Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

DC (CJ) 206509

1 Kayla is investigating the lengths of the leaves of a certain type of tree found in two forests *X* and *Y*. She chooses a random sample of 40 leaves of this type from forest *X* and records their lengths, *x* cm. She also records the lengths, *y* cm, for a random sample of 60 leaves of this type from forest *Y*. Her results are summarised as follows.

$$\sum x = 242.0 \qquad \sum x^2 = 1587.0 \qquad \sum y = 373.2 \qquad \sum y^2 = 2532.6$$

Find a 90% confidence interval for the difference between the population mean lengths of leaves in forests *X* and *Y*. [7]

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2 Metal rods produced by a certain factory are claimed to have a median breaking strength of 200 tonnes. For a random sample of 9 rods, the breaking strengths, measured in tonnes, were as follows.

210 186 188 208 184 191 215 198 196

A scientist believes that the median breaking strength of metal rods produced by this factory is less than 200 tonnes.

(a) Use a Wilcoxon signed-rank test, at the 5% significance level, to test whether there is evidence to support the scientist's belief. [6]

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(b) Give a reason why a Wilcoxon signed-rank test is preferable to a sign test, when both are valid. [1]

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3 Apples are sold in bags of 5. Based on her previous experience, Freya claims that the probability of any apple weighing more than 100 grams is 0.35, independently of other apples in the bag.

The apples in a random sample of 150 bags are checked and the number, x, in each bag weighing more than 100 grams is recorded. The results are shown in the following table.

x	0	1	2	3	4	5
Frequency	12	39	46	37	12	4

Carry out a goodness of fit test at the 5% significance level and hence comment on Freya's claim. [7]

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4 Members of the Sprints athletics club have been taking part in an intense training scheme, aimed at reducing their times taken to run 400 m. For a random sample of 9 athletes from the club, the times taken, in seconds, before and after the training scheme are given in the following table.

Athlete	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Time before	48.8	48.2	50.3	49.6	49.4	48.9	47.6	50.3	48.4
Time after	47.9	47.8	49.6	49.1	49.6	48.9	47.7	49.1	48.1

The organiser of the training scheme claims that on average an athlete's time will be reduced by at least 0.3 seconds.

Test at the 10% significance level whether the organiser's claim is justified, stating any assumption that you make. [8]

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5 Keira has two unbiased coins. She tosses both coins. The number of heads obtained by Keira is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [1]

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Hassan has three coins, two of which are biased so that the probability of obtaining a head when the coin is tossed is $\frac{1}{3}$. The corresponding probability for the third coin is $\frac{1}{4}$. The number of heads obtained by Hassan when he tosses these three coins is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [3]

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The random variable Z is the total number of heads obtained by Keira and Hassan.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $\mathrm{E}(Z)$. [2]

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(e) Use the probability generating function of Z to find the most probable value of Z. [1]

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6 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{60}(16x - x^2) & 0 \leqslant x \leqslant 6, \\ 1 & x > 6. \end{cases}$$

(a) Find the interquartile range of X. [4]

(b) Find $\mathrm{E}(X^3)$. [4]

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The random variable Y is such that $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [3]

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Additional Page

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