



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

**9231/04**

Paper 4 Further Probability & Statistics

**For examination from 2020**

SPECIMEN PAPER

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

- 1 (a) State briefly the circumstances in which a parametric test is significant and be used rather than a parametric test. [ ]

The level of pollution in rivers was measured at 10 sites along the headwaters. The results, in table 1, are shown with the help of a scatter plot showing greater biodiversity.

6 3 5 6 6 6 5 8 3 5 6 6 9 9

- (b) Use a Wilcoxon signed rank test to test whether the average price per litre in the river is more than \$1. Use a 5% significance level. [6]

## 3

- 2 Each  $\theta$  is identified by its value in the interval in which it lies and the number of observations falling in each interval is summarised in the following table.

Number of classes	1	2	3	4	5	6	$\geq 7$
Frequency	4	3	2	3	5	1	0

Carry out a goodness of fit test, at the 5% significance level, to test whether  $G(\theta)$  is a satisfactory model for the data. [7]

- 3 Employees work at a particular company between working hours each day, from 9am to 4pm. To try to reduce absence, the company decided to introduce 'flex-time' and allow employees to work their seesn hours each day at any time between 7am and 9pm. For a random sample of 10 employees, the members of hours of absence in the year before and the year after the introduction of flex-time are given in the following table.

Employee	A	B	C	D	E	F	G	H	I	J
Before	2	3	0	4	0	5	8	4	6	0
After	3	2	0	2	3	2	6	5	0	0

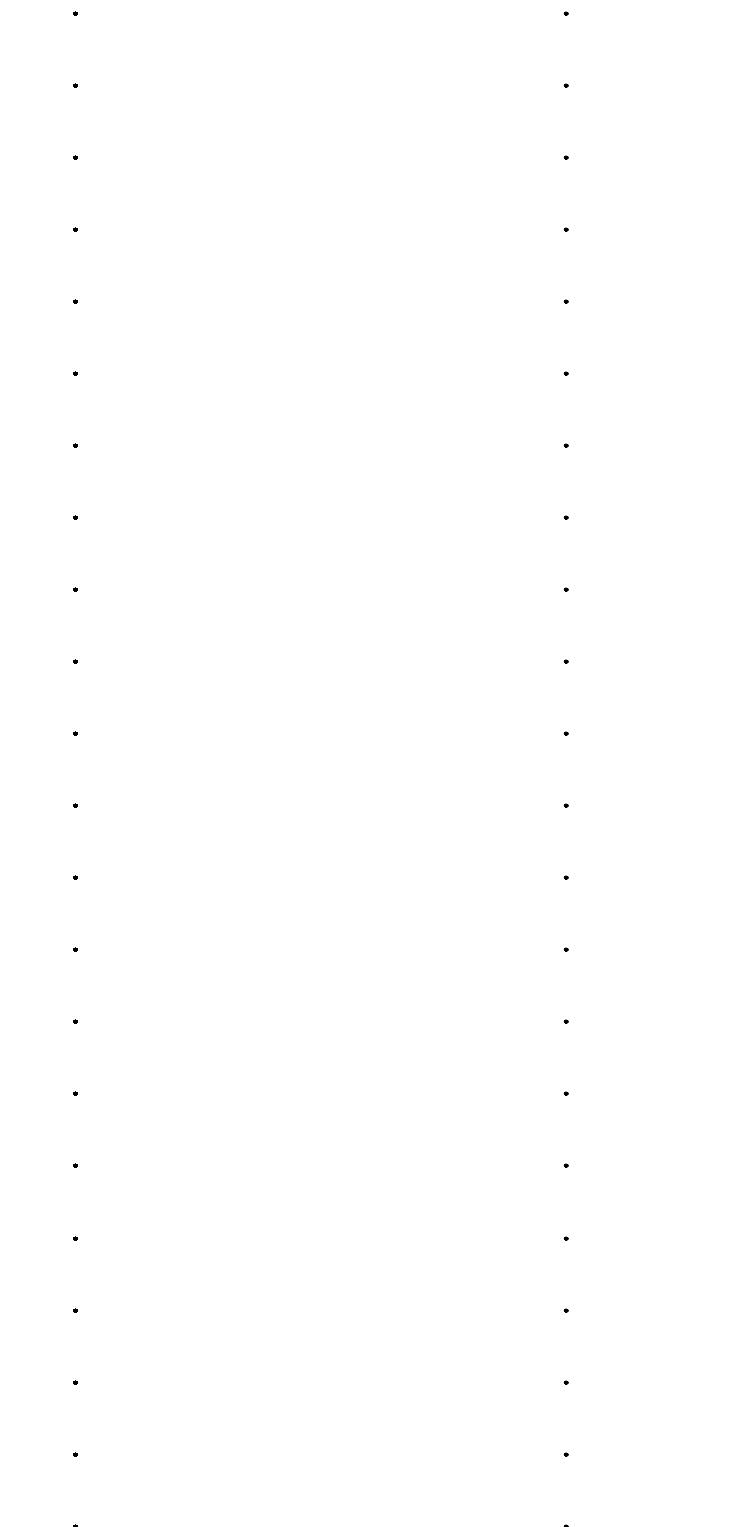
Test, at the 5% significance level, whether the proportion mean birth rates decreased below its limit over time, statistically assuming proportion to be known.



- 4 The mean,  $\bar{x}$ , of a certain type of sea shell was measured at 6 random sites, each 1 metre square, along the coastline in country A. The mean,  $\bar{y}$ , of the same type of sea shell was measured at 6 random sites, each 1 metre square, along the coastline in country B. The results are summarised as follows, where  $\bar{x}$  and  $\bar{y}$  denote the sample means of  $x$  and  $y$  respectively.

$$\bar{x} = 2 \quad \Sigma(x - \bar{x})^2 = 6 \quad \bar{y} = 4 \quad \Sigma(y - \bar{y})^2 = 10$$

Find a 95% confidence interval for the difference between the mean numbers of sea shells, per square metre, along coastlines in country A and in country B. [7]





**5** The ~~ctn~~ is rather a reliable X has problems dealing with its ~~governor~~

$$f(x) = \begin{cases} 0 & x < 0, \\ \frac{6}{5}x & 0 \leq x \leq 1, \\ \frac{6}{5}x^{-4} & x > 1. \end{cases}$$

(a)  $\text{FidP} \quad (X > 1)$ .

[1]

(b) Find the median and the 6<sup>th</sup> X.

[2]

- (c) Given that  $E(X) = 10$ , find the variance of  $X$ . [3]

- (d) **FinE**  $\left(\sqrt{X}\right)$ . [2]

10

- 6 Aish has a bag containing 3 red balls and 3 white balls. She selects a ball at random, notes its color and returns it to the bag. The same process is repeated twice more. The number of red balls selected by Aish is denoted by  $X$ .

(a) Find the probability of rating  $x(t)$  for  $X$ .

[2]

Basan also has a bag containing 3 red balls and 3 white balls. He selects three balls at random, with replacement, from his bag. The number of red balls selected by Basan is denoted by  $Y$ .

(b) Find the probability of rating  $\gamma^{(t)}$  for  $Y$ .

[3]

## 11

The random variable  $Z$  is the total marks scored by Aisha and Asan.

- (c) Find the probability of a rating of 4 stars for  $Z$ , expressing the answer as a fraction. [3]

. . . . .  
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- (d) Use the probability rating of 4 stars for  $Z$  to find  $E(Z)$  and  $\text{Var}(Z)$ . [5]

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**Additional page**

If you need the following lined paper to complete the answer(s) to any question(s), the question in blank(s) must be clearly written.

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