



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2022

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ ms}^{-2}$ .

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.









- 4 One end of a light inextensible string of length  $a$  is attached to a fixed point  $O$ . A particle of mass  $m$  is attached to the other end of the string and is held with the string taut at the point  $A$ . At  $A$  the string makes an angle  $\theta$  with the upward vertical through  $O$ . The particle is projected perpendicular to the string in a downward direction from  $A$  with a speed  $u$ . It moves along a circular path in the vertical plane.

When the string makes an angle  $\alpha$  with the downward vertical through  $O$ , the speed of the particle is  $2u$  and the magnitude of the tension in the string is 10 times its magnitude at  $A$ .

It is given that  $u = \sqrt{\frac{2}{3}ga}$ .

- (a) Find, in terms of  $m$  and  $g$ , the magnitude of the tension in the string at  $A$ . [6]

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(b) Find the value of  $\cos \alpha$ . [2]

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5 A particle  $P$  of mass  $4 \text{ kg}$  is moving in a horizontal straight line. At time  $t \text{ s}$  the velocity of  $P$  is  $v \text{ m s}^{-1}$  and the displacement of  $P$  from a fixed point  $O$  on the line is  $x \text{ m}$ . The only force acting on  $P$  is a resistive force of magnitude  $(4e^{-x} + 12)e^{-x} \text{ N}$ . When  $t = 0$ ,  $x = 0$  and  $v = 4$ .

(a) Show by integration that  $v = \frac{1 + 3e^x}{e^x}$ . [4]

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(b) Find an expression for  $x$  in terms of  $t$ .

[4]

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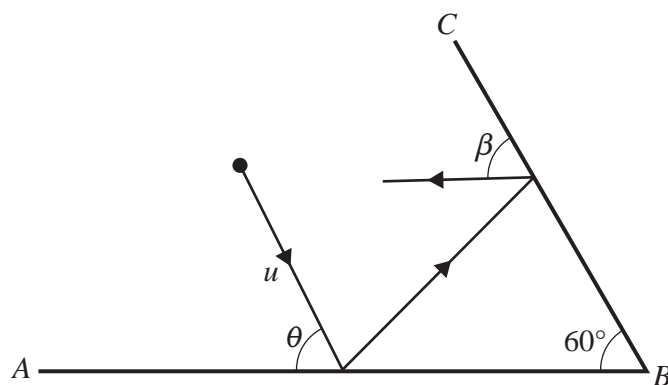
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$AB$  and  $BC$  are two fixed smooth vertical barriers on a smooth horizontal surface, with angle  $ABC = 60^\circ$ . A particle of mass  $m$  is moving with speed  $u$  on the surface. The particle strikes  $AB$  at an angle  $\theta$  with  $AB$ . It then strikes  $BC$  and rebounds at an angle  $\beta$  with  $BC$  (see diagram). The coefficient of restitution between the particle and each barrier is  $e$  and  $\tan \theta = 2$ .

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of  $e$ . [4]

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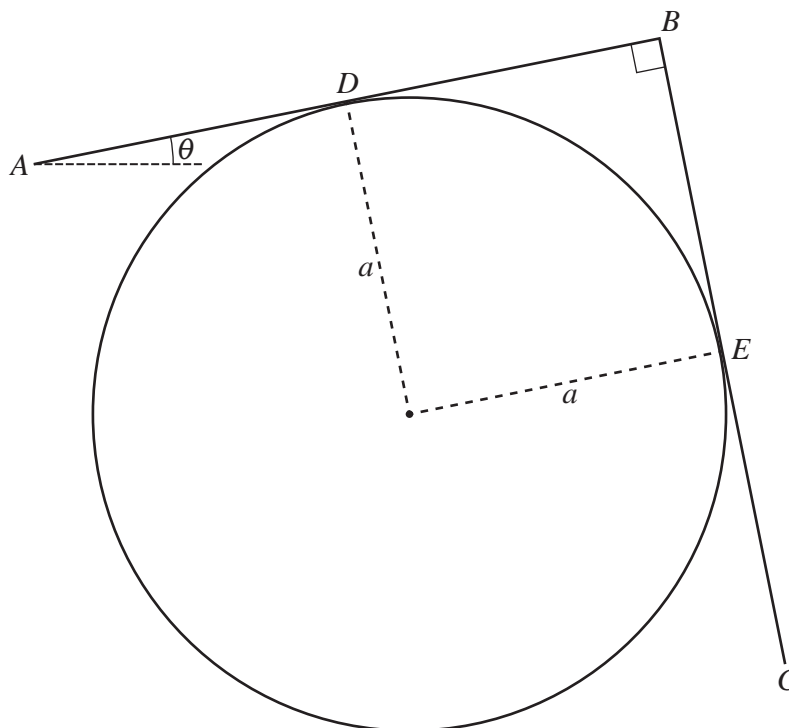
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A uniform cylinder with a rough surface and of radius  $a$  is fixed with its axis horizontal. Two identical uniform rods  $AB$  and  $BC$ , each of weight  $W$  and length  $2a$ , are rigidly joined at  $B$  with  $AB$  perpendicular to  $BC$ . The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with  $AB$  at an angle  $\theta$  to the horizontal.  $D$  and  $E$  are the midpoints of  $AB$  and  $BC$  respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is  $\mu$ .

The normal reaction between  $AB$  and the cylinder is  $R$  and the normal reaction between  $BC$  and the cylinder is  $N$ .

- (a) Find the ratio  $R : N$  in terms of  $\mu$ . [6]

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(b) Given that  $\mu = \frac{1}{3}$ , find the value of  $\tan \theta$ . [3]

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