A-level
MATHEMATICS
7357/1
Paper 1

Mark scheme

June 2024

Version: 1.0 Final

246A7357/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
ISW	ignore subsequent working
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

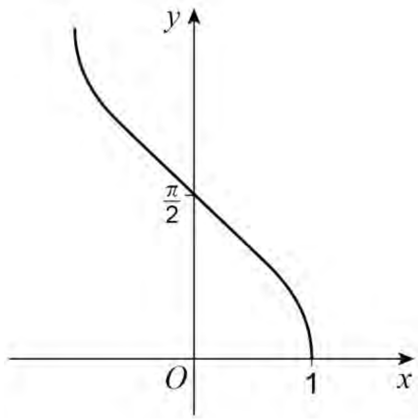
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the 1st answer	1.1b	B1	–5
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks the 1st box	1.1b	B1	$f^{-1}(x)=\ln(x-1)$
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the 2nd answer	1.1b	B1	4
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the bottom-right box	1.2	B1	
	Question 4 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5	Obtains $\sin x = \pm 1$ Or Obtains $\cos x = 0$ PI by a correct value for x Condone radians and values outside of range	1.1a	M1	$\sin x = \pm 1$ $x = 90^\circ, 270^\circ$
	Obtains 90 or 270 Or Obtains both $\frac{\pi}{2}$ and $\frac{3\pi}{2}$	1.1b	A1	
	Obtains 90 and 270 and no other values in the range.	1.1b	A1	
	Question 5 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6	Uses the chain rule to obtain $7\left(x^3+5x\right)^6 \mathrm{f}(x)$ Or $\mathrm{g}(x)\left(3x^2+5\right)$ Where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are polynomials of degree at least 1. Condone incorrect or missing brackets around their $3x^2+5$	1.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 7\left(3x^2+5\right)\left(x^3+5x\right)^6$
	Obtains $7\left(3x^2+5\right)\left(x^3+5x\right)^6$ OE ISW	1.1b	A1	
	Question 6 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7				
	Simplifies $\sqrt{8n}$ to $2\sqrt{2n}$	1.1b	B1	$\dfrac{3+\sqrt{8n}}{1+\sqrt{2n}}$
	Multiplies by $\dfrac{1-\sqrt{2n}}{1-\sqrt{2n}}$ or $\dfrac{\sqrt{2n}-1}{\sqrt{2n}-1}$	1.1a	M1	$=\dfrac{3+\sqrt{8n}}{1+\sqrt{2n}}\times\dfrac{1-\sqrt{2n}}{1-\sqrt{2n}}$ $=\dfrac{3-3\sqrt{2n}+\sqrt{8n}-\sqrt{16n^2}}{1-2n}$
	Obtains correct single fraction with denominator of $1-2n$ or $2n-1$	1.1b	A1	$=\dfrac{3-3\sqrt{2n}+2\sqrt{2n}-4n}{1-2n}$
	Completes reasoned argument to obtain $\dfrac{4n-3+\sqrt{2n}}{2n-1}$ AG Condone eg $\sqrt{8}n$ or $\sqrt{2}n$ except if seen on their final line.	2.1	R1	$=\dfrac{3-\sqrt{2n}-4n}{1-2n}$ $=\dfrac{4n-3+\sqrt{2n}}{2n-1}$
	Question 7 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Obtains the correct constant term 32	1.1b	B1	$(2+kx)^5 = 32+80kx+80k^2x^2+...$
	Obtains $5\times16kx$ or $10\times8(kx)^2$ OE PI by $\frac{5k}{2}x$ or $\frac{5\times4}{2!}\left(\frac{kx}{2}\right)^2$	1.1a	M1	
	Obtains $32+80kx+80k^2x^2\ (+...)$ Accept list of correct terms. **No ISW** If more terms are given it must be obvious which are their first three terms.	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Forms the equation their $Ak = 4\times$their Bk^2 OE May recover if x is initially included.	3.1a	M1	$80k = 4\times80k^2$ $k = 0$ or $\frac{1}{4}$ $k = \frac{1}{4}$ Since $k > 0$
	Deduces $k = \frac{1}{4}$ only Or their $k =$ their $\frac{A}{4B}$ Justification of rejection k = 0 not required.	2.2a	A1F	
	Subtotal		**2**	

