



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

BLANK PAGE

1 Prove by mathematical induction that $2^{4n} + 31^n - 2$ is divisible by 15 for all positive integers n . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that

$$\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)} = \frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [1]

.....

.....

- (b) Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 The matrix **M** represents the sequence of two transformations in the x - y plane given by a rotation of 60° anticlockwise about the origin followed by a one-way stretch in the x -direction, scale factor d ($d \neq 0$).

(a) Find **M** in terms of d . [4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

(b) The unit square in the x - y plane is transformed by **M** onto a parallelogram of area $\frac{1}{2}d^2$ units².
Show that $d = 2$. [2]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

The matrix \mathbf{N} is such that $\mathbf{MN} = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

(c) Find \mathbf{N} . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by \mathbf{MN} . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 The curve C has polar equation $r = a \cot\left(\frac{1}{3}\pi - \theta\right)$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{6}\pi$.

It is given that the greatest distance of a point on C from the pole is $2\sqrt{3}$.

(a) Sketch C and show that $a = 2$. [3]

.....

(b) Find the exact value of the area of the region bounded by C , the initial line and the half-line $\theta = \frac{1}{6}\pi$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....
.....
.....
.....
.....
.....

(c) Show that C has Cartesian equation $2(x + y\sqrt{3}) = (x\sqrt{3} - y)\sqrt{x^2 + y^2}$. [3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

6 Let t be a positive constant.

The line l_1 passes through the point with position vector $t\mathbf{i} + \mathbf{j}$ and is parallel to the vector $-2\mathbf{i} - \mathbf{j}$. The line l_2 passes through the point with position vector $\mathbf{j} + t\mathbf{k}$ and is parallel to the vector $-2\mathbf{j} + \mathbf{k}$.

It is given that the shortest distance between the lines l_1 and l_2 is $\sqrt{21}$.

(a) Find the value of t . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The plane Π_1 contains l_1 and is parallel to l_2 .

(b) Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [1]

.....

.....

.....

.....

.....

13

The plane Π_2 has Cartesian equation $5x - 6y + 7z = 0$.

- (c) Find the acute angle between l_2 and Π_2 . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) Find the acute angle between Π_1 and Π_2 . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The curve C has equation $y = \frac{x^2 + x + 9}{x + 1}$.

(a) Find the equations of the asymptotes of C . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the coordinates of the stationary points on C . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Sketch C , stating the coordinates of any intersections with the axes. [3]

-
- (d) Sketch the curve with equation $y = \left| \frac{x^2 + x + 9}{x + 1} \right|$ and find the set of values of x for which $2|x^2 + x + 9| > 13|x + 1|$. [5]

