MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

A-level
MATHEMATICS
Paper 2

Wednesday 13 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Which of these statements is correct?

Tick **one** box.

[1 mark]

$x = 2 \Rightarrow x^2 = 4$ ☑

$x^2 = 4 \Rightarrow x = 2$ ☐

$x^2 = 4 \Leftrightarrow x = 2$ ☐

$x^2 = 4 \Rightarrow x = -2$ ☐

2 Find the coefficient of x^2 in the expansion of $(1 + 2x)^7$

Circle your answer.

[1 mark]

42 4 21 84

$\binom{7}{2} \times 2^2 = 84$

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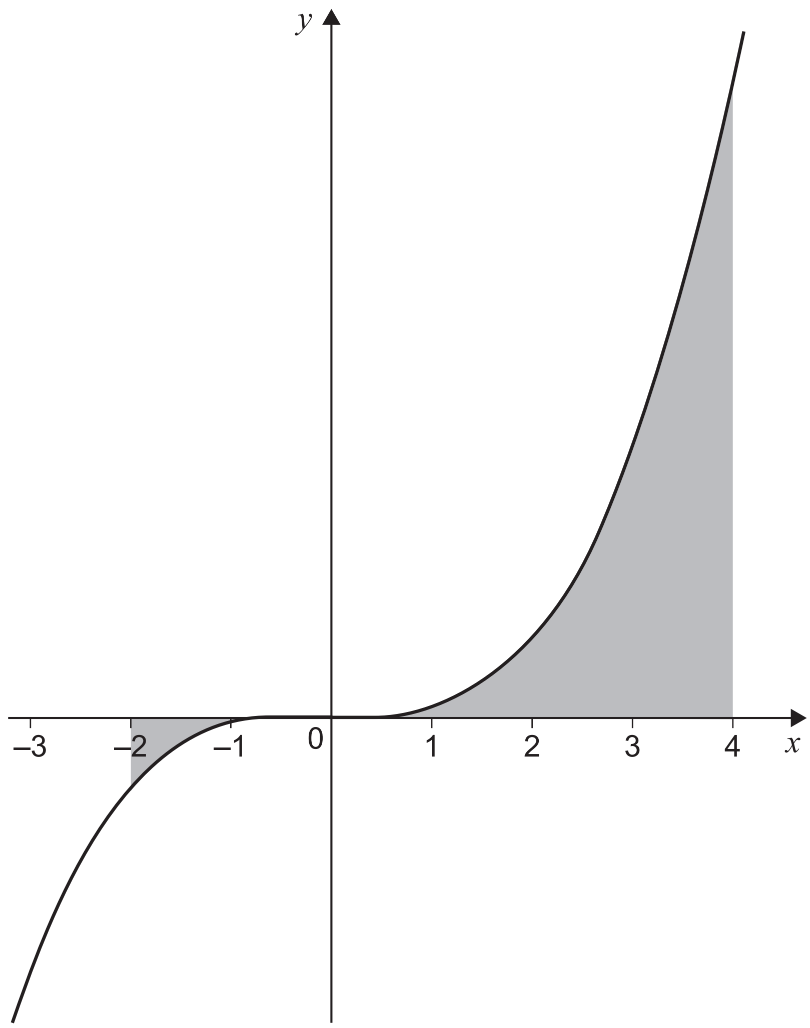
3 The graph of $y = x^3$ is shown.



Find the total shaded area.

Circle your answer.