	Question 8 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Substitutes at least one small angle identity correctly into $\cos 4\theta + 2\sin 3\theta - \tan 2\theta$	1.1a	M1	$\cos 4\theta + 2\sin 3\theta - \tan 2\theta \approx 1 - \frac{(4\theta)^2}{2} + 2(3\theta) - (2\theta)$ $= 1 + 4\theta - 8\theta^2$
	Obtains a correct expression in terms of θ ACF	1.1b	A1	
	Completes argument to obtain $1 + 4\theta - 8\theta^2$	2.1	R1	
	Subtotal		**3**	

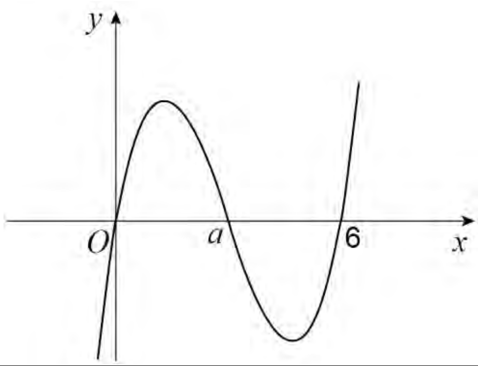
Q	Marking instructions	AO	Marks	Typical solution
9(b)	Substitutes $\theta = 0.07$ into their $1 + 4\theta - 8\theta^2$	3.1a	M1	$1 + 4 \times 0.07 - 8 \times 0.07^2 = 1.2408$ ≈ 1.241
	Obtains AWRT 1.241 CSO	1.1b	A1	
	Subtotal		**2**	

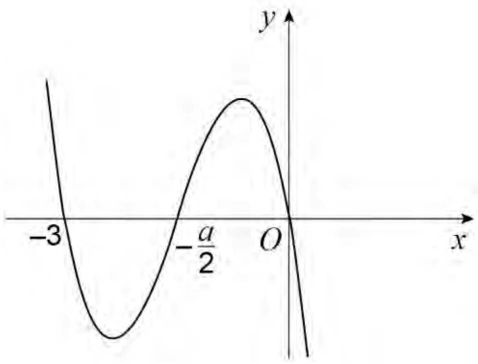
	Question 9 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Substitutes $n=300, a=-7$ and $l=32$ Into $S_n=\frac{n}{2}(a+l)$ Or Substitutes $n=300, a=-7$ and $d=\frac{39}{299}=\frac{3}{23}$ into $S_n=\frac{n}{2}(2a+(n-1)d)$ Condone n = 299 or 301 and $d=$AWRT 0.13	3.1a	M1	$S_{300}=\frac{300}{2}(-7+32)$ $=3750$
	Obtains 3750	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Forms an equation using $S_9=1260$ Might see $\frac{9}{2}(2a+8d)=1260 \Rightarrow a+4d=140$	3.4	M1	$\frac{9}{2}(a+l)=1260 \Rightarrow a+l=280$ $l=6a$ $7a=280$ $a=40, l=240$ Value of top prize = £240
	Forms an equation using the relationship between the highest and least values. eg $a+8d=6a$ or $l=6a$ OE Might see $l=\frac{1}{6}a$ which may indicate the candidate is correctly working from the highest term to the lowest term.	3.4	M1	
	Obtains and solves an equation in one variable having formed one equation using $S_9=1260$ OR used the relationship between the highest and least values.	3.1a	M1	
	Obtains £240 Must have correct units. CAO	3.2a	A1	
	Subtotal		**4**	

	Question 10 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Draws cubic graph with exactly two turning points	1.1a	M1	
	Draws cubic graph of correct orientation passing **through** the origin and positive x-axis at two points.	1.1b	A1	
	Draws fully correct sketch with x-axis intercepts correctly labelled a and 6. Ignore labelling on the y-axis.	1.1b	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Draws cubic graph of correct orientation passing **through** the origin and negative x-axis at two points. Or Substitutes $-2x$ into $f(x)$ Or Describes the reflection in the y-axis and a stretch of scale factor $\frac{1}{2}$ in the x-direction or a stretch scale factor $-\frac{1}{2}$ in the x-direction.	3.1a	M1	
	Draws fully correct sketch with x-axis intercepts correctly labelled $-\frac{a}{2}$ and -3.	2.2a	R1	
	Subtotal		**2**	

