MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______

Forename(s) ______

Candidate signature ______

A-level
MATHEMATICS
Paper 2

Wednesday 12 June 2019 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

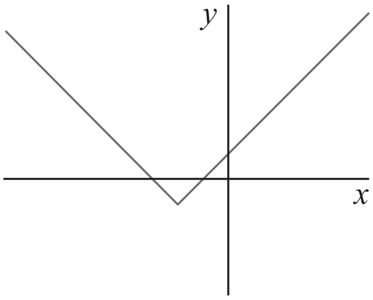
Answer **all** questions in the spaces provided.

1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) **one** box.

[1 mark]

A



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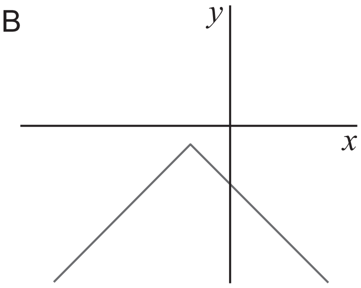

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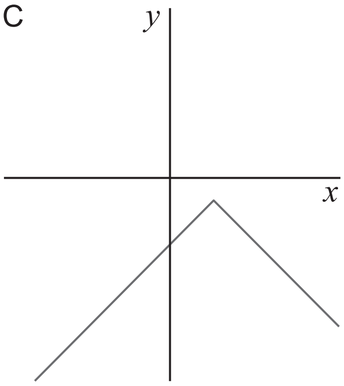

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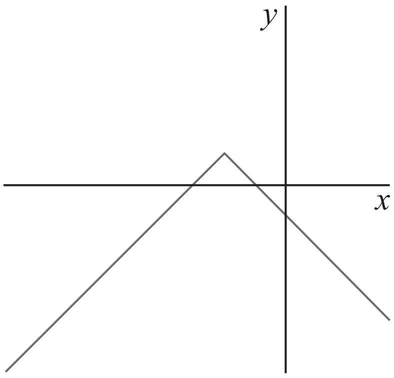

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2 Simplify $\sqrt{a^{\frac{2}{3}} \times a^{\frac{2}{5}}}$

Circle your answer.

[1 mark]

$a^{\frac{2}{15}}$ $a^{\frac{4}{15}}$ $a^{\frac{8}{15}}$ $a^{\frac{16}{15}}$

$\sqrt{a^{\frac{2}{3}} \times a^{\frac{2}{5}}} = (a^{\frac{2}{3}} \times a^{\frac{2}{5}})^{\frac{1}{2}} = (a^{\frac{16}{15}})^{\frac{1}{2}} = a^{\frac{8}{15}}$

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does **not** have an inverse?

Circle your answer.

[1 mark]

$f(x) = x^3$ $f(x) = 2x + 1$ $f(x) = x^2$ $f(x) = e^x$

Turn over for the next question

Do not write outside the box

4 $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor $(x + 2)$

Show that $2(d - b) = e - c$

Fully justify your answer.

[4 marks]

Since $(x+2)$ is a factor, by the factor theorem $f(-2) = 0$ for each of the functions given.

So

$(-2)^2 + b(-2) + c = 0 \Rightarrow 4 - 2b + c = 0$ ①

$(-2)^2 + d(-2) + e = 0 \Rightarrow 4 - 2d + e = 0$ ②

Equating ① and ②: $4 - 2b + c = 4 - 2d + e$

$2d - 2b = e - c$

$2(d - b) = e - c$.

