



## Cambridge International AS & A Level

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### FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 Find the Maclaurin's series for  $e^x \tan x$  from first principles up to and including the term in  $x^2$ . [5]

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2 The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of  $\mathbf{A}$  to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where  $p$  and  $q$  are integers to be determined.

[6]

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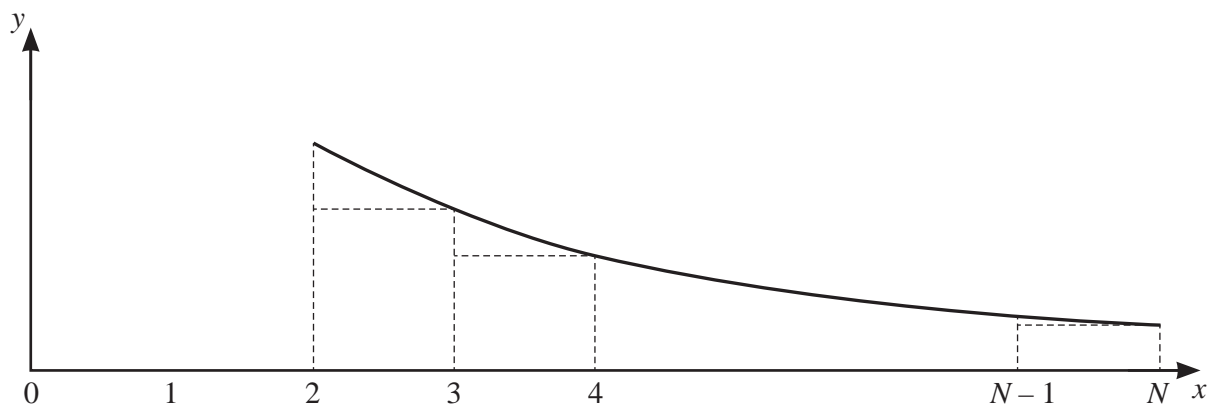
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4



The diagram shows the curve with equation  $y = \frac{\ln x}{x^2}$  for  $x \geq 2$ , together with a set of  $(N - 2)$  rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2 + 3 \ln 2}{4} - \frac{1 + \ln N}{N}. \quad [7]$$

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(b) Use a similar method to find, in terms of  $N$ , a lower bound for  $\sum_{r=1}^N \frac{\ln r}{r^2}$ . [3]

Dotted lines for writing the solution.

5 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\cos x,$$

given that, when  $x = 0$ ,  $y = -4$  and  $\frac{dy}{dx} = 3$ .

[11]

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6 (a) Use de Moivre's theorem to show that

cosec 5θ = ( cosec^5 θ ) / ( 5 cosec^4 θ - 20 cosec^2 θ + 16 ) . [6]

Series of horizontal dotted lines for handwritten solution.



7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$$

is  $x + \sqrt{x^2 - 1}$ .

[4]

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- 8 (a) Starting from the definition of  $\cosh$  in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1. \quad [3]$$

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The curve  $C$  has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when  $C$  is rotated through  $2\pi$  radians about the  $y$ -axis is denoted by  $A$ .

- (b) (i) Show that  $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt$ . [4]

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**Additional Page**

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