	Question 11 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Substitutes $u_1 = 3$ into $\frac{-6}{u_n}$ PI $u_2 = -2$	1.1a	M1	$u_2 = -2$ $u_3 = 3$ $u_4 = -2$
	Obtains $u_2 = -2, u_3 = 3, u_4 = -2$ Condone missing labels if order is obvious.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	States 2	2.2a	B1	2
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Shows that pairs of consecutive terms sum to 1 in a series Or Considers a sum of 3s and a sum of ± 2 s	3.1a	M1	$\sum_{n=1}^{101} u_n = \underbrace{\underbrace{3-2}_{=1}+3-2+\ldots+3-2}_{=50}+3$ $= 53$
	Deduces $\sum_{n=1}^{101} u_n = 53$	2.2a	R1	
	Subtotal		**2**	

	Question 12 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Substitutes $x=-\frac{1}{2}$ into $\mathrm{P}(x)$ and obtains zero. Must see $-\frac{1}{2}$ bracketed correctly. If bracket(s) missing must see a further step to indicate correct evaluation eg $-\frac{4}{8}+\frac{8}{4}-\frac{11}{2}+4=0$ or better.	1.1a	M1	$\mathrm{P}\left(-\frac{1}{2}\right)=4\left(-\frac{1}{2}\right)^3+8\left(-\frac{1}{2}\right)^2+11\left(-\frac{1}{2}\right)+4$ $=0$ $\therefore (2x+1)$ is a factor of $\mathrm{P}(x)$
	Completes factor theorem argument by showing $\mathrm{P}\left(-\frac{1}{2}\right)=0$ and stating $\therefore (2x+1)$ is a factor of $\mathrm{P}(x)$ OE	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Obtains two correct coefficients of $2x^2+3x+4$	1.1a	M1	$\mathrm{P}(x)=(2x+1)\left(2x^2+3x+4\right)$
	Obtains $(2x+1)\left(2x^2+3x+4\right)$	1.1b	A1	
	Subtotal		**2**	

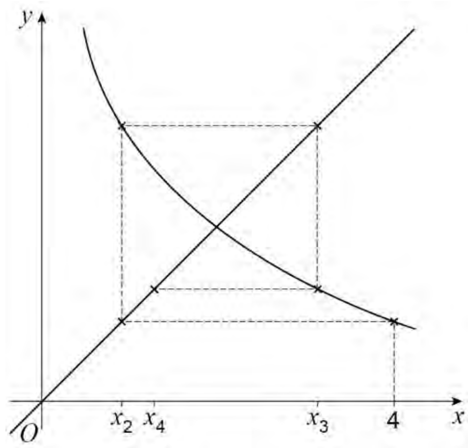
Q	Marking instructions	AO	Marks	Typical solution
13(c)	Begins argument by explaining that either $(2n+1)\neq 1$ Or $(an^2+bn+c)\neq 1$ Or $(2n+1)\neq$ the cubic expression Or $(an^2+bn+c)\neq$ the cubic expression Condone x instead of n	2.1	M1	$4n^3+8n^2+11n+4=(2n+1)(2n^2+3n+4)$ There are two factors. Both factors are integers not equal to 1 so $4n^3+8n^2+11n+4$ is never prime.
	States that Either **both** $(2n+1)\neq 1$ **and** their $(an^2+bn+c)\neq 1$ Or **both** $(2n+1)$ is not equal to $4n^3+8n^2+11n+4$ **and** their (an^2+bn+c) is not equal to $4n^3+8n^2+11n+4$ Or $(2n+1)\neq 1$ **and** $(2n+1)$ is not equal to $4n^3+8n^2+11n+4$ Or their $(an^2+bn+c)\neq 1$ **and** their (an^2+bn+c) is not equal to $4n^3+8n^2+11n+4$	2.2a	R1F	
	Subtotal		**2**	