5 Solve the differential equation

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t} \quad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = f(x)$

[7 marks]

$\frac{dt}{dx} = \frac{\ln x}{x^2 t}$

$\int t \, dt = \int \frac{\ln x}{x^2} dx$

$\frac{t^2}{2} + c_1 = \int \frac{\ln x}{x^2} dx$

Integration by parts: $u = \ln x$ $v' = x^{-2}$

$u' = \frac{1}{x}$ $v = -x^{-1}$

$\frac{t^2}{2} + c_1 = \int \frac{\ln x}{x^2} dx$

$\frac{t^2}{2} + c_1 = -x^{-1}\ln x - \int \frac{1}{x}(-x^{-1}) dx$

$\frac{t^2}{2} + c_1 = -x^{-1}\ln x + \int x^{-2} dx$

$\frac{t^2}{2} + c_1 = -x^{-1}\ln x - x^{-1} + c_2$

$\frac{t^2}{2} = -x^{-1}\ln x - x^{-1} + c$, where $c = c_2 - c_1$.

When $t = 2$, $x = 1$: $\frac{(2)^2}{2} = -(1)^{-1}\ln(1) - 1^{-1} + c$

$2 = -1 + c$

$c = 3$

So, $\frac{t^2}{2} = -x^{-1}\ln x - x^{-1} + 3$

$t^2 = 6 - 2\left(\frac{1 + \ln x}{x}\right)$

Turn over for the next question

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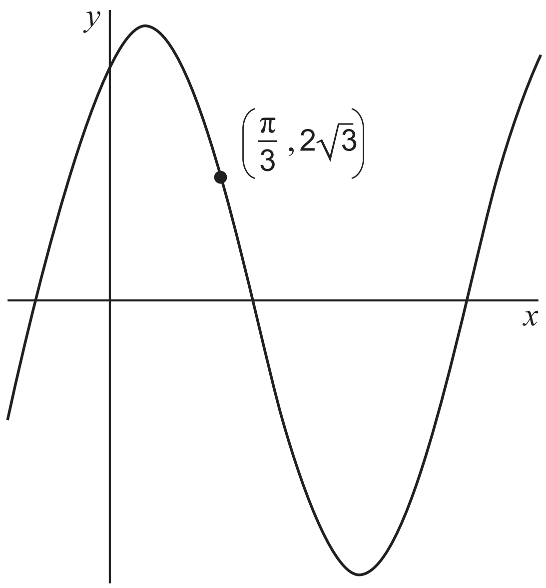
6 A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

[6 marks]

$R\sin(x+\alpha) = a\sin x + b\cos x$

The maximum value is 4 so $R = 4$.

$4\sin(x+\alpha) = a\sin x + b\cos x$

$4\sin(x+\alpha) = 4\sin x\cos\alpha + 4\cos x\sin\alpha$

$4\sin\left(\frac{\pi}{3}+\alpha\right) = 4\sin\left(\frac{\pi}{3}\right)\cos\alpha + 4\cos\left(\frac{\pi}{3}\right)\sin\alpha = 2\sqrt{3}$

$a\sin\left(\frac{\pi}{3}\right) + b\cos\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

$a = 4\cos\alpha$

$b = 4\sin\alpha$

$4\sin\left(\frac{\pi}{3}+\alpha\right) = 2\sqrt{3} \Rightarrow \sin\left(\frac{\pi}{3}+\alpha\right) = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 0$ or $\alpha = \frac{\pi}{3}$.

From the diagram, we can see that $\alpha = \frac{\pi}{3}$.

