



## Cambridge International AS & A Level

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### FURTHER MATHEMATICS

9231/42

Paper 4 Further Probability & Statistics

May/June 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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- 3 The continuous random variable  $X$  has cumulative distribution function  $F$  given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leq x \leq 9, \\ 1 & x > 9. \end{cases}$$

- (a) Find  $E(\sqrt{X})$ . [3]

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- (b) Find  $\text{Var}(\sqrt{X})$ . [2]

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(c) The random variable  $Y$  is given by  $Y^3 = X$ . Find the probability density function of  $Y$ . [3]

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- 4 A scientist is investigating the lengths of the leaves of birch trees in different regions. He takes a random sample of 50 leaves from birch trees in region *A* and a random sample of 60 leaves from birch trees in region *B*. He records their lengths in cm,  $x$  and  $y$ , respectively. His results are summarised as follows.

$$\Sigma x = 282 \quad \Sigma x^2 = 1596 \quad \Sigma y = 328 \quad \Sigma y^2 = 1808$$

The population mean lengths of leaves from birch trees in regions *A* and *B* are  $\mu_A$  cm and  $\mu_B$  cm respectively.

Carry out a test at the 5% significance level to test the null hypothesis  $\mu_A = \mu_B$  against the alternative hypothesis  $\mu_A \neq \mu_B$ . [8]

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5 Georgio has designed two new uniforms  $X$  and  $Y$  for the employees of an airline company. A random sample of 11 employees are each asked to assess each of the two uniforms for practicality and appearance, and to give a total score out of 100. The scores are given in the table.

Employee	$A$	$B$	$C$	$D$	$E$	$F$	$G$	$H$	$I$	$J$	$K$
Uniform $X$	82	74	42	59	60	73	94	98	62	36	50
Uniform $Y$	78	75	63	56	67	82	99	90	72	48	61

(a) Give a reason why a Wilcoxon signed-rank test may be more appropriate than a  $t$ -test for investigating whether there is any evidence of a preference for one of the uniforms. [1]

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(b) Carry out a Wilcoxon matched-pairs signed-rank test at the 10% significance level. [7]

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6 Tanji has a bag containing 4 red balls and 2 blue balls. He selects 3 balls at random from the bag, without replacement. The number of red balls selected by Tanji is denoted by  $X$ .

(a) Find the probability generating function  $G_X(t)$  of  $X$ . [2]

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Tanji also has two coins, each biased so that the probability of obtaining a head when it is thrown is  $\frac{1}{4}$ . He throws the two coins at the same time. The number of heads obtained is denoted by  $Y$ .

(b) Find the probability generating function  $G_Y(t)$  of  $Y$ . [2]

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The random variable  $Z$  is the sum of the number of red balls selected by Tanji and the number of heads obtained.

(c) Find the probability generating function of  $Z$ , expressing your answer as a polynomial. [3]

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**(d)** Use the probability generating function of  $Z$  to find  $E(Z)$  and  $\text{Var}(Z)$ . [5]

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