Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4597910349

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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1 **(a)** By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]

(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2}\,dx$, giving your answer as a rational fraction in its lowest terms. [2]

2 The variables x and y are related by the differential equation

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

(a) Find the general solution for y in terms of x. [6]

(b) State an approximate solution for large positive values of x. [1]

3 **(a)** Show that the system of equations

$$x - 2y - 4z = 1,$$
$$x - 2y + kz = 1,$$
$$-x + 2y + 2z = 1,$$

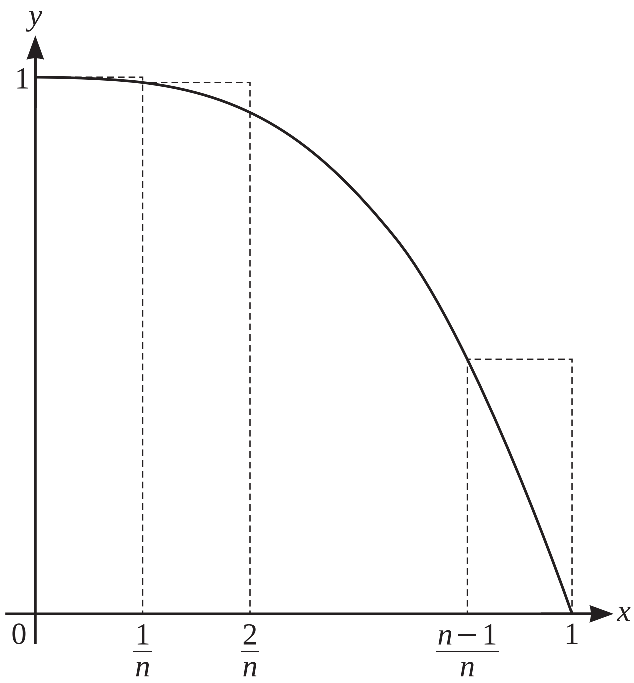
where k is a constant, does not have a unique solution. [2]

(b) Given that $k = -4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

(c) Given instead that $k = -2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

(d) For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

4



The diagram shows the curve with equation $y = 1 - x^3$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3)\,dx \leqslant \frac{3n^2 + 2n - 1}{4n^2}.$$ [4]

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(b) Use a similar method to find, in terms of n, a lower bound for $\int_0^1 (1-x^3)\,dx$. [4]

5 It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t, show that $\dfrac{dy}{dt} = -\dfrac{1}{\sqrt{1-t^2}}$. [4]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer. [5]

6 **(a)** Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [5]

(b) Find the solution of the differential equation

$$\frac{dy}{d\theta} + y\cot\theta = \sin^3\theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$. [6]

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7 The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of **P**. [1]

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(b) Use the characteristic equation of **P** to find $\mathbf{P}^{-1}$. [4]

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The 3×3 matrix $\mathbf{A}$ has distinct eigenvalues b, -1, 1 with corresponding eigenvectors

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \begin{pmatrix}4\\-1\\0\end{pmatrix}, \quad \begin{pmatrix}2\\1\\2\end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}$ in terms of b. [4]

8 **(a)** Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

(b) Starting from the definitions of coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1.$$ [3]

The curve C has equation $y = \ln\coth\left(\frac{1}{2}x\right)$ for $x > 0$.

(c) Show that $\dfrac{dy}{dx} = -\operatorname{cosech} x$. [3]

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(d) It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a. [7]

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Additional Page

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