



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.



- 2 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2 \sinh^2 x + 1. \quad [3]$$

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- (b) Find the set of values of  $k$  for which  $\cosh 2x = k \sinh x$  has two distinct real roots. [5]

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3 The variables  $t$  and  $x$  are related by the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for  $x$  in terms of  $t$ . [6]

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(b) Deduce an approximate value of  $\frac{d^2x}{dt^2}$  for large positive values of  $t$ . [2]

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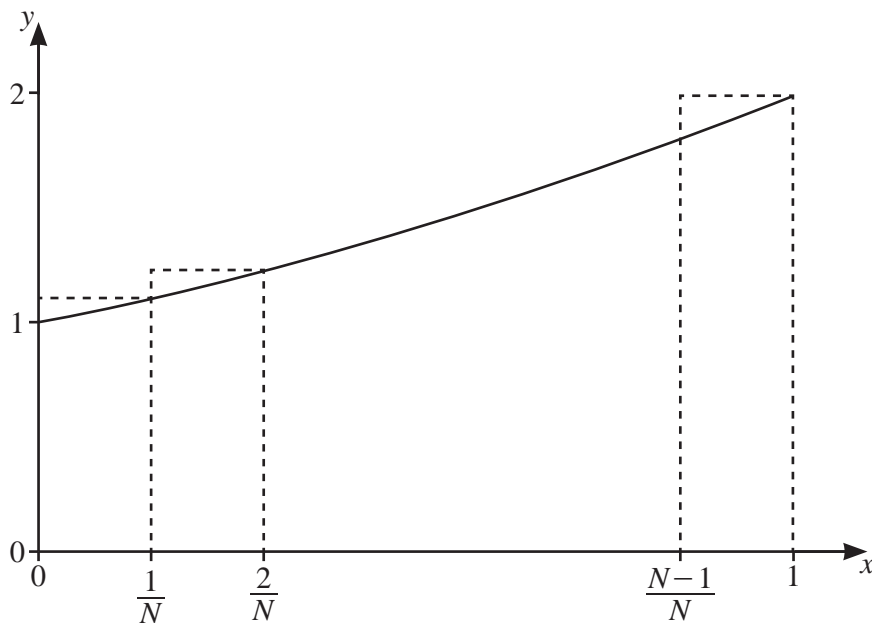
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- 4 The diagram shows the curve with equation  $y = 2^x$  for  $0 \leq x \leq 1$ , together with a set of  $N$  rectangles each of width  $\frac{1}{N}$ .



- (a) By considering the sum of the areas of these rectangles, show that  $\int_0^1 2^x dx < U_N$ , where

$$U_N = \frac{2^{\frac{1}{N}}}{N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

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- (b) Use a similar method to find, in terms of  $N$ , a lower bound  $L_N$  for  $\int_0^1 2^x dx$ . [4]

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- (c) Find the least value of  $N$  such that  $U_N - L_N < 10^{-4}$ . [2]

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**5** The variables  $x$  and  $y$  are such that  $y = 0$  when  $x = 0$  and

$$(x+1)y + (x+y+1)^3 = 1.$$

**(a)** Show that  $\frac{dy}{dx} = -\frac{3}{4}$  when  $x = 0$ . [3]

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**(b)** Find the Maclaurin's series for  $y$  up to and including the term in  $x^2$ . [7]

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(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form  $\operatorname{cosec} q\pi$ , where  $q$  is rational.

[5]

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- 8 (a) Find the value of  $a$  for which the system of equations

$$\begin{aligned} 3x + ay &= 0, \\ 5x - y &= 0, \\ x + 3y + 2z &= 0, \end{aligned}$$

does not have a unique solution.

[2]

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The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

- (b) Find a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{A}^2 = \mathbf{PDP}^{-1}$ .

[7]

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(c) Use the characteristic equation of  $\mathbf{A}$  to show that

$$(\mathbf{A} + 6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where  $b$  is an integer to be determined.

[4]

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**Additional page**

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