



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}) & 1 \leq x \leq 27, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the cumulative distribution function of X . [3]

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The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

- (b) Find the probability density function of Y . [3]

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(c) Find the exact value of the median of Y . [2]

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- 2 Shane is studying the lengths of the tails of male red kangaroos. He takes a random sample of 14 male red kangaroos and measures the length of the tail, x m, for each kangaroo. He then calculates a 90% confidence interval for the population mean tail length, μ m, of male red kangaroos. He assumes that the tail lengths are normally distributed and finds that $1.11 \leq \mu \leq 1.14$.

Find the values of $\sum x$ and $\sum x^2$ for this sample. [6]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

- 3 A large number of students took two test papers in mathematics. The teacher believes that the marks obtained in Paper 1 will be higher than the marks obtained in Paper 2. She chooses a random sample of 9 students and compares their marks. The marks are shown in the table.

Student	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
Paper 1	46	73	55	64	86	42	66	68	60
Paper 2	41	66	61	63	90	40	58	42	70

- (a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the teacher's belief. [7]

[illegible]

[illegible]

(b) State an assumption that you have made in carrying out the test in part **(a)**. [1]

[illegible]

- 4 An inspector is checking the lengths of metal rods produced by two machines, X and Y . These rods should be of the same length, but the inspector suspects that those made by machine X are shorter, on average, than those made by machine Y . The inspector chooses a random sample of 80 rods made by machine X and a random sample of 60 rods made by machine Y . The lengths of these rods are x cm and y cm respectively. Her results are summarised as follows.

$$\Sigma x = 164.0 \quad \Sigma x^2 = 338.1 \quad \Sigma y = 124.8 \quad \Sigma y^2 = 261.1$$

- (a)** Test at the 10% significance level whether the data supports the inspector's suspicion. [8]

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[illegible]

- (b)** Give a reason why it is not necessary to make any assumption about the distributions of the lengths of the rods. [1]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- 5 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1 + 3t + 4t^2),$$

where k is a constant.

- (a) Show that $E(X) = \frac{11}{8}$. [3]

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The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{1}{3}t^2(1 + 2t).$$

The random variables X and Y are independent and $Z = X + Y$.

- (b) Find the probability generating function of Z , expressing your answer as a polynomial in t . [2]

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- (c) Use your answer to part (b) to find the value of $\text{Var}(Z)$. [3]

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- (d) Write down the most probable value of Z . [1]

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The scientist decides instead to use three grades: grade *A* for 16 or more correct, grade *B* for 10 to 15 correct and grade *C* for fewer than 10 correct. The results are shown in the following table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade <i>A</i>	25	16	19
Grade <i>B</i>	12	27	11
Grade <i>C</i>	16	18	6

With this second set of data, the test statistic is calculated as 10.91.

- (b) Complete the χ^2 -test at the 2.5% significance level for this second set of data. [2]

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- (c) State, with a reason, whether you would prefer to use the result from part (a) or part (b) to investigate whether the ability to remember depends on age. [1]

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Additional page

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