$a = 4\cos\left(\frac{\pi}{3}\right) \Rightarrow a = 2$

$b = 4\sin\left(\frac{\pi}{3}\right) \Rightarrow b = 2\sqrt{3}$

Turn over for the next question

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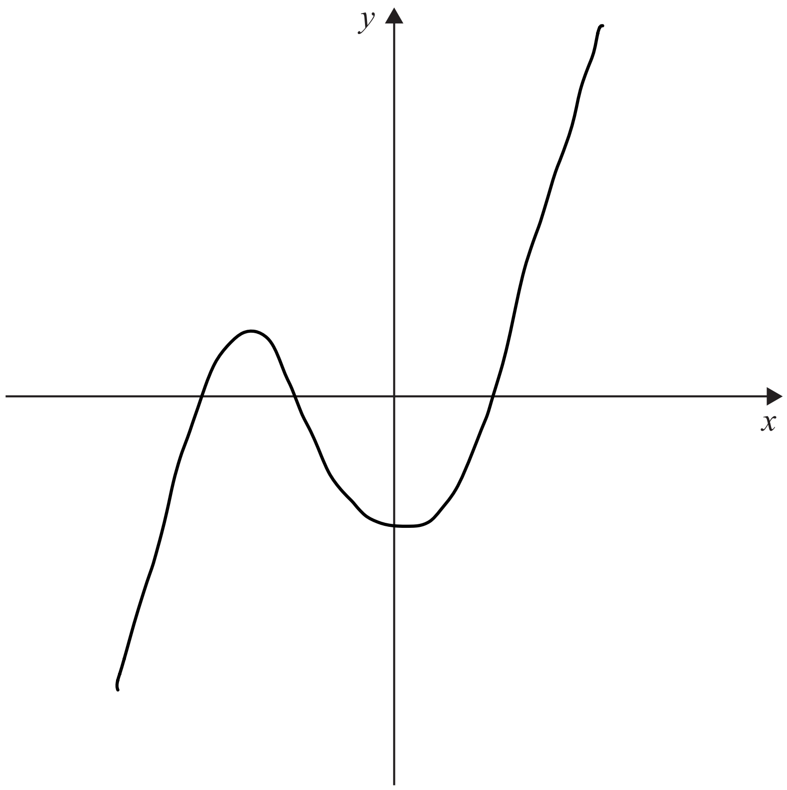
7 (a) Sketch the graph of any cubic function that has **both** three distinct real roots **and** a positive coefficient of x^3

[2 marks]



7 (b) The function $f(x)$ is defined by

$$f(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$

7 (b) (i) Show that there is a turning point where the curve crosses the y-axis.

[3 marks]

$f(x) = x^3 + 3px^2 + q$

Turning points occur when $f'(x) = 0$:

$f'(x) = 3x^2 + 6px$

$3x^2 + 6px = 0$

$3x(x + 2p) = 0$

$x = 0$ or $x = -2p$

Since there is a root at $x=0$, there is a turning point on the y axis.

Do not write outside the box

7 (b) (ii) The equation $f(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p, the range of possible values of q.

[5 marks]

$x^3 + 3px^2 + q = 0$, $p > 0$

Since $p > 0$, $x = -2p < 0$.

By consulting the diagram, we can see that there is a maximum at $x = -2p$ and a minimum at $x = 0$.

$f(0) = q$ so minimum at $(0, q)$.

$f(-2p) = (-2p)^3 + 3p(-2p)^2 + q$

$= -8p^3 + 12p^3 + q$

$= 4p^3 + q$ so maximum at $(-2p, 4p^3 + q)$.

$(-2p, 4p^3 + q)$ must be above x axis: $4p^3 + q > 0 \Rightarrow -4p^3 < q$

$(0, q)$ must be below x axis: $q < 0$

So, $-4p^3 < q < 0$.

Turn over for the next question

Do not write outside the box

8 Theresa bought a house on 2 January 1970 for £8000.

The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuations are shown in the following table.

Year	1970	1980	1990	2000	2010
Valuation price	£8 000	£19 000	£36 000	£82 000	£205 000

The valuation price of the house can be modelled by the equation

$$V = pq^t$$

where V pounds is the valuation price t years after 2 January 1970 and p and q are constants.

8 (a) Show that $V = pq^t$ can be written as $\log_{10} V = \log_{10} p + t \log_{10} q$

[2 marks]

