifying your answer, state an eigenvalue of $\mathbf{A} + \mathbf{B}$. [3]

The matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 6 & -1 & -6 \\ 1 & 0 & -2 \\ 3 & -1 & -3 \end{pmatrix},$$

has eigenvectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. Find the corresponding eigenvalues. [4]

The matrix **B**, where

$$\mathbf{B} = \begin{pmatrix} 8 & -2 & -8 \\ 2 & 0 & -4 \\ 4 & -2 & -4 \end{pmatrix},$$

also has eigenvectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, for which -2, 2, 4, respectively, are corresponding eigenvalues. The matrix **M** is given by $\mathbf{M} = \mathbf{A} + \mathbf{B} - 5\mathbf{I}$, where **I** is the 3×3 identity matrix. State the eigenvalues of **M**. [1]

Find matrices **R** and **S** and a diagonal matrix **D** such that $\mathbf{M}^5 = \mathbf{RDS}$. [6]

[You should show clearly all the elements of the matrices **R**, **S** and **D**.]