



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/02

Paper 2 Further Pure Mathematics 2

For examination from 2020

SPECIMEN PAPER

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **18** pages. Blank pages are indicated.

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3

1 Find a general solution to the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 72t^2.$$

[6]

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3 Find a solution to the differential equation

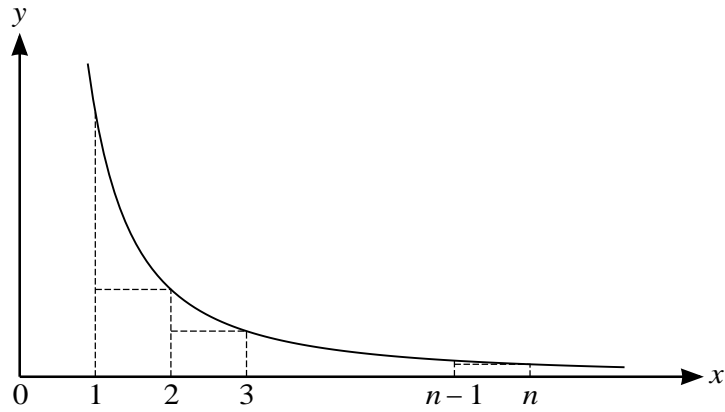
$$x \frac{dy}{dx} + 3y = \frac{\sin x}{x}$$

for which $y = 0$ when $x = \frac{1}{2}\pi$. Give your answer in the form $y = f(x)$.

[8]

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The diagram shows the curve with equation $y = \frac{1}{x^2}$ for $x > 0$ together with a set of $(n-1)$ rectangles drawn in it with height

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{r^2} < \frac{2n-1}{n}. \quad [5]$$

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(b) Use a similar method to find, in terms of n , a formula for $\sum_{r=1}^n \frac{1}{r^2}$. [3]



7 (a) Starting from the definition of \tanh in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$. [3]

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(b) Given that $y = \tanh^{-1}\left(\frac{1-x}{2+x}\right)$, show that $(2x+1) \frac{dy}{dx} + 1 = 0$. [4]

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(c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1-x}{2+x}\right)$ in the form

$$a \ln 3 + bx + cx^2,$$

where a , b and c are constants to be determined.

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8 (a) (i) Find the set of values of a for which the system of equations

$$\begin{aligned} x - 2y - 2z + 7 &= 0 \\ 2x + (a - 9)y - 0z + 1 &= 0 \\ 3x - 6y + 2az + 9 &= 0 \end{aligned}$$

has a unique solution

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(b) The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

(i) Find the eigenvalues of \mathbf{A} .

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(ii) Use the characteristic equation to find A^{-1} .

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