Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 The times taken by members of a large quiz club to complete a challenge have a normal distribution with mean μ minutes. The times, x minutes, are recorded for a random sample of 8 members of the club. The results are summarised as follows, where $\overline{x}$ is the sample mean.

$$\overline{x} = 33.8 \qquad \sum(x - \overline{x})^2 = 94.5$$

Find a 95% confidence interval for μ. [4]

2 A scientist is investigating the size of shells at various beach locations. She selects four beach locations and takes a random sample of shells from each of these beaches. She classifies each shell as large or small. Her results are summarised in the following table.

		Beach location				
		A	*B*	*C*	*D*	Total
Size of shell	Large	68	69	96	81	314
	Small	28	55	64	39	186
	Total	96	124	160	120	500

Test, at the 10% significance level, whether the size of shell is independent of the beach location. [7]

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3 George throws two coins, A and B, at the same time. Coin A is biased so that the probability of obtaining a head is a. Coin B is biased so that the probability of obtaining a head is b, where $b < a$. The probability generating function of X, the number of heads obtained by George, is $G_X(t)$. The coefficients of t and t^2 in $G_X(t)$ are $\frac{5}{12}$ and $\frac{1}{12}$ respectively.

(a) Find the value of a. [2]

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The random variable Y is the sum of two independent observations of X.

(b) Find the probability generating function of Y, giving your answer as a polynomial in t. [3]

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(c) Find Var(Y). [3]

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4 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{3}{8}\left(1+\frac{1}{x^2}\right) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $E\left(\sqrt{X}\right)$. [3]

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The random variable Y is given by $Y = X^2$.

(b) Find the probability density function of Y. [4]

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(c) Find the 40th percentile of Y. [3]

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5 A manager claims that the lengths of the rubber tubes that his company produces have a median of 5.50 cm. The lengths, in cm, of a random sample of 11 tubes produced by this company are as follows.

5.56 5.45 5.47 5.58 5.54 5.52 5.60 5.35 5.59 5.51 5.62

It is required to test at the 10% significance level the null hypothesis that the population median length is 5.50 cm against the alternative hypothesis that the population median length is not equal to 5.50 cm.

Show that both a sign test and a Wilcoxon signed-rank test give the same conclusion and state this conclusion. [9]

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6 A company has two machines, *A* and *B*, which independently fill small bottles with a liquid. The volumes of liquid per bottle, in suitable units, filled by machines *A* and *B* are denoted by x and y respectively. A scientist at the company takes a random sample of 40 bottles filled by machine *A* and a random sample of 50 bottles filled by machine *B*. The results are summarised as follows.

$$\sum x = 1120 \qquad \sum x^2 = 31400 \qquad \sum y = 1370 \qquad \sum y^2 = 37600$$

The population means of the volumes of liquid in the bottles filled by machines *A* and *B* are denoted by μ_A and μ_B.

(a) Test at the 2% significance level whether there is any difference between μ_A and μ_B. [8]

(b) Find the set of values of α for which there would be evidence at the α% significance level that $\mu_A - \mu_B$ is greater than 0.25. [4]

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Additional page

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