[1 mark]

-68 60 68 128

$$\int_0^4 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_0^4 = 64$$

$$\int_{-2}^0 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_{-2}^0 = 0 - 4 = -4$$

The second value is negative because it is under the x axis. However, its area cannot be negative so its area is 4.
Total area $= 64 + 4 = 68$

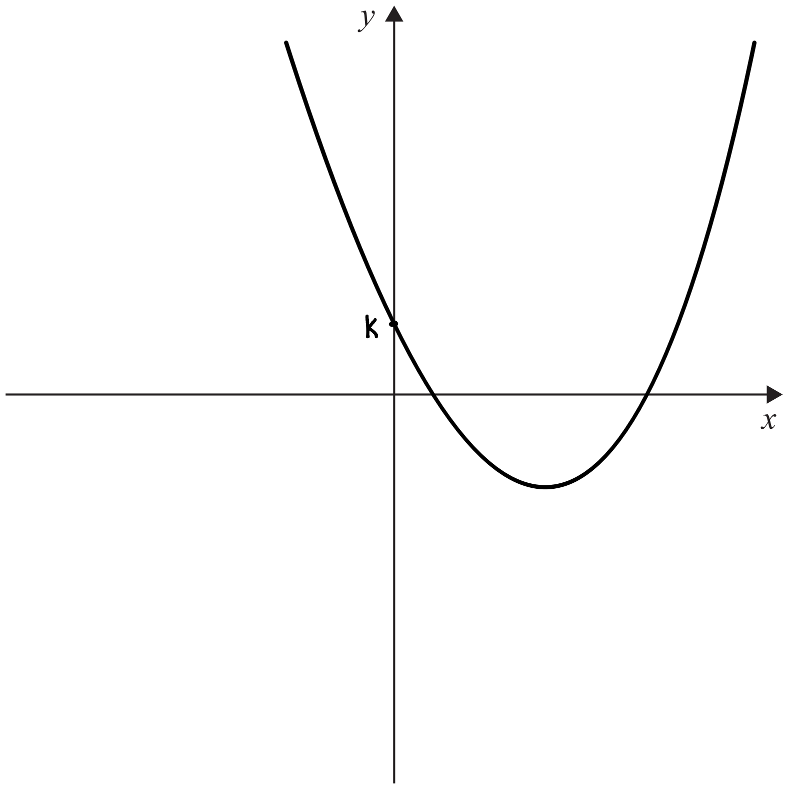
Turn over ▶

4 A curve, *C*, has equation $y = x^2 - 6x + k$, where k is a constant.

The equation $x^2 - 6x + k = 0$ has two distinct positive roots.

4 (a) Sketch *C* on the axes below.

[2 marks]



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4 (b) Find the range of possible values for k.

Fully justify your answer.

[4 marks]

The roots are distinct so the discriminant is greater than zero:

$$b^2 - 4ac > 0$$

$$36 - 4(1)(k) > 0$$

$$36 - 4k > 0$$

$$4k < 36$$

$$k < 9$$

The roots are positive so k must also be greater than zero.

So,

$$0 < k < 9.$$

Turn over for the next question

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5 Prove that 23 is a prime number.

[2 marks]

$\sqrt{23} \approx 4.8$ so we need to check if 2 and 3 are factors.

23 is odd so 2 is not a factor.

23 is not a multiple of 3.

So 23 is prime.

Do not write outside the box

6 Find the coordinates of the stationary point of the curve with equation

$$(x + y - 2)^2 = e^y - 1$$

[7 marks]

$(x+y-2)^2 = e^y - 1$

Differentiate implicitly: $2\left(1 + \frac{dy}{dx}\right)(x+y-2) = \frac{dy}{dx}e^y$

For the stationary point we need $\frac{dy}{dx} = 0$:

