AQA

A-LEVEL
MATHEMATICS
7357/2

Paper 2

Mark scheme
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186A73572/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

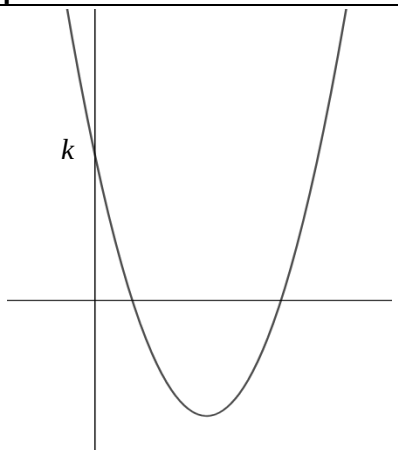
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Ticks correct option	AO2.5	B1	$x = 2 \Rightarrow x^2 = 4$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	84
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	AO1.1b	B1	68
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4a	Sketches graph recalling correct u shape	AO1.2	B1	
	Deduces correct relative positions of intersections with axes and with k labelled	AO2.2a	B1	
b	Shows evidence of discriminant being used or completing the square to find vertex	AO1.1a	M1	For distinct roots $b^2 - 4ac > 0$ $(-6)^2 - 4 \times 1 \times k > 0$ $36 - 4k > 0$ $k < 9$
	Obtains $k < 9$ Condone $k \leq 9$	AO1.1b	A1	
	Explains that positive roots and the u shape of the graph (OE) mean the graph must cross the y-axis above 0 or $k > 0$.	AO2.4	E1	k is the y-intercept and for positive roots the intercept must be positive
	States correct range of values for k	AO2.2a	R1	$0 < k < 9$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Begins checking for factors to start proof by exhaustion or makes a statement about numbers which don't need to be checked	AO3.1a	M1	$\sqrt{23} \approx 4.8$ so only need to check 2 and 3 23 is odd so no need to check 2. 23 is not a multiple of 3 $\therefore$ 23 is prime.
	Completes rigorous argument, for example: Only need to check primes less than $\sqrt{23}$ 23 is not divisible by 2 or 3 therefore 23 is prime or checks all possible factors or checks more factors than necessary, but argument must be complete.	AO2.1	R1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Selects appropriate technique to differentiate	AO3.1a	M1	$2\left(x+y-2\right)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)=e^{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow x+y-2=0$ $\Rightarrow 0=e^{y}-1$ $y=0$ $x=2$
	Differentiates term involving e^{y} correctly	AO1.1b	B1	
	Differentiates fully correctly	AO1.1b	A1	
	Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=0$	AO1.1a	M1	
	Eliminates x or y from the equation of the curve	AO1.1a	M1	
	Obtains correct y CAO	AO1.1b	A1	
	Obtains correct x CAO	AO1.1b	A1	
	Total		**7**	

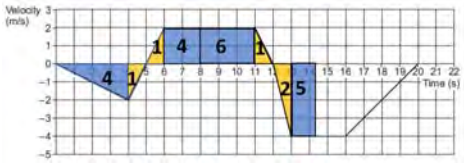
Q	Marking Instructions	AO	Marks	Typical Solution
7	Integrates using integration by parts	AO3.1a	M1	$y = \int (x-1)\mathrm{e}^x \mathrm{d}x$
	Applies integration by parts formula correctly to either of $(x-1)\mathrm{e}^x$ or $x\mathrm{e}^x$	AO1.1a	M1	$u = x-1 \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = 1$ $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \qquad v = \mathrm{e}^x$
	Obtains fully correct integral, condone missing constant.	AO1.1b	A1	$y = (x-1)\mathrm{e}^x - \int \mathrm{e}^x \mathrm{d}x$ $y = (x-1)\mathrm{e}^x - \mathrm{e}^x + c$
	Explains clearly why the minimum y value is e with reference to the range of the function OE	AO2.4	E1	Range $\geq \mathrm{e} \Rightarrow$ at min $y = \mathrm{e}$
	Uses $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to find x coordinate of minimum	AO1.1a	M1	Min point when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \therefore x = 1$ So curve passes through $(1, \mathrm{e})$
