Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9117044394

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The cubic equation $x^3+bx^2+cx+d=0$, where b, c and d are constants, has roots α, β, γ. It is given that $\alpha\beta\gamma=-1$.

(a) State the value of d. [1]

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(b) Find a cubic equation, with coefficients in terms of b and c, whose roots are $\alpha+1$, $\beta+1$, $\gamma+1$. [3]

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(c) Given also that $\gamma+1=-\alpha-1$, deduce that $(c-2b+3)(b-3)=b-c$. [4]

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2 Prove by mathematical induction that $7^{2n} - 1$ is divisible by 12 for every positive integer n. [5]

3 **(a)** By simplifying $\left(x^n - \sqrt{x^{2n}+1}\right)\left(x^n + \sqrt{x^{2n}+1}\right)$, show that $\dfrac{1}{x^n - \sqrt{x^{2n}+1}} = -x^n - \sqrt{x^{2n}+1}$. [1]

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Let $u_n = x^{n+1} + \sqrt{x^{2n+2}+1} + \dfrac{1}{x^n - \sqrt{x^{2n}+1}}$.

(b) Use the method of differences to find $\sum_{n=1}^{N} u_n$ in terms of N and x. [3]

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(c) Deduce the set of values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

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4 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by **A**. [1]

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(b) Give full details of the geometrical transformation in the x-y plane represented by **B**. [2]

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The triangle DEF in the x-y plane is transformed by **AB** onto triangle PQR.

(c) Show that the triangles DEF and PQR have the same area. [3]

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(d) Find the matrix which transforms triangle *PQR* onto triangle *DEF*. [2]

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(e) Find the equations of the invariant lines, through the origin, of the transformation in the *x*-*y* plane represented by **AB**. [5]

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5 The curve C has polar equation $r = \ln(1+\pi-\theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Using the substitution $u = 1+\pi-\theta$, or otherwise, show that the area of the region enclosed by C and the initial line is

$$\tfrac{1}{2}(1+\pi)\ln(1+\pi)\big(\ln(1+\pi)-2\big)+\pi.$$ [6]

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(c) Show that, at the point of C furthest from the initial line,

$$(1+\pi-\theta)\ln(1+\pi-\theta)-\tan\theta=0$$

and verify that this equation has a root between 1.2 and 1.3. [5]

6 Let a be a positive constant.

(a) The curve C_1 has equation $y = \frac{x-a}{x-2a}$. [2]

Sketch C_1.

The curve C_2 has equation $y = \left(\frac{x-a}{x-2a}\right)^2$. The curve C_3 has equation $y = \left|\frac{x-a}{x-2a}\right|$.

(b) **(i)** Find the coordinates of any stationary points of C_2. [3]

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(ii) Find also the coordinates of any points of intersection of C_2 and C_3. [3]

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(c) Sketch C_2 and C_3 on a single diagram, clearly identifying each curve. Hence find the set of values of x for which $\left(\frac{x-a}{x-2a}\right)^2 \leqslant \left|\frac{x-a}{x-2a}\right|$. [5]

7 The points A, B, C have position vectors

$$-2\mathbf{i}+2\mathbf{j}-\mathbf{k}, \quad -2\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{j}+\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

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(b) Find the acute angle between the planes OBC and ABC. [4]

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The point D has position vector $t\mathbf{i} - \mathbf{j}$.

(c) Given that the shortest distance between the lines AB and CD is $\sqrt{10}$, find the value of t. [6]

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Additional Page

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