Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 7 2 2 6 3 8 2 7 6 6 *

FURTHER MATHEMATICS **9231/12**

Paper 1 **May/June 2019**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **26** printed pages and **2** blank pages.

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Cambridge Assessment International Education

[Turn over

1 A curve C has equation $\cos y = x$, for $-\pi < x < \pi$.

(i) Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -\cot y \left(\frac{dy}{dx}\right)^2.$$ [4]

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(ii) Hence find the exact value of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{1}{2}, \frac{1}{3}\pi\right)$ on C. [2]

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2 Let $u_n = \dfrac{4\sin\left(n - \frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1) + \cos 1}$.

(i) Using the formulae for $\cos P \pm \cos Q$ given in the List of Formulae MF10, show that

$$u_n = \frac{1}{\cos n} - \frac{1}{\cos(n-1)}.$$ [2]

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(ii) Use the method of differences to find $\sum_{n=1}^{N} u_n$. [2]

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(iii) Explain why the infinite series $u_1 + u_2 + u_3 + \ldots$ does not converge. [1]

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3 The lines l_1 and l_2 have equations $\mathbf{r} = 6\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + \mu(-6\mathbf{j} + \mathbf{k})$ respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the position vectors of P and Q. [8]

4 It is given that, for $n \geqslant 0$,

$$I_n = \int_0^1 x^n e^{x^3} \, dx.$$

(i) Show that $I_2 = \frac{1}{3}(e - 1)$. [2]

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(ii) Show that, for $n \geqslant 3$,

$$3I_n = e - (n-2)I_{n-3}.$$ [3]

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(iii) Hence find the exact value of I_8. [3]

5 A curve C is defined parametrically by

$$x = \frac{2}{e^t + e^{-t}} \quad \text{and} \quad y = \frac{e^t - e^{-t}}{e^t + e^{-t}},$$

for $0 \leqslant t \leqslant 1$. The area of the surface generated when C is rotated through 2π radians about the x-axis is denoted by S.

(i) Show that $S = 4\pi \int_0^1 \frac{e^t - e^{-t}}{(e^t + e^{-t})^2}\, dt$. [5]

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(ii) Using the substitution $u = \mathrm{e}^t + \mathrm{e}^{-t}$, or otherwise, find S in terms of π and e. [3]

6 The equation

$$x^3 - x + 1 = 0$$

has roots α, β, γ.

(i) Use the relation $x = y^{\frac{1}{3}}$ to show that the equation

$$y^3 + 3y^2 + 2y + 1 = 0$$

has roots $\alpha^3, \beta^3, \gamma^3$. Hence write down the value of $\alpha^3 + \beta^3 + \gamma^3$. [3]

Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(ii) Find the value of S_{-3}. [2]

(iii) Show that $S_6 = 5$ and find the value of S_9. [4]

7 Find the particular solution of the differential equation

$$10\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} - x = t + 2,$$

given that when $t = 0$, $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$. [10]

8 **(i)** Prove by mathematical induction that, for $z \neq 1$ and all positive integers n,

$$1 + z + z^2 + \ldots + z^{n-1} = \frac{z^n - 1}{z - 1}.$$ [5]

(ii) By letting $z = \frac{1}{2}(\cos\theta + \mathrm{i}\sin\theta)$, use de Moivre's theorem to deduce that

$$\sum_{m=1}^{\infty} \left(\tfrac{1}{2}\right)^m \sin m\theta = \frac{2\sin\theta}{5 - 4\cos\theta}.$$ [5]

9 It is given that $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{A}$, with corresponding eigenvalue λ.

(i) Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^2$, with corresponding eigenvalue λ^2. [2]

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The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{pmatrix} n & 1 & 3 \\ 0 & 2n & 0 \\ 0 & 0 & 3n \end{pmatrix} \quad \text{and} \quad \mathbf{B} = (\mathbf{A} + n\mathbf{I})^2,$$

where $\mathbf{I}$ is the 3×3 identity matrix and n is a non-zero integer.

(ii) Find, in terms of n, a non-singular matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{B} = \mathbf{PDP}^{-1}$. [8]

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10 The curves C_1 and C_2 have equations

$$y = \frac{ax}{x+5} \quad \text{and} \quad y = \frac{x^2 + (a+10)x + 5a + 26}{x+5}$$

respectively, where a is a constant and $a > 2$.

(i) Find the equations of the asymptotes of C_1. [2]

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(ii) Find the equation of the oblique asymptote of C_2. [2]

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(iii) Show that C_1 and C_2 do not intersect. [2]

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(iv) Find the coordinates of the stationary points of C_2. [3]

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(v) Sketch C_1 and C_2 on a single diagram. [You do not need to calculate the coordinates of any points where C_2 crosses the axes.] [3]

11 Answer only **one** of the following two alternatives.

EITHER

The curve C_1 has polar equation $r^2 = 2\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

$$2\theta \tan \theta = 1$$

and verify that this equation has a root between 0.6 and 0.7. [5]

The curve C_2 has polar equation $r^2 = \theta \sec^2 \theta$, for $0 \leqslant \theta < \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O, and at another point Q.

(ii) Find the exact value of θ at Q. [2]

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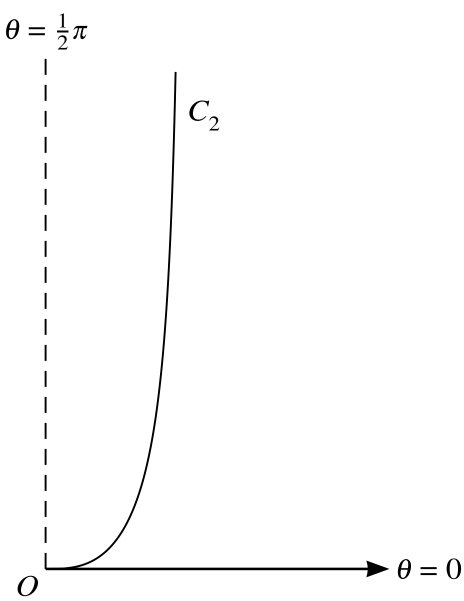
(iii) The diagram below shows the curve C_2. Sketch C_1 on this diagram. [2]



(iv) Find, in exact form, the area of the region OPQ enclosed by C_1 and C_2. [5]

OR

The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -1 & 2 & 3 & 4 \\ 1 & 0 & 1 & -1 \\ 1 & -2 & -3 & a \\ 1 & 2 & 5 & 2 \end{pmatrix}.$$

(i) For $a \neq -4$, the range space of T is denoted by V.

(a) Find the dimension of V and show that

$$\begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 0 \\ -2 \\ 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 4 \\ -1 \\ a \\ 2 \end{pmatrix}$$

form a basis for V. [5]

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(b) Show that if $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ belongs to V then $x + 2y = t$. [4]

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(ii) For $a = -4$, find the general solution of

$$\mathbf{Mx} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}. \qquad [5]$$

Additional Page

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