	Deduces that the curve passes through the point $(1, \mathrm{e})$	AO2.2a	A1	$\mathrm{e} = (1-1)\mathrm{e}^1 - \mathrm{e}^1 + c$ $c = 2\mathrm{e}$
	Uses their minimum point to find their c	AO1.1a	M1	$\therefore \mathrm{f}(x) = (x-2)\mathrm{e}^x + 2\mathrm{e}$
	States the correct equation in any correct form Condone y instead of $\mathrm{f}(x)$ CAO	AO1.1b	A1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Compares with $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$	AO3.1a	M1	$\sqrt{3}\sin x-3\cos x\equiv R\sin(x-\alpha)$ $\equiv R\sin x\cos\alpha-R\cos x\sin\alpha$
	Obtains two correct equations for R and α for example $R\cos\alpha=\sqrt{3}$ $R\sin\alpha=3$ Must be explicitly seen	AO3.1a	M1	$R\cos\alpha=\sqrt{3}$ $R\sin\alpha=3$
	Obtains correct R Condone AWRT 3.46 PI by description of stretch	AO1.1b	B1	$R=\sqrt{12}=2\sqrt{3}$
	Obtains correct α in radians or degrees PI by description of translation	AO1.1b	B1	$\tan\alpha=\sqrt{3}$ $\alpha=\frac{\pi}{3}$
	Interprets their values of R and α to form an equation of the form $y=R\sin(x\pm\alpha)+4$ or $y=R\cos(x\pm\alpha)+4$	AO3.2a	B1F	$y=2\sqrt{3}\sin(x-\frac{\pi}{3})+4$ Translation $\begin{pmatrix}\frac{\pi}{3}\\0\end{pmatrix}$ Stretch in the y-direction scale factor $2\sqrt{3}$
	Interprets 'their' equation to identify a transformation	AO3.2a	E1F	
	Identifies all required transformations in a correct order CAO	AO3.2a	A1	Translation $\begin{pmatrix}0\\4\end{pmatrix}$
(b)(i)	Deduces the least value occurs when their $\sin(x-\frac{\pi}{3})=1$ Using 'their' values of R and α PI by sight of $\frac{1}{2\sqrt{3}+4}$	AO2.2a	M1	$\frac{1}{\sqrt{3}\sin x-3\cos x+4}=\frac{1}{2\sqrt{3}\sin(x-\frac{\pi}{3})+4}$ Least value when $\sin(x-\frac{\pi}{3})=1$ ∴ least value is given by
	Completes rigorous argument to obtain $\frac{1}{2\sqrt{3}+4}$ and then the given answer	AO2.1	R1	$\frac{1}{2\sqrt{3}+4}=\frac{2-\sqrt{3}}{2}$
(b)(ii)	Deduces the greatest value Using 'their' values of R and α ACF $\frac{1}{-2\sqrt{3}+4}=\frac{2+\sqrt{3}}{2}$	AO2.2a	B1F	Greatest value = $\frac{2+\sqrt{3}}{2}$
	Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Translates proportionality into a differential equation involving $\frac{dx}{dt}$, *t*, *x* and a constant of proportionality.	AO3.3	B1	$\frac{dx}{dt} = \frac{k(8-t)}{x}$
	Substitutes $t = 2$, $x = 336$, $\frac{dx}{dt} = 72$ to find *k*	AO1.1a	M1	$72 = \frac{k(8-2)}{336}$ $k = 4032$ $\frac{dx}{dt} = \frac{4032(8-t)}{x}$
	Obtains correct value of *k* shows given result **AG**	AO2.1	R1	$x\frac{dx}{dt} = 4032(8 - t)$
(b)	Integrates one side correctly	AO1.1a	M1	$\int x\mathrm{d}x = \int 4032(8-t)\mathrm{d}t$ $\frac{1}{2}x^2 = 4032\left(8t - \frac{t^2}{2}\right) + c$
	Integrates both sides correctly condone missing *c*	AO1.1b	A1	$\frac{1}{2}\times 336^2 = 4032\left(8\times 2 - \frac{2^2}{2}\right) + c$ $c = 0$ $x^2 = 64512t - 4032t^2$
	Uses conditions to show *c*=0 and correctly obtains given result.**AG**	AO2.1	R1	$\therefore x^2 = 4032t(16-t)$
(c)(i)	Translates rate of growth into $\frac{\mathrm{d}x}{\mathrm{d}t} = 24$ and uses in model for rate of sales	AO3.3	M1	$24x = 4032(8-t)$ $x = 168(8-t)$ $(168(8-t))^2 = 4032t(16-t)$