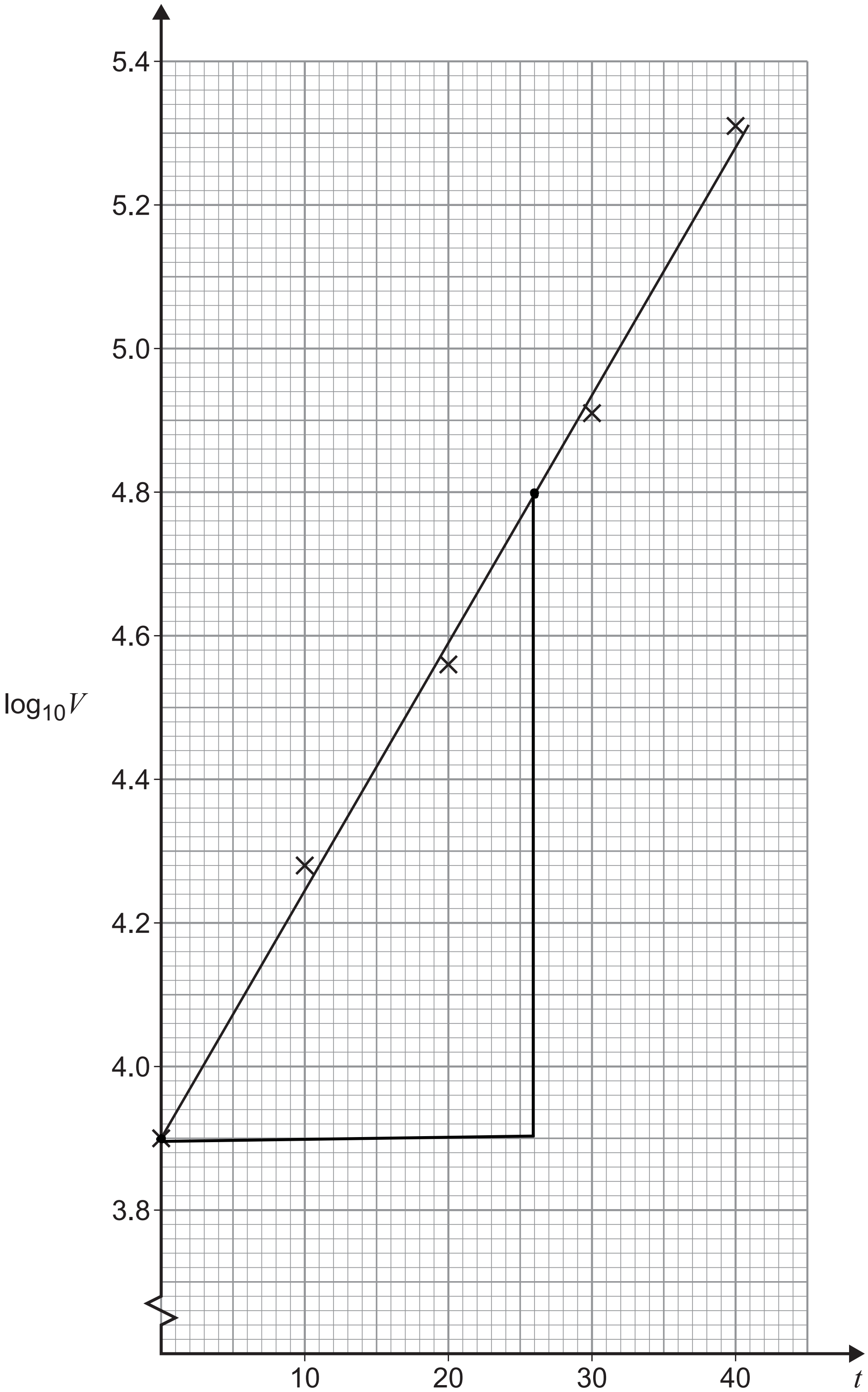
$\log_{10} V = \log_{10} pq^t$

$\log_{10} V = \log_{10} p + \log_{10} q^t$

$\log_{10} V = \log_{10} p + t \log_{10} q$

8 (b) The values in the table of $\log_{10} V$ against t have been plotted and a line of best fit has been drawn on the graph below.

t	0	10	20	30	40
$\log_{10} V$	3.90	4.28	4.56	4.91	5.31



Question 8 continues on the next page

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Using the given line of best fit, find estimates for the values of p and q.

Give your answers correct to three significant figures.

