Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

7639958140

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **May/June 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (LK) 187468/2

1 Let a be a positive constant.

(a) Sketch the curve with equation $y = \frac{ax}{x+7}$. [2]

(b) Sketch the curve with equation $y = \left|\dfrac{ax}{x+7}\right|$ and find the set of values of x for which $\left|\dfrac{ax}{x+7}\right| > \dfrac{a}{2}$.

[4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

2 The cubic equation $6x^3 + px^2 - 3x - 5 = 0$, where p is a constant, has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^2, \beta^2, \gamma^2$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) It is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$.

(i) Find the value of p. [3]

....

....

....

....

....

....

....

....

(ii) Find the value of $\alpha^3+\beta^3+\gamma^3$. [2]

3 The curve C has equation $y = \dfrac{x^2}{2x+1}$.

(a) Find the equations of the asymptotes of C. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Find the coordinates of the stationary points on C. [3]

....

....

....

....

....

....

....

....

....

....

....

....

(c) Sketch C. [3]

4 **(a)** By first expressing $\frac{1}{r^2-1}$ in partial fractions, show that

$$\sum_{r=2}^{n} \frac{1}{r^2-1} = \frac{3}{4} - \frac{an+b}{2n(n+1)},$$

where a and b are integers to be found. [5]

(b) Deduce the value of $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$. [1]

....

....

....

....

....

....

....

....

....

....

(c) Find the limit, as $n \to \infty$, of $\sum_{r=n+1}^{2n} \frac{n}{r^2-1}$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

5 The lines l_1 and l_2 have equations $\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{j} + 6\mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π contains l_1 and is parallel to the vector $\mathbf{i}+\mathbf{k}$.

(b) Find the equation of Π, giving your answer in the form $ax+by+cz=d$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Find the acute angle between l_2 and Π. [3]

....

....

....

....

....

....

....

....

....

....

....

6 Let $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) The transformation in the x-y plane represented by $\mathbf{A}^{-1}$ transforms a triangle of area $30\,\text{cm}^2$ into a triangle of area $d\,\text{cm}^2$.

Find the value of d. [3]

....

....

....

....

....

(b) Prove by mathematical induction that, for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}.$$ [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) The line $y = 2x$ is invariant under the transformation in the x-y plane represented by $\mathbf{A}^n\mathbf{B}$, where $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}$.

Find the value of n. [5]

7 The curve C_1 has polar equation $r = \theta\cos\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

$$2\theta\tan\theta - 1 = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

The curve C_2 has polar equation $r = \theta\sin\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O, and at another point Q.

(b) Find the polar coordinates of Q, giving your answers in exact form. [2]

(c) Sketch C_1 and C_2 on the same diagram. [3]

(d) Find, in terms of π, the area of the region bounded by the arc OQ of C_1 and the arc OQ of C_2. [7]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.