$2(1+0)(x+y-2) = 0(e^y)$

$2(x+y-2) = 0$

$x+y-2 = 0$

So if we substitute this condition back into the original equation we get

$0 = e^y - 1$

$e^y = 1$

$y = 0$

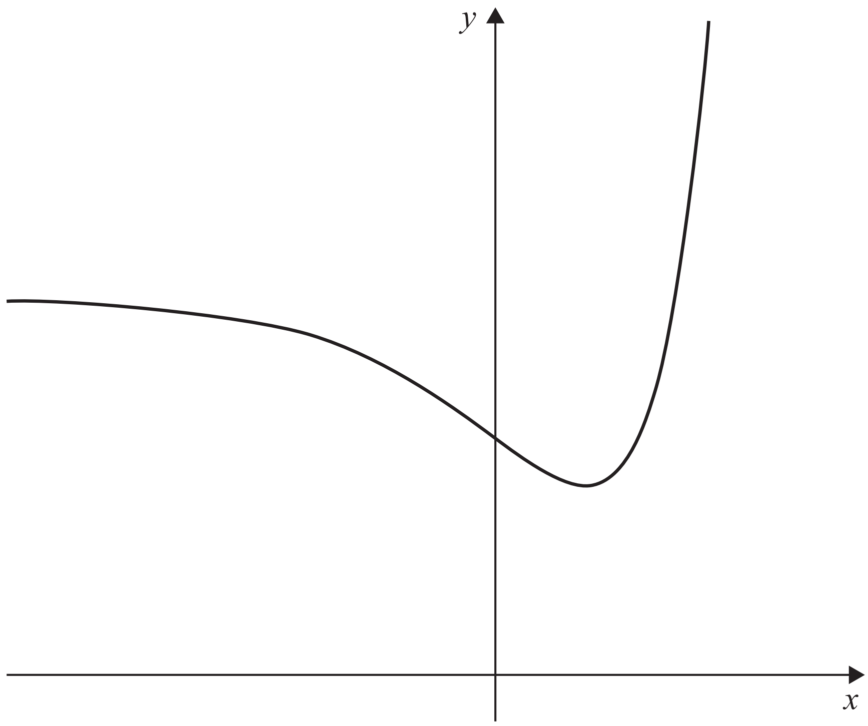
When $y=0$, $x = 2 - y = 2$.

So the stationary point is $(2,0)$.

Do not write outside the box

7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

$\dfrac{dy}{dx} = (x-1)e^x$

$\int dy = \int (x-1)e^x \, dx$

$y = \int xe^x \, dx - \int e^x \, dx$

$y = xe^x - \int e^x \, dx - e^x$

$y = xe^x - e^x - e^x + c$

$y = xe^x - 2e^x + c$

The range of $f(x)$ is $y \geq e$. This means that the minimum point on the graph is at $y = e$, and $\dfrac{dy}{dx} = 0$ here because it is a stationary point.

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So $\frac{dy}{dx} = 0 \Rightarrow (x-1)e^x = 0$

$\Rightarrow \; x-1=0$

$\Rightarrow \; x=1$

So the curve passes through the minimum point $(1,e)$.

We can now use these conditions to find the value for c:

$y = xe^x - 2e^x + c$

$e = e - 2e + c$

$c = 2e$

Therefore, $f(x) = xe^x - 2e^x + 2e$.

Turn over for the next question

Do not write outside the box

8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

Use the identity $R\sin(x-\alpha) \equiv R\sin x\cos\alpha - R\sin\alpha\cos x$.

Set $R\sin(x-\alpha) = \sqrt{3}\sin x - 3\cos x$.

$R\sin x\cos\alpha - R\sin\alpha\cos x = \sqrt{3}\sin x - 3\cos x$

$R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 3$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{\sqrt{3}}$

$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$R = \frac{\sqrt{3}}{\cos\alpha} = \frac{\sqrt{3}}{\cos\frac{\pi}{3}} = \frac{\sqrt{3}}{1/2} = 2\sqrt{3}$

So, $\sqrt{3}\sin x - 3\cos x = 2\sqrt{3}\sin(x-\frac{\pi}{3})$

This is a translation of $\frac{\pi}{3}$ in the positive x direction: $\begin{pmatrix}\frac{\pi}{3}\\0\end{pmatrix}$

Followed by a stretch in the y direction of scale factor $2\sqrt{3}$.

Followed by a translation of 4 in the positive y direction: $\begin{pmatrix}0\\4\end{pmatrix}$.