[4 marks]

The intercept is $\log_{10} p$ so : $\log_{10} p = 3.90$

$p = 10^{3.9} = 7943.28\ldots$

$p = 7940$ (3.s.f)

The gradient is $\log_{10} q$ so: $\log_{10} q = \dfrac{4.8 - 3.9}{26 - 0} = \dfrac{9}{260}$

$q = 10^{\left(\frac{9}{260}\right)} = 1.0829\ldots$

$q = 1.08$ (3.s.f)

Do not write outside the box

8 (c) Determine the year in which Theresa's house will first be worth half a million pounds.
[3 marks]

$500000 = 7940 \times 1.08^t$

$1.08^t = \frac{25000}{397}$

$\log_{10} 1.08^t = \log_{10}\left(\frac{25000}{397}\right)$

$t \log_{10} 1.08 = \log_{10}\left(\frac{25000}{397}\right)$

$t = \frac{\log_{10}\left(\frac{25000}{397}\right)}{\log_{10} 1.08}$

$t = 53.8284...$ $1970 + 53 = 2023$

The house will first be worth half a million pounds during 2023.

8 (d) Explain whether your answer to part **(c)** is likely to be reliable.
[2 marks]

The model is based on data between 1970 and 2010.

Houseprices may not continue to grow.

Turn over for the next question

Do not write outside the box

9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4-2x^2}$ are

$$2-\frac{x^2}{2}$$

[2 marks]

$\sqrt{4-2x^2} = \sqrt{4(1-\frac{x^2}{2})} = 2\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$

$\approx 2\left(1+\frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$

$\approx 2-\frac{x^2}{2}$

9 (b) State the range of values of x for which the expansion found in part **(a)** is valid.

[2 marks]

$\left|-\frac{x^2}{2}\right| < 1$

$\left|\frac{x^2}{2}\right| < 1$

$|x^2| < 2$

$|x|^2 < 2$

$|x| < \sqrt{2}$

Do not write outside the box

9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x}\, dx$$

giving your answer to five decimal places.

Fully justify your answer.

[4 marks]

Since 0.4 is small, $\cos x \approx 1 - \frac{x^2}{2}$.

$$\int_0^{0.4} \sqrt{\cos x}\, dx \approx \int_0^{0.4} \sqrt{1 - \frac{x^2}{2}}\, dx$$

$$\approx \frac{1}{2} \int_0^{0.4} \left(2 - \frac{x^2}{2}\right) dx$$

$$\approx \frac{1}{2} \left[2x - \frac{x^3}{6}\right]_0^{0.4}$$

$$\approx \frac{1}{2} \left[\left(\frac{296}{375}\right) - (0)\right]$$

$$\approx \frac{148}{375}$$

$$\approx 0.39467 \quad (5.\text{d.p})$$

9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x}\, dx$$

Explain why this may not be a suitable method.

[1 mark]

As 1.4 is not a small angle, the approximation of $\cos x \approx 1 - \frac{x^2}{2}$ is no longer suitable.