	Eliminates *x* to form quadratic equation or inequality in *t*	AO3.1a	M1	$t^2-16t+56 = 0$
	Obtains correct equation or inequality in *t* Need not be simplified	AO1.1b	A1	$t = 5.171..$ or $10.828…$
	Obtains *t*=5.17	AO1.1b	A1	5 hours 10 minutes
	Converts their *t* into hours and minutes	AO3.4	A1	Earliest time 14:40
	Interprets the closing time as 14:40 09:30+their converted time	AO3.2a	R1F	
(c)(ii)	Explains in context that when the stall opens there will be zero sales	AO3.5a	E1	When the stall opens there are zero sales
	Explains that when *x*=0 the model is undefined	AO3.5a	E1	When x=0 $\frac{dx}{dt}$ is undefined as the denominator is zero
		Total	**14**	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Circles correct answer	AO1.1b	B1	0.0071 m s^{-2}
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	AO1.1b	B1	1.6 m
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Finds the steepest gradient. Ignore units. Do not allow -4.	AO1.1b	B1	4 m s^{-2}
12(b)	Shows evidence of determining areas above and below the time axis for values of t between 7 and 9 inclusive Evidence may include values or indication on a diagram	AO3.1b	M1	
	Deduces t_1 value correctly.	AO2.2a	A1	$t_1 = 8$
	Shows evidence of determining areas above and below the time axis for values of t between 13 and 15 inclusive Evidence may include values or indication on a diagram	AO1.1a	M1	$t_2 = 14.25$
	Deduces t_2 value correctly	AO2.2a	A1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Uses model for maximum friction $= \mu mg$	AO3.3	B1	$F_{\max} = \mu mg$ $= 0.85 \times 20 \times 9.8$ $= 166.6\text{N}$
	Makes an appropriate comparison	AO1.1a	M1	
	Explains clearly why crate remains stationary	AO2.4	E1	$150 < 166.6$ $\therefore$ crate does not move
13(b)	Forms an equation by resolving vertically Condone one of sign error or cos error	AO3.1b	M1	$20g = R + 150\sin 15°$ $R = 157.177\text{N}$
	Obtains correct reaction force	AO1.1b	A1	$F_{\max} = \mu \times 157.177$ $= 133.6\text{N}$
	Uses maximum friction $= \mu R$ With 'their' reaction force Must identify maximum or limiting friction	AO1.1b	B1F	$150\cos 15° = 145\text{N}$ $145 > 133.6$ $\therefore$ crate begins to move
	Compares $150\cos 15°$ with 'their' maximum friction	AO1.1a	M1	
	Explains, using their values, why the crate begins to move.	AO2.4	E1F	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Obtains correct vector	AO1.1b	B1	$\begin{pmatrix}-4\\-3\\6\end{pmatrix}$
14(b)	Obtains one other edge as vector	AO1.1a	M1	$\overrightarrow{BC} = \begin{pmatrix}1\\5\\-1\end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix}1\\5\\-1\end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix}-4\\-3\\6\end{pmatrix}$ $AB = \sqrt{(-4)^2 + (-3)^2 + 6^2}$ $= \sqrt{61}$ $AD = \sqrt{1^2 + 5^2 + (-1)^2}$ $= 3\sqrt{3}$ $\overrightarrow{AB} = \overrightarrow{DC}$ ABCD must be a parallelogram $AB \neq AD$ ABCD is not a rhombus
	Obtains $\overrightarrow{DC}$ correctly Or Obtains correctly both $\overrightarrow{BC}$ and $\overrightarrow{AD}$ Or Obtains correctly both $\overrightarrow{CB}$ and $\overrightarrow{DA}$	AO1.1b	A1	
	Obtains length of one edge (or its square)	AO1.1a	M1	
	Obtains two correct lengths of different edges	AO1.1b	A1	
