



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.

- 4 A company has two different machines, X and Y , each of which fills empty cups with coffee. The manager is investigating the volumes of coffee, x and y , measured in appropriate units, in the cups filled by machines X and Y respectively. She chooses a random sample of 50 cups filled by machine X and a random sample of 40 cups filled by machine Y . The volumes are summarised as follows.

$$\Sigma x = 15.2 \quad \Sigma x^2 = 5.1 \quad \Sigma y = 13.4 \quad \Sigma y^2 = 4.8$$

The manager claims that there is no difference between the mean volume of coffee in cups filled by machine X and the mean volume of coffee in cups filled by machine Y .

Test the manager's claim at the 10% significance level.

[9]

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A series of 25 horizontal dotted lines for writing.

5 A large number of children are competing in a throwing competition. The distances, in metres, thrown by a random sample of 8 children are as follows.

19.8 22.1 24.4 21.5 20.8 26.3 23.7 25.0

- (a) Assuming that distances are normally distributed, test, at the 5% significance level, whether the population mean distance thrown is more than 22.0 metres. [7]

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(b) Find a 95% confidence interval for the population mean distance thrown. [3]

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6 A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by X .

(a) Find the probability generating function, $G_X(t)$, of X . [2]

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Rassa also tosses two coins. One coin is biased so that the probability of a head is $\frac{2}{3}$. The other coin is biased so that the probability of a head is p . The probability generating function of Y , the number of heads obtained by Rassa, is $G_Y(t)$. The coefficient of t in $G_Y(t)$ is $\frac{7}{12}$.

(b) Find $G_Y(t)$. [3]

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The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

(c) Find the probability generating function of Z , expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $E(Z)$. [2]

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Additional Page

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