Turn over for Section B

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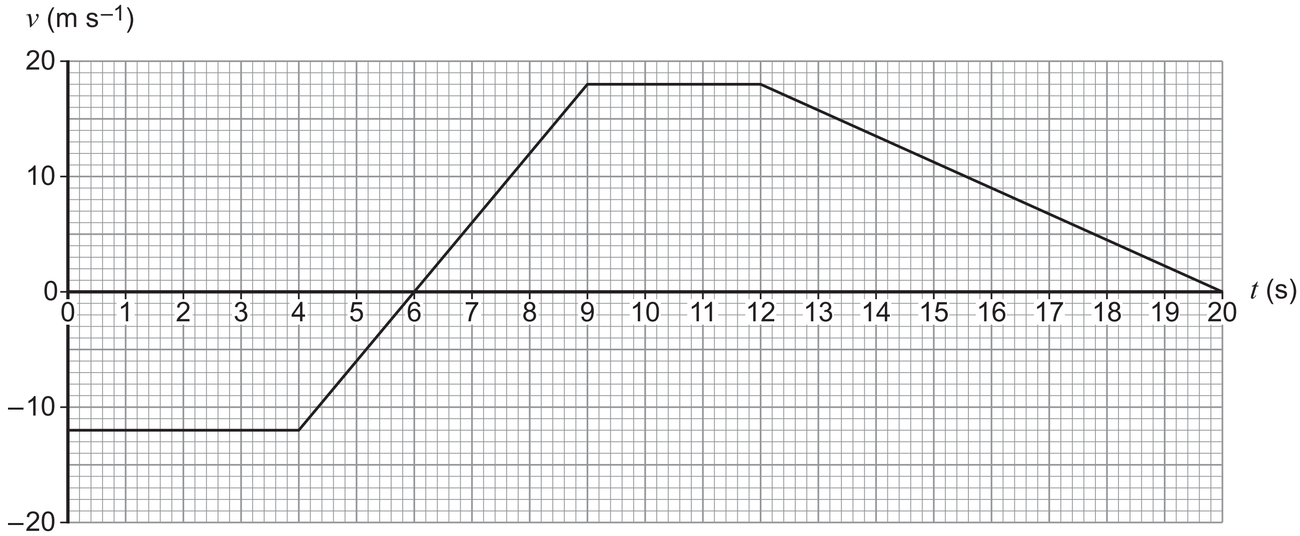
Section B

Answer **all** questions in the spaces provided.

10 The diagram below shows a velocity-time graph for a particle moving with velocity $v\,\text{m s}^{-1}$ at time t seconds.



Which statement is correct?

Tick (✓) **one** box.

[1 mark]

- [ ] The particle was stationary for $9 \leq t \leq 12$
- [x] The particle was decelerating for $12 \leq t \leq 20$
- [ ] The particle had a displacement of zero when $t = 6$
- [ ] The particle's speed when $t = 4$ was $-12\,\text{m s}^{-1}$

Do not write outside the box

11 A wooden crate rests on a rough horizontal surface.

The coefficient of friction between the crate and the surface is 0.6

A forward force acts on the crate, parallel to the surface.

When this force is 600 N, the crate is on the point of moving.

Find the weight of the crate.

$F = \mu R$

$600 = 0.6R \Rightarrow R = 1000$

Circle your answer.

[1 mark]

(1000 N) 100 kg 360 N 36 kg

12 A particle, under the action of two constant forces, is moving across a perfectly smooth horizontal surface at a constant speed of $10\,\text{m s}^{-1}$

The first force acting on the particle is $(400\mathbf{i} + 180\mathbf{j})$ N.

The second force acting on the particle is $(p\mathbf{i} - 180\mathbf{j})$ N.

Find the value of p.

Circle your answer.

[1 mark]

(−400) −390 390 400

$F_1 + F_2 = 0 \Rightarrow p = -400$

Turn over for the next question

Do not write outside the box

13 In a school experiment, a particle, of mass m kilograms, is released from rest at a point h metres above the ground.

At the instant it reaches the ground, the particle has velocity v m s^{-1}

13 (a) Show that

$$v = \sqrt{2gh}$$

[2 marks]

u=0, s=h, a=g

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2gh$

$v^2 = 2gh$

$v = \sqrt{2gh}$

13 (b) A student correctly used $h = 18$ and measured v as 20

The student's teacher claims that the machine measuring the velocity must have been faulty.

Determine if the teacher's claim is correct.

Fully justify your answer.

[3 marks]

When g=9.8 and h=18,

$v = \sqrt{2 \times 9.8 \times 18}$

$v = 18.8$

As $18.8 < 20$, the machine is faulty.