	Question 13 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Rearranges the given equation to equal zero and evaluates their non-zero expression in the interval [0,4] at least once. **Must have equated to zero.**	1.1a	M1	$x^3 = e^{6-2x} \Rightarrow x^3 - e^{6-2x} = 0$ Let $f(x) = x^3 - e^{6-2x}$ $f(0) = -403 < 0$ $f(4) = 63.86 > 0$ Hence α lies between 0 and 4
	Completes argument with two **correct** evaluations of their correct expression in the interval [0,4] either side of the solution, with comparison to zero or a comment about change of sign. **AND** concludes that the solution α lies between 0 and 4 Evaluations must be correct to at least two significant figures rounded or truncated. Accept exact evaluation at x = 0.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Uses natural logs to correctly remove exponential.	1.1a	M1	$x^3 = e^{6-2x}$ $\ln x^3 = 6 - 2x$ $3\ln x = 6 - 2x$ $2x = 6 - 3\ln x$ $x = 3 - \frac{3}{2}\ln x$
	Uses log rule or cube roots to remove power of 3.	1.1a	M1	
	Completes reasoned argument to show $x = 3 - \frac{3}{2}\ln x$ AG	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)(i)	Obtains any correct value to at least 3 decimal places, ignoring labels.	1.1a	M1	$x_2 = 0.921$ $x_3 = 3.124$ $x_4 = 1.291$
	Obtains x_2, x_3 and x_4 correct to at least 3 decimal places If no labels only accept the three correct answers in the correct order with no extras seen beyond x_4 $x_2 = 0.92055...$ $x_3 = 3.12416...$ $x_4 = 1.29125...$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)(ii)	Draws correct cobweb diagram Condone missing vertical line at $x = 4$	1.1a	M1	
	Shows positions of x_2, x_3 and x_4 on the x-axis Accept correct values in place of x_n AWRT 0.92, 3.12 and 1.29 Do not accept labels on $y = x$ without indication on x-axis	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)(iii)	Explains that (it is not possible to evaluate x_2 as) ln 0 has no value or that y is undefined OE	2.4	E1	It is not possible to evaluate x_2 as ln 0 has no value
	Subtotal		**1**	

	Question 14 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Recalls the identity for $\sin 2\theta = 2\sin\theta\cos\theta$ or an identity for $\cos 2\theta$ eg $\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ This mark could be scored later if compound angle formula is used with a completely correct argument.	1.2	B1	$\sin 2\theta \operatorname{cosec}\theta + \cos 2\theta \sec\theta$ $= 2\sin\theta\cos\theta\operatorname{cosec}\theta + \left(2\cos^2\theta - 1\right)\sec\theta$ $= 2\sin\theta\cos\theta\operatorname{cosec}\theta + 2\cos^2\theta\sec\theta - \sec\theta$ $= 2\cos\theta + 2\cos\theta - \sec\theta$ $= 4\cos\theta - \sec\theta$
	Substitutes $A\sin\theta\cos\theta$ and a correct identity for $\cos 2\theta$ OR Substitutes $2\sin\theta\cos\theta$ and an identity for $\cos 2\theta$ with sign errors condoned provided $\cos 2\theta$ is not replaced with an expression equivalent to a constant.	3.1a	M1	
	Simplifies $B\sin\theta\operatorname{cosec}\theta$ to B *Or* $D\cos^2\theta\sec\theta$ to $D\cos\theta$	1.1a	M1	
	Completes reasoned argument to obtain $4\cos\theta - \sec\theta$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Explains that $\operatorname{cosec}\theta$ is undefined when $\cos\theta = 1$ Or explains $\cos\theta = 1$ would mean that $\sin\theta = 0$ or uses $\sin\theta \neq 0$ to show that $\cos\theta = 1$ should be rejected	2.4	E1	$\cos\theta \neq 1$ as $\operatorname{cosec}\theta$ is undefined
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	Obtains 104.5, 255.5 CAO	2.2a	B1	$\therefore \theta = 104.5^\circ, 255.5^\circ$
	Subtotal		**1**	

