



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 (a) Show that the system of equations

$$x + 2y + 3z = 1,$$

$$4x + 5y + 6z = 1,$$

$$7x + 8y + 9z = 1,$$

does not have a unique solution.

[2]

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(b) Show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- 2 Use the substitution $z = x + y$ to find the solution of the differential equation

$$\frac{dy}{dx} = \frac{1 + 3x + 3y}{3x + 3y - 1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a \ln(x+y) + b(x-y) + c = 0$, where a , b and c are constants to be determined. [7]

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3 (a) By considering the binomial expansion of $(z+z^{-1})^4$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that $\cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)$. [5]

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(b) Use the substitution $x = \sin \theta$ to find the exact value of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$. [3]

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4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n dx$.

(a) By considering $\frac{d}{dx}(x(1+x^5)^n)$, or otherwise, show that

$$(5n + 1)I_n = 2^n + 5nI_{n-1}. \tag{5}$$

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(b) Find the exact value of I_3 . [4]

A series of 25 horizontal dotted lines for writing.

7 (a) Use the substitution $u = x^2 - 1$ to find $\int \frac{x}{\sqrt{x^2 - 1}} dx$. [3]

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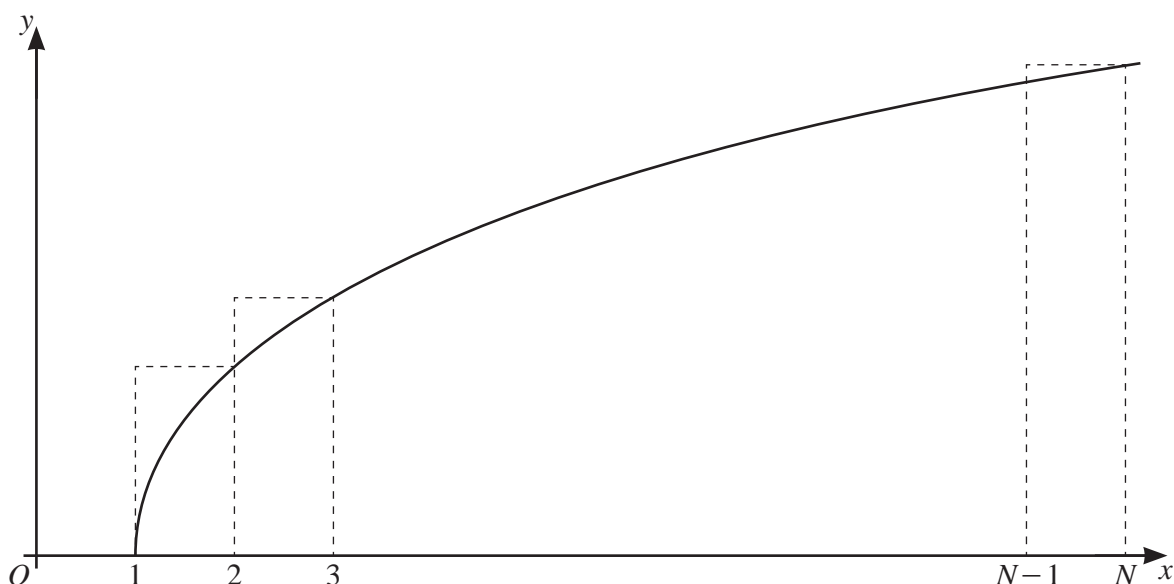
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The diagram shows the curve with equation $y = \cosh^{-1}x$ together with a set of $(N - 1)$ rectangles of unit width.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=2}^N \ln(r + \sqrt{r^2 - 1}) > N \ln(N + \sqrt{N^2 - 1}) - \sqrt{N^2 - 1}. \quad [5]$$

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8 (a) Starting from the definitions of sech and tanh in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t. \quad [3]$$

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The curve C has parametric equations

$$x = \frac{1}{2} \tanh^2 t + \ln \operatorname{sech} t, \quad y = 1 + \tanh^4 t, \quad \text{for } t > 0.$$

(b) Show that $\frac{dy}{dx} = -4 \operatorname{sech}^2 t.$ [5]

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- (c) Find the coordinates of the point on C with $\frac{d^2y}{dx^2} = -\frac{9}{2}$, giving your answer in the form $(a + \ln b, c)$ where a, b and c are rational numbers. [6]

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