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Turn over for the next question

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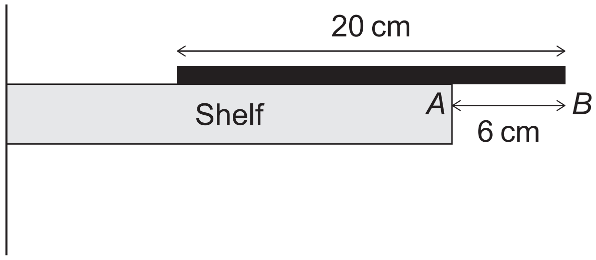
14 A metal rod, of mass m kilograms and length 20 cm, lies at rest on a horizontal shelf.

The end of the rod, B, extends 6 cm beyond the edge of the shelf, A, as shown in the diagram below.

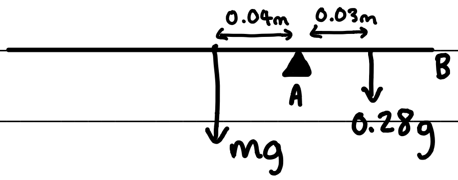


14 (a) The rod is in equilibrium when an object of mass 0.28 kilograms hangs from the midpoint of AB.

Show that $m = 0.21$

[3 marks]

Take moments about A:



$(0.28g \times 0.03) - (mg \times 0.04) = 0$

$0.0084g = 0.04mg$

$0.0084 = 0.04m$

$m = \frac{0.0084}{0.04}$

$m = 0.21$

Do not write outside the box

14 (b) The object of mass 0.28 kilograms is removed.

A number, n, of identical objects, each of mass 0.048 kg, are hung from the rod all at a distance of 1 cm from *B*.

Find the maximum value of n such that the rod remains horizontal.

[4 marks]

$(0.048g \times 0.05 \times n) - (0.21g \times 0.04) = 0$

$0.0024gn = 0.0084g$

$0.0024n = 0.0084$

$n = 3.5$

So maximum is $n = 3$ as n must be an integer.

14 (c) State one assumption you have made about the rod.

[1 mark]

The rod is uniform.

Turn over for the next question

Turn over ▶

Do not write outside the box

15 Four buoys on the surface of a large, calm lake are located at A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 410 \\ 710 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -210 \\ 530 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} -340 \\ -310 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 590 \\ -40 \end{bmatrix}$$

All values are in metres.

15 (a) Prove that the quadrilateral $ABCD$ is a trapezium but **not** a parallelogram.

[5 marks]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{bmatrix} -210 \\ 530 \end{bmatrix} - \begin{bmatrix} 410 \\ 710 \end{bmatrix} = \begin{bmatrix} -620 \\ -180 \end{bmatrix}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = \begin{bmatrix} 590 \\ -40 \end{bmatrix} - \begin{bmatrix} -340 \\ -310 \end{bmatrix} = \begin{bmatrix} 930 \\ 270 \end{bmatrix}$

Since $\overrightarrow{CD} = -1.5 \times \overrightarrow{AB}$, AB and CD are parallel but not of the same length.

For a parallelogram, parallel sides are the same length.

Therefore, ABCD is a trapezium but not a parallelogram.

Do not write outside the box

15 (b) A speed boat travels directly from *B* to *C* at a constant speed in 50 seconds.

Find the speed of the boat between *B* and *C*.

[4 marks]

velocity = $\frac{\text{displacement}}{\text{time}}$

$\vec{BC} = \vec{OC} - \vec{OB} = \begin{bmatrix} -340 \\ -310 \end{bmatrix} - \begin{bmatrix} -210 \\ 530 \end{bmatrix} = \begin{bmatrix} -130 \\ -840 \end{bmatrix}$

$V = \frac{1}{50} \times \begin{bmatrix} -130 \\ -840 \end{bmatrix} = \begin{bmatrix} -2.6 \\ -16.8 \end{bmatrix}$

Speed = $|V| = \sqrt{2.6^2 + 16.8^2} = 17\,ms^{-1}$

Turn over for the next question

Do not write outside the box

16 An elite athlete runs in a straight line to complete a 100-metre race.

During the race, the athlete's velocity, $v\,\text{m}\,\text{s}^{-1}$, may be modelled by

$$v = 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t}$$

where t is the time, in seconds, after the starting pistol is fired.