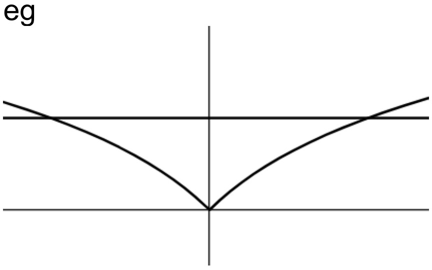
	Question 15 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
16	Uses the symmetry of the curve. Evidenced by doubling area from x = 0 to x = 2 Or considering the whole region from x = – 2 to x = 2	3.1a	M1	$\text{Area} \approx \frac{0.4}{2}(3+0+2[2.943+2.752+2.353+1.572])$ $= 4.448$ $\text{Total area} = 8.896$ $\text{Volume} = 8.896 \times 150$ $= 1334.4\,cm^3$ $\approx 1300\,cm^3$
	States or uses $h = 0.4$ OE Accept 0.2 as the multiplier. PI by 4.448 or 8.896 Accept use of h=0.8 or multiplier of 0.4 provided their answer is not then doubled.	2.2a	B1	
	Substitutes given y values or absolute y values to achieve $3+0+2[2.943+2.752+2.353+1.572]$ or $-3+0+2[-2.943-2.752-2.353-1.572]$ or $0+0+2\begin{bmatrix}2.943+2.752+2.353+1.572\\+3+2.943+2.752+2.353+1.572\end{bmatrix}$ or $0+0+2\begin{bmatrix}-2.943-2.752-2.353-1.572\\-3-2.943-2.752-2.353-1.572\end{bmatrix}$ Condone missing or misplaced zeros PI by ± 22.24 or ± 44.48	1.1a	M1	
	Obtains ± 8.896 or ± 4.448 Do not award this mark if they go on to obtain ± 17.792	1.1b	A1	
	Obtains AWRT 1300cm^3 Or AWRT 0.0013m^3 Must include units	3.2a	A1	
	Question 16 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)	Deduces correct region eg $\mathrm{f}(x) \geq 1$ or $y \geq 1$ Condone $\mathrm{f}(x) > 1$ or $y > 1$	2.2a	M1	$\{y : y \geq 1\}$
	Obtains correct answer in set notation eg $\{x : x \geq 1\}$ $\{\mathrm{f}(x) : \mathrm{f}(x) \geq 1\}$ $[1, \infty)$	2.5	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)	Obtains $\{x : x > 0\}$ or $(0, \infty)$ Accept $\{y : y > 0\}$ but not $y > 0$	1.1b	B1	$x > 0$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(c)(i)	Obtains $\ln(\lvert x\rvert+1)$ Accept $\ln\lvert\lvert x\rvert+1\rvert$ ISW	1.1b	B1	$\mathrm{h}(x)=\ln(\lvert x\rvert+1)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(c)(ii)	States that h does not have an inverse **And** States that h is not one-to-one or that h is many-to-one	2.2a	E1	
	Explains why h is not one-to-one or why h is many-to-one For example Gives two x values such that $\mathrm{h}(x_1)=\mathrm{h}(x_2)$ Or Explains that using the positive and negative of the same x-value will result in the same y-value Or Sketches a graph of y=h(x) with a horizontal line meeting the curve in two places. eg  Or States that f is many-to-one and that h is a function of f.	2.4	E1	The function h does not have an inverse as h is not one-to-one. For example $\mathrm{h}(1)=\ln 2$ and $\mathrm{h}(-1)=\ln 2$
	Subtotal		**2**	

	Question 17 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Selects the substitution $u = 2x+1$ and differentiates or uses it to replace $2x+1$ in the integrand.	3.1a	B1	Let $u = 2x+1$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 2 \Rightarrow dx = \frac{1}{2}du$ $4x+1 = 2u-1$ $\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x$ $= \int_1^9 (2u-1)(u)^{\frac{1}{2}}\frac{1}{2}\mathrm{d}u$ $= \frac{1}{2}\int_1^9 \left(2u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$
	Differentiates their substitution and uses the result to replace $\mathrm{d}x$ in the integral.	1.1a	M1	
	Makes a complete substitution to write the integrand in terms of u leading to an integrand of the form $A(2u-k)u^{\frac{1}{2}}$ Or FT their substitution $u = (2x+1)^{\frac{1}{2}}$ or $u^2 = 2x+1$ leading to an integrand of the form $A(2u^2-1)u^2$	3.1a	M1	
	Obtains correct lower limit for their substitution	1.1a	M1	
	Completes a reasoned argument to show the required result with $a = 1$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Integrates to obtain $\frac{1}{2}\frac{4u^{\frac{5}{2}}}{5}$ or $\frac{4u^{\frac{5}{2}}}{5}$ or $-\frac{1}{2}\frac{2u^{\frac{3}{2}}}{3}$ or $\frac{2u^{\frac{3}{2}}}{3}$	1.1a	M1	$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx$ $=\frac{1}{2}\int_1^9\left(2u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$ $=\frac{1}{2}\left[\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}\right]_1^9$ $=\frac{1}{2}\left[\left(\frac{4\times9^{\frac{5}{2}}}{5}-\frac{2\times9^{\frac{3}{2}}}{3}\right)-\left(\frac{4\times1^{\frac{5}{2}}}{5}-\frac{2\times1^{\frac{3}{2}}}{3}\right)\right]$ $=\frac{1322}{15}$
	Obtains $\frac{1}{2}\left(\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}\right)$ or $\frac{4u^{\frac{5}{2}}}{5}-\frac{2u^{\frac{3}{2}}}{3}$	1.1b	A1	
	Substitutes limits explicitly into their integrated expression of the form $Au^{\frac{5}{2}}-Bu^{\frac{3}{2}}$ Where A and B are both positive FT their non-zero a Condone omission of powers on substitution of 1	1.1a	M1	
	Completes argument to show the given result with no unrecovered slips.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Explains that increasing the number of rectangles will lead to an increased value so the (improved) approximation will be greater than A	2.4	E1	Since the area of the rectangles is an underestimate, increasing their number will give an increased total $> A$ No matter how many rectangles are used their total will be less than the exact area so $< \frac{1322}{15}$
	Gives a reason why the approximation will be an under-estimate so will be less than $\frac{1322}{15}$ AG	2.4	E1	
	Subtotal		**2**	