Do not write outside the box

8 (b) (i) Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

$$\frac{1}{\sqrt{3}\sin x - 3\cos x + 4} = \frac{1}{2\sqrt{3}\sin(x-\frac{\pi}{3}) + 4}$$

This is smallest when the denominator is biggest, so when $\sin(x-\frac{\pi}{3}) = 1$:

$$\frac{1}{2\sqrt{3}+4} = \frac{1}{2\sqrt{3}+4} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$

$$= \frac{4-2\sqrt{3}}{16-12} = \frac{4-2\sqrt{3}}{4} = \frac{2-\sqrt{3}}{2}.$$

8 (b) (ii) Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

Now we want the denominator to be smallest, so let $\sin(x-\frac{\pi}{3}) = -1$:

$$\frac{1}{-2\sqrt{3}+4} = \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{4} = \frac{2+\sqrt{3}}{2}.$$

Turn over for the next question

9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8-t)$$

[3 marks]

$\frac{dx}{dt} \propto \frac{8-t}{x}$

$\frac{dx}{dt} = \frac{k(8-t)}{x}$

At $t=2$, $\frac{dx}{dt} = 72$ and $x = 336$:

$72 = \frac{k(8-2)}{336} \Rightarrow 24192 = 6k \Rightarrow k = 4032$

Therefore, $\frac{dx}{dt} = \frac{4032(8-t)}{x} \Rightarrow x\frac{dx}{dt} = 4032(8-t)$.

Do not write outside the box

9 (b) Hence, show that

$$x^2 = 4032t(16 - t)$$

[3 marks]

$x\,dx = 4032(8-t)\,dt$

$\int x\,dx = 4032\int(8-t)\,dt$

$\frac{1}{2}x^2 = 4032\left(8t - \frac{t^2}{2}\right) + c$

$x^2 = 4032(16t - t^2) + c$

$x^2 = 4032t(16-t) + c$

At $t=2$, $x = 336$:

$336^2 = 4032(2)(16-2) + c$

$112896 = 112896 + c \Rightarrow c = 0$

Therefore, $x^2 = 4032t(16-t)$.

Question 9 continues on the next page

Turn over ▶

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9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

We want to find the point when the rate becomes £24, so when $\frac{dx}{dt} = 24$. Substitute this into the equation from a):

$$24x = 4032(8-t)$$

$$x = 168(8-t)$$

Now substitute this into the equation from b):

$$(168(8-t))^2 = 4032t(16-t)$$

$$7(8-t)^2 = t(16-t)$$

$$448 - 112t + 7t^2 = 16t - t^2$$

$$8t^2 - 128t + 448 = 0$$

$$t^2 - 16t + 56 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{16 \pm \sqrt{16^2 - 4(56)}}{2} = \frac{16 \pm \sqrt{32}}{2} = \frac{16 \pm 4\sqrt{2}}{2} = 8 \pm 2\sqrt{2}$$

So, $t = 5.172...$ or $t = 10.828...$

The earliest time is when $t = 5.712$.

This is 5 hours plus $0.172 \times 60 = 10.29$ minutes.

So 5 hours 10 minutes.

This is at 14:40.

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9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

As soon as the stall opens there are zero sales, so $x = 0$.

$\frac{dx}{dt}$ is now undefined as the denominator is zero.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

10 A garden snail moves in a straight line from rest to 1.28 cm s^{-1}, with a constant acceleration in 1.8 seconds.

Find the acceleration of the snail.

Circle your answer.

