



## Cambridge International AS & A Level

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### FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Let  $a$  be a positive constant.

(a) Use the method of differences to find  $\sum_{r=1}^n \frac{1}{(ar+1)(ar+a+1)}$  in terms of  $n$  and  $a$ . [4]

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(b) Find the value of  $a$  for which  $\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}$ . [3]

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- (b) Find the perpendicular distance from  $O$  to the plane  $ABC$ . [2]

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- (c) The point  $D$  has position vector  $2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ .  
Find the coordinates of the point of intersection of the line  $OD$  with the plane  $ABC$ . [3]

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**(b)** Show that  $u_{n+1} < u_n$  for  $n \geq 1$ . [3]

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4 The cubic equation  $2x^3 + 5x^2 - 6 = 0$  has roots  $\alpha, \beta, \gamma$ .

(a) Find a cubic equation whose roots are  $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$ . [3]

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(b) Find the value of  $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$ . [3]

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(c) Find also the value of  $\frac{1}{\alpha^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9}$ . [2]

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5 The curve  $C$  has equation  $y = \frac{2x^2 - x - 1}{x^2 + x + 1}$ .

(a) Show that  $C$  has no vertical asymptotes and state the equation of the horizontal asymptote of  $C$ . [3]

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(b) Find the coordinates of the stationary points on  $C$ . [4]

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- (c) Sketch  $C$ , stating the coordinates of the intersections with the axes. [3]

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- (d) Sketch the curve with equation  $y = \left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right|$  and state the set of values of  $k$  for which  $\left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right| = k$  has 4 distinct real solutions. [2]

6 The curve  $C$  has polar equation  $r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right)$ , where  $0 \leq \theta \leq 2$ .

(a) Sketch  $C$  and state, in exact form, the greatest distance of a point on  $C$  from the pole. [3]

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(b) Find the exact value of the area of the region bounded by  $C$  and the half-line  $\theta = 2$ . [5]

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Now consider the part of  $C$  where  $0 \leq \theta \leq \frac{1}{2}\pi$ .

(c) Show that, at the point furthest from the half-line  $\theta = \frac{1}{2}\pi$ ,

$$(\theta^2 + 4)\tan^{-1}\left(\frac{1}{2}\theta\right)\sin\theta - \cos\theta = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

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7 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix}$ .

(a) Find the set of values of  $k$  for which  $\mathbf{A}$  is non-singular. [3]

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(b) Given that  $\mathbf{A}$  is non-singular, find, in terms of  $k$ , the entries in the top row of  $\mathbf{A}^{-1}$ . [4]

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- (c) Given that  $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ , give an example of a matrix  $\mathbf{C}$  such that  $\mathbf{BAC} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ . [4]

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