	Question 18 Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
19	Uses implicit differentiation, with $Ay^2\frac{dy}{dx}$ or $2\frac{dy}{dx}$ seen.	3.1a	M1	$y^3e^{2x}+2y-16x=k$ $3y^2e^{2x}\frac{dy}{dx}+2y^3e^{2x}+2\frac{dy}{dx}-16=0$ $\frac{dy}{dx}=0, x=0$ $2y^3-16=0$ $y=2$ $2^3e^0+4-16\times0=k$ $k=12$
	Uses product rule to differentiate y^3e^x and obtains $Ay^2e^{2x}\frac{dy}{dx}+By^3e^{2x}$	3.1a	M1	
	Obtains correctly $3y^2e^{2x}\frac{dy}{dx}+2y^3e^{2x}+2\frac{dy}{dx}-16=0$	1.1b	A1	
	Substitutes $\frac{dy}{dx}=0, x=0$ into their differentiated equation or rearranged equation to obtain a value for y. Their equation needs to have contained either $Ay^2\frac{dy}{dx}$ or $2\frac{dy}{dx}$ and involve e^{2x}	3.1a	M1	
	Obtains y = 2 Must have achieved M1M1A1M1 so far. PI substituting $y=\frac{2}{e^{\frac{2x}{3}}}$ and x=0 into $y^3e^{2x}+2y-16x$	1.1b	A1	
	Substitutes $x=0$ and their y = 2 into $y^3e^{2x}+2y-16x$ to obtain a value of k	3.1a	M1	
	Deduces k =12 Must have achieved all previous marks	2.2a	R1	
	Question 19 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
20(a)	Models the rate of change of depth using an equation of the form $\frac{dh}{dt} = \pm k(h-5)$	3.3	M1	$\frac{dh}{dt} = -k(h-5)$ when $h = 130$, $\frac{dh}{dt} = -1.5$ $\Rightarrow -1.5 = -k \times 125$ $k = 0.012$ $\frac{dh}{dt} = -0.012(h-5)$
	Substitutes $h = 130$ and $\frac{dh}{dt} = \pm 1.5$ into $\frac{dh}{dt} = \pm k(h-5)$	3.1b	M1	
	Completes argument with no sign slips to show $\frac{dh}{dt} = -0.012(h-5)$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
20(b)	Separates variables to obtain an equation of the form $\int \frac{A}{h-5}\, dh = \int B\, dt$ PI by $\ln(h-5) = -0.012t$ OE	3.1a	M1	$\frac{1}{h-5}\frac{dh}{dt} = -0.012$ $\int \frac{1}{h-5}\, dh = \int -0.012\, dt$ $\ln(h-5) = -0.012t + c$ $h - 5 = Ae^{-0.012t}$ $t = 0, h = 130 \Rightarrow A = 125$ $h = 5 + 125e^{-0.012t}$
	Integrates one of their integrals of the form $\int \frac{A}{h-5}\, dh$ or $\int B\, dt$ correctly. PI by $\ln(h-5) = -0.012t$ OE	1.1a	M1	
	Obtains correct integrated equation. Condone missing $+c$	1.1b	A1	
	Uses $t = 0, h = 130$ to obtain their constant of integration.	3.1b	M1	
	Obtains $5 + 125e^{-0.012t}$ OE Accept $5 + e^{-0.012t+p}$ where p =ln125 or AWRT 4.83	3.3	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
20(c)	Uses h = 65 in their answer from part (b) and obtains a final positive value	3.4	M1	$5+125\mathrm{e}^{-0.012t}=65$ $t=61.164$ 61 minutes
	Obtains AWRT 61 minutes Accept 62 minutes Must have correct units	3.2a	A1	
	Subtotal		**2**	

	Question 20 Total		**10**	

	Question Paper Total		**100**	