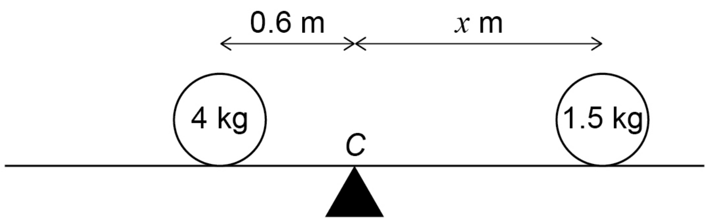
[1 mark]

2.30 m s^{-2} 0.71 m s^{-2} 0.0071 m s^{-2} 0.023 m s^{-2}

1.28 cm = 0.0128 m

acceleration $= \frac{0.0128}{1.8} = 0.0071$ ms^{-2}

11 A uniform rod, *AB*, has length 4 metres.
The rod is resting on a support at its midpoint *C*.
A particle of mass 4 kg is placed 0.6 metres to the left of *C*.
Another particle of mass 1.5 kg is placed x metres to the right of *C*, as shown.



The rod is balanced in equilibrium at *C*.

Find x.

Circle your answer.

[1 mark]

1.8 m 1.5 m 1.75 m 1.6 m

$\frac{2.4}{1.5} = 1.6$m

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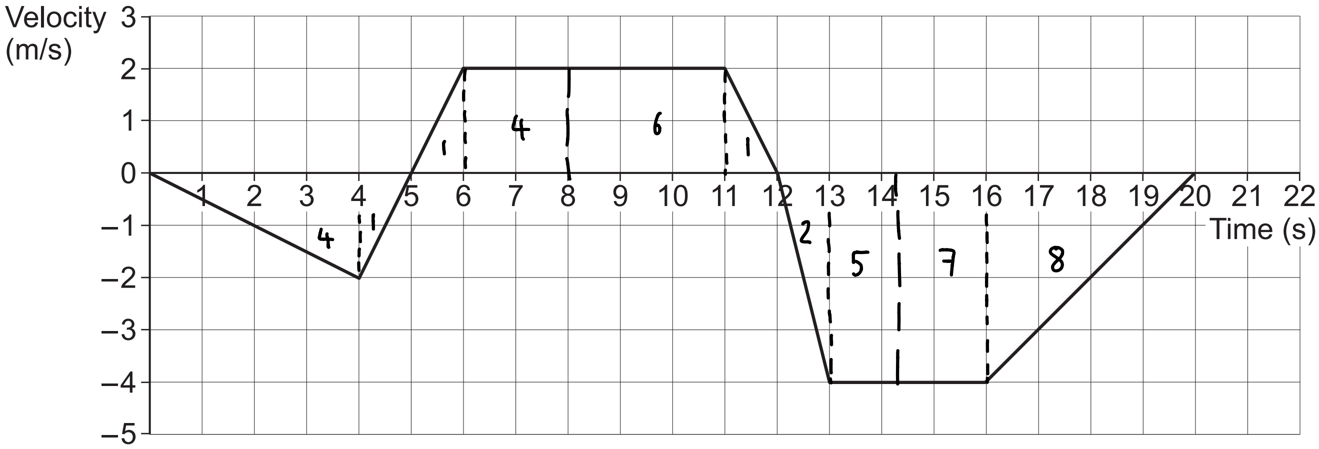
12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

The line is steepest from 12–13 seconds.

The acceleration here is $\frac{-4}{1} = -4\ ms^{-2}$.

This line has a magnitude of $4\ ms^{-2}$.

12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

It is back at its stationary position when the area above the x axis is equal to the area below.

At t = 8, the area below is 4 + 1 and the area above is 4 + 1 so they are equal. Hence, $t_1 = 8$.

At t = 14.25, the area below is 4 + 1 + 2 + 5 = 12 and the area above is 1 + 4 + 6 + 1 = 12 so they are equal. Hence, $t_2 = 14.25$.

Turn over ▶

Do not write outside the box

13 **In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.