	Completes rigorous argument to show ABCD is a parallelogram and not a rhombus	AO2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Integrates 0.138 t^2 twice	AO3.4	M1	$v = \int 0.138t^2 \mathrm{d}t$
	Finds the correct expression for displacement condone no consideration of *c*	AO1.1b	A1	$= 0.046t^3 + c$ $t = 0, v = 0 \Rightarrow c = 0$
	Demonstrates at least one constant of integration is zero	AO1.1b	B1	$s = \int 0.046t^3 \mathrm{d}t$ $= 0.0115t^4 + k$ $t = 0, s = 0 \Rightarrow k = 0$
	Finds the correct time for minibus A	AO1.1b	A1	$0.0115t^4 = 100$ $t = 9.657$
15(b)	Integrates 0.024 t^3 twice	AO1.1a	M1	$v = \int 0.024t^3 \mathrm{d}t$
	Finds the correct expression for displacement condone no consideration of *c*	AO1.1b	A1	$= 0.006t^4 + c$ $t = 0, v = 0 \Rightarrow c = 0$
	Finds correct time for minibus B	AO1.1b	A1	$s = \int 0.006t^4 \mathrm{d}t$ $= 0.0012t^5 + k$
	States correct choice consistent with 'their' answers Must have integrated twice in both parts	AO3.2a	E1F	$t = 0, s = 0 \Rightarrow k = 0$ $0.0012\, t^5 = 100$ $t = 9.642$ 9.642 < 9.657 company chooses minibus B
15(c)	Explains how reaction times of each driver could change the outcome	AO3.5b	E1	If Driver B's reaction time is greater than Driver A's then A could travel 100 metres faster than B
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Uses $v = u + at$ with $v = 0$ for the vertical motion Condone cos or sign error	AO3.4	M1	0 = u sin 35 – 9.81 x 1.5 $u = 25.7$ m s^{-1}
	Obtains correct equation	AO1.1b	A1	
	Obtains correct u to 3 significant figures **CAO**	AO1.1b	A1	
16(b)	Uses $s = ut + \frac{1}{2}at^2$ with $s = -10$ and their u for vertical motion Condone cos or sign error	AO3.4	M1	$-10 = (25.7\sin 35)t - \frac{1}{2} \times 9.81t^2$ t = 3.571 Time in flight is 3.57 seconds
	Obtains correct equation	AO1.1b	A1F	
	Obtains correct time of flight with units AWRT 3.6 **CAO**	AO3.2a	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)(i)	Forms equation of motion with four correct terms Condone sign error	AO3.4	M1	300 – 140 – *R* = 482 x 0.2 R = 63.6 N
	Obtains correct equation.	AO1.1b	A1	
	Obtains correct value of *R*.	AO1.1b	A1	
17(a)(ii)	Forms equation of motion with correct terms Condone sign error	AO1.1a	M1	*T* – 63.6 = 72 x 0.2 *T* = 78 N
	Obtains correct equation Follow through their *R*	AO1.1b	A1F	
	Obtains correct value of *T*	AO1.1b	A1	
17(b)	States appropriate assumption **NOT friction or air resistance**	AO3.3	E1	Rope has no mass **or** is horizontal **or** is inextensible
17(c)(i)	Forms equation of motion for skater using 'their' *R* Condone sign error	AO3.1b	M1	-63.6 = 72*a*
	Finds correct acceleration for 'their' *R*	AO1.1b	A1F	*a* = -0.883… m s^{-2}
	Uses a suitable constant acceleration formula with 'their' *a*	AO1.1a	M1	u = 6 v =0 a = -0.883
	Obtains s when $v = 0$ Or Obtains v or positive v^2 when $s = 20$	AO1.1b	A1F	$0 = 6^2 - 2 \times 0.883s$ s = 20.4 m
	Explains that the skater hits buggy using correct values	AO3.2a	E1	20.4 > 20 Skater hits buggy
17(c)(ii)	Explains that the tension is removed from the buggy	AO2.4	E1	The rope is released so there is no tension acting on the buggy, so there is a higher resultant force. The driver will notice an increase in acceleration.
	Explains that the driver notices an increase in acceleration	AO2.4	E1	
	Total		**14**	

	TOTAL		**100**	