16 (a) Find the maximum value of v, giving your answer to one decimal place.

Fully justify your answer.

[8 marks]

$V = 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t}$

$\frac{dv}{dt} = 10.512e^{-0.9t} - 0.009e^{0.3t}$

Maximum V occurs when $\frac{dv}{dt} = 0$: $10.512e^{-0.9t} - 0.009e^{0.3t} = 0$

$10.512 - 0.009e^{1.2t} = 0$

$0.009e^{1.2t} = 10.512$

$e^{1.2t} = \frac{10.512}{0.009} = 1168$

$1.2t = \ln(1168)$

$t = \frac{5}{6}\ln(1168)$

$t = 5.886$ (3.d.p)

$V = 11.71 - 11.68e^{-0.9\times 5.886} - 0.03e^{0.3\times 5.886}$

$V = 11.5$

This is the maximum value as it is the only value which relates to $\frac{dv}{dt} = 0$.

Do not write outside the box

16 (b) Find an expression for the distance run in terms of *t*.

[6 marks]

$s = \int v \, dt$

$s = \int 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t} \, dt$

$S = 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} + c$

When $t=0$, $s=0$: $12.978 - 0.1 + c = 0$

$c = -12.878$

Distance $= 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} - 12.878$

Question 16 continues on the next page

Do not write outside the box

16 (c) The athlete's actual time for this race is 9.8 seconds.

Comment on the accuracy of the model.

[2 marks]

$S = 11.71(9.8) + 12.978e^{-0.9 \times 9.8} - 0.1e^{0.3 \times 9.8} - 12.878 = 99.99\text{m}$

The model is accurate as it predicts the distance to be 99.99m which is very close to 100m.

Turn over for the next question

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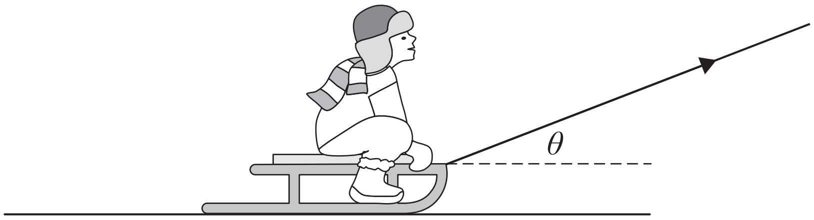
17 Lizzie is sat securely on a wooden sledge.

The combined mass of Lizzie and the sledge is M kilograms.

The sledge is being pulled forward in a straight line along a horizontal surface by means of a light inextensible rope, which is attached to the front of the sledge.

This rope stays inclined at an acute angle θ above the horizontal and remains taut as the sledge moves forward.



The sledge remains in contact with the surface throughout.

The coefficient of friction between the sledge and the surface is μ and there are no other resistance forces.

Lizzie and the sledge move forward with constant acceleration, $a\,\text{m s}^{-2}$

The tension in the rope is a constant T Newtons.

17 (a) Show that

$$T = \frac{M(a + \mu g)}{\cos\theta + \mu\sin\theta}$$

[7 marks]

Resolve vertically: $R + T\sin\theta = Mg$

Using $F=ma$: $T\cos\theta - F = Ma$

Substitute $F = \mu R$: $T\cos\theta - \mu R = Ma$

$T\cos\theta - \mu(Mg - T\sin\theta) = Ma$

$T(\cos\theta + \mu\sin\theta) = Ma + \mu Mg$

$T = \dfrac{M(a+\mu g)}{\cos\theta + \mu\sin\theta}$

17 (b) It is known that when $M = 30$, $\theta = 30°$, and $T = 40$, the sledge remains at rest.

Lizzie uses these values with the relationship formed in part **(a)** to find the value for μ

Explain why her value for μ may be incorrect.

[2 marks]

The sledge is at rest so the relationship may not be valid as friction may not be its limiting value.

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