[3 marks]

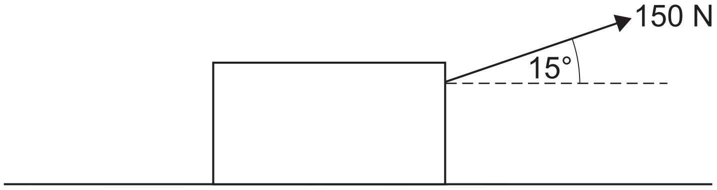
$F_{max} = \mu R = 0.85 \times 20g$

$= 166.6\ \mathrm{N}$

So, he would need to apply at least 166.6 N to the box to move it. Since $150 < 166.6$, it will not move.

Do not write outside the box

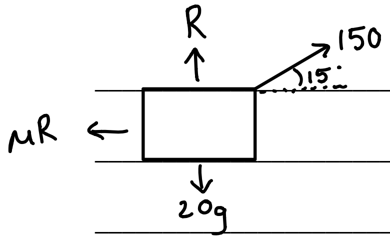
13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]



$R(\uparrow)$: $R + 150\sin 15 = 20g$

$R = 20g - 150\sin 15$

$R = 157.177$

$F_{max} = \mu R = 0.85 \times 157.177$

$= 133.6$

Horizontal component of his force $= 150\cos 15$

$= 145\,N$

$145 > 133.6$ so it will move.

Do not write outside the box

14 A quadrilateral has vertices A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$$

14 (a) Write down the vector $\overrightarrow{AB}$

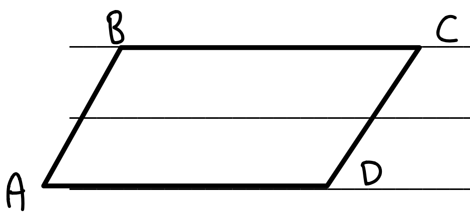
[1 mark]

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$= -\begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix}$

14 (b) Show that $ABCD$ is a parallelogram, but not a rhombus.

[5 marks]



$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix}$, from (a).

So we have two sets of parallel sides. If they are all the same length then it is a rhombus, otherwise it is a parallelogram.

$|AB| = \sqrt{(-4)^2 + (-3)^2 + 6^2} = \sqrt{61}$

$|AD| = \sqrt{1^2 + 5^2 + (-1)^2} = \sqrt{27}$

Since $\sqrt{61} \neq \sqrt{27}$, it is not a rhombus, hence it is a parallelogram.

15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in m s^{-2} of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

$$\text{The acceleration of A} = 0.138\,t^2$$
$$\text{The acceleration of B} = 0.024\,t^3$$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

Velocity $= \int a \, dt = \int 0.138t^2 \, dt = 0.046t^3 + c_1$

At $t=0$, $v=0$: $0 = 0 + c_1$

$c_1 = 0$

Displacement $= \int v \, dt = \int 0.046t^3 \, dt = 0.0115t^4 + c_2$

At $t=0$, $s=0$: $0 = 0 + c_2$

$c_2 = 0$

So, $s = 0.0115t^4$

When $s=100$: $100 = 0.0115t^4$

$t^4 = 8695.65$

$t = 9.657$

Question 15 continues on the next page

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15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

We do the same again but for minibus B:

$v = \int 0.024t^3 \, dt = 0.006t^4 + k_1$

At $t=0$, $v=0$: $0 = 0 + k_1 \Rightarrow k_1 = 0$

$s = \int 0.006t^4 \, dt = 0.0012t^5 + k_2$

At $t=0$, $s=0$: $0 = 0 + k_2 \Rightarrow k_2 = 0$

So, $s = 0.0012t^5$

At $s=100$: $100 = 0.0012t^5$

$t^5 = 83333.33 \Rightarrow t = 9.642$

$9.642 < 9.657$ so minibus B is better.

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

The times are also dependent on the driving and reaction times of the drivers. Driver B could have a faster reaction time than Driver A, meaning B looks faster than it actually is.

16 **In this question use $g = 9.81\,\text{m}\,\text{s}^{-2}$**

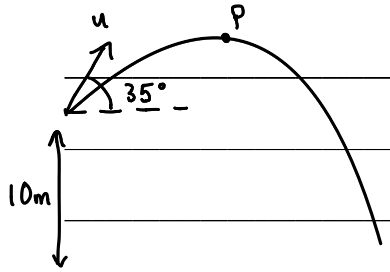
A particle is projected with an initial speed u, at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u.

[3 marks]



At P, upwards velocity is equal to zero.

$s = -,\ u = u\sin 35,\ v = 0,\ a = -9.81,\ t = 1.5$

$v = u + at$

$0 = u\sin 35 - 9.81(1.5)$

$u = \frac{14.715}{\sin 35}$

$u = 25.6548$

$u = 25.7\ \text{ms}^{-1}$

16 (b) Find the total time that the particle is in flight.

[3 marks]

$s = -10,\ u = 25.7\sin 35 = 14.715,\ v = -,\ a = -9.81,\ t = t$

$s = ut + \frac{1}{2}at^2$

$-10 = 14.715t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 - 14.715t - 10 = 0$

$t = \frac{14.715 \pm \sqrt{14.715^2 - 4(4.905)(-10)}}{2 \times 4.905}$

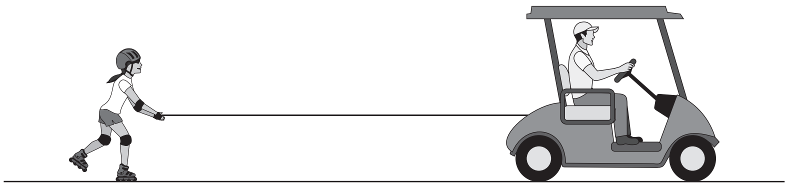
$t = \frac{14.715 \pm \sqrt{412.73}}{9.81} \Rightarrow t = 3.57$ or $t = -0.571$

$\Rightarrow t = 3.57$ seconds

Turn over ▶

Do not write outside the box

17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.



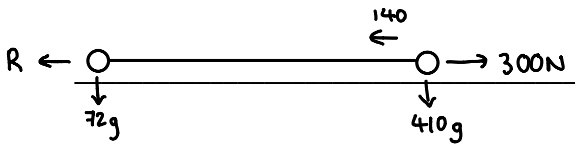
The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\mathrm{m\,s^{-2}}$

17 (a) (i) Find R.

[3 marks]



$R(\rightarrow)$: $300 - 140 - R = (410 + 72)(0.2)$

$R = 300 - 140 - 482(0.2)$

$R = 63.6$ N

Do not write outside the box

17 (a) (ii) Find the tension in the rope.

[3 marks]

Look just at the roller-skater:

$\twoheadrightarrow 0.2$

$63.6 \leftarrow \bigcirc \rightarrow T$ $\qquad$ $T - 63.6 = 72(0.2)$

$\downarrow$

$72g$ $\qquad$ $T = 14.4 + 63.6$

$T = 78$ N

17 (b) State a necessary assumption that you have made.

[1 mark]

Rope is inextensible.

Question 17 continues on the next page

Do not write outside the box

17 (c) The roller-skater releases the rope at a point *A*, when she reaches a speed of $6\,\text{m}\,\text{s}^{-1}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from *A*.

17 (c) (i) Determine whether the roller-skater will stop before reaching the stationary buggy.

Fully justify your answer.

[5 marks]

Using $F = ma$:

63.6 ← ○ $-63.6 = 72a$

72g $a = -0.8833...$

$s = s$ $v^2 = u^2 + 2as$

$u = 6$ $0 = 6^2 + 2(-0.8833)s$

$v = 0$ $1.767s = 36$

$a = -0.8833$ $s = 20.377$

$t = -$ $s = 20.4$ m

$20.4 > 20$ so the skater does not stop in time and hits the buggy.

17 (c) (ii) Explain the change in motion that the driver noticed.

[2 marks]

The driver will start accelerating faster because there is no tension in the rope.

END OF QUESTIONS

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