MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

A-level
MATHEMATICS
Paper 1

Wednesday 6 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 $y = \frac{1}{x^2}$

Find an expression for $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = \frac{0}{2x}$ $\qquad$ $\frac{dy}{dx} = x^{-2}$ $\qquad$ $\frac{dy}{dx} = -\frac{2}{x}$ $\qquad$ $\frac{dy}{dx} = -\frac{2}{x^3}$

2 The graph of $y = 5^x$ is transformed by a stretch in the y-direction, scale factor 5

State the equation of the transformed graph.

Circle your answer.

[1 mark]

$y = 5 \times 5^x$ $\qquad$ $y = 5^{\frac{x}{5}}$ $\qquad$ $y = \frac{1}{5} \times 5^x$ $\qquad$ $y = 5^{5x}$

Do not write outside the box

3 A periodic sequence is defined by $U_n = \sin\left(\frac{n\pi}{2}\right)$

State the period of this sequence.

Circle your answer.

[1 mark]

8 $\quad$ 2π $\quad$ 4 $\quad$ π

Period $= \dfrac{2\pi}{(\pi/2)} = 4$

4 The function f is defined by $f(x) = e^{x-4}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

[3 marks]

Let $x = e^{y-4}$

$\ln x = y - 4$

$y = \ln x + 4$

$f^{-1}(x) = \ln x + 4 \quad , \quad x > 0$

Turn over for the next question

Do not write outside the box

5 A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^{t} - 5$$

5 (a) Show that $\dfrac{dy}{dx} = -\dfrac{3}{4} \times 2^{2t}$

[3 marks]

$x = 4 \times 2^{-t} + 3 = 4\left(\frac{1}{2}\right)^t + 3$

$y = 3 \times 2^t - 5$

$\frac{dx}{dt} = 4\left(\frac{1}{2}\right)^t \ln\left(\frac{1}{2}\right) = -\ln 2 \times 4 \times 2^{-t}$

$\frac{dy}{dt} = 3 \times 2^t \times \ln 2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3 \times 2^t \times \ln 2}{-\ln 2 \times 4 \times 2^{-t}} = -\frac{3}{4} \times 2^{2t}$

5 (b) Find the Cartesian equation of the curve in the form $xy + ax + by = c$, where a, b and c are integers.

[3 marks]

$x = 4 \times 2^{-t} + 3 \Rightarrow \frac{x-3}{4} = 2^{-t} \Rightarrow 2^t = \frac{4}{x-3}$

$y = 3 \times 2^t - 5$

$y = 3 \times \left(\frac{4}{x-3}\right) - 5$

$y = \frac{12}{x-3} - 5 \Rightarrow y(x-3) = 12 - 5(x-3)$

$xy - 3y = 12 - 5x + 15$

$xy + 5x - 3y = 27$

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6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of $\frac{1}{\sqrt{4+x}}$

[3 marks]

$\frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}} = \left(4\left(1+\frac{1}{4}x\right)\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left(1+\frac{1}{4}x\right)^{-\frac{1}{2}}$

$\frac{1}{2}\left(1+\frac{1}{4}x\right)^{-\frac{1}{2}} \approx \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(\frac{1}{4}x\right) + \frac{(-1/2)(-3/2)}{2}\left(\frac{1}{4}x\right)^2\right]$

$= \frac{1}{2}\left[1 - \frac{1}{8}x + \frac{3}{128}x^2\right]$

$= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2$

6 (b) Hence, find the first three terms of the binomial expansion of $\frac{1}{\sqrt{4-x^3}}$

[2 marks]

Replace x with $-x^3$:

$\frac{1}{\sqrt{4-x^3}} \approx \frac{1}{2} - \frac{1}{16}(-x^3) + \frac{3}{256}(-x^3)^2$

$= \frac{1}{2} + \frac{1}{16}x^3 + \frac{3}{256}x^6$

Question 6 continues on the next page

Turn over ▶

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6 (c) Using your answer to part **(b)**, find an approximation for $\int_0^1 \frac{1}{\sqrt{4-x^3}}\,\mathrm{d}x$, giving your answer to seven decimal places.

[3 marks]

$$\int_0^1 \frac{1}{\sqrt{4-x^3}}\,dx \approx \int_0^1 \frac{1}{2} + \frac{1}{16}x^3 + \frac{3}{256}x^6 \,dx$$

$$= \left[\frac{1}{2}x + \frac{1}{64}x^4 + \frac{3}{1792}x^7\right]_0^1$$

$$= \frac{1}{2} + \frac{1}{64} + \frac{3}{1792}$$

$$= 0.5172991$$

6 (d) (i) Edward, a student, decides to use this method to find a more accurate value for the integral by increasing the number of terms of the binomial expansion used.

Explain clearly whether Edward's approximation will be an overestimate, an underestimate, or if it is impossible to tell.

[2 marks]

Each term of the expansion is positive so adding more terms will increase the estimated value. This means the approximation will be an underestimate because there will always be more positive terms you could add.

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6 (d) (ii) Edward goes on to use the expansion from part **(b)** to find an approximation for $\int_{-2}^{0} \frac{1}{\sqrt{4-x^3}}\,dx$

Explain why Edward's approximation is invalid.

[2 marks]

The expansion is valid for : $\left|\frac{1}{4}x^3\right| < 1$

$|x^3| < 4$

$|x| < \sqrt[3]{4}$

$-\sqrt[3]{4} < x < \sqrt[3]{4}$

-2 does not satisfy this so is invalid.

Turn over for the next question

Do not write outside the box

7 Three points A, B and C have coordinates A (8, 17), B (15, 10) and C (−2, −7)

7 (a) Show that angle ABC is a right angle.

[3 marks]

$AB^2 = (15-8)^2 + (10-17)^2 = 7^2 + 7^2 = 98$

$AC^2 = (8--2)^2 + (17--7)^2 = 10^2 + 24^2 = 676$

$BC^2 = (15--2)^2 + (10--7)^2 = 17^2 + 17^2 = 578$

If it is a right angled triangle then : $AB^2 + CB^2 = AC^2$

$98 + 578 = 676$

$676 = 676$

Since $AB^2 + CB^2 = AC^2$ is satisfied, we have a right angled triangle.

7 (b) A, B and C lie on a circle.

7 (b) (i) Explain why AC is a diameter of the circle.

[1 mark]

The angle subtended by the diameter is 90°, so AC must be the diameter.

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7 (b) (ii) Determine whether the point D $(-8, -2)$ lies inside the circle, on the circle or outside the circle.

Fully justify your answer.

[4 marks]

Length of radius $= \frac{AC}{2} = \frac{\sqrt{676}}{2} = 13$

Centre $= P = \left(\frac{8-2}{2}, \frac{17-7}{2}\right) = (3, 5)$

So, if D lies on the circle we would need the distance from D to P to be 13.

$DP = \sqrt{(3--8)^2 + (5--2)^2} = \sqrt{11^2 + 7^2} = \sqrt{170}$

$\sqrt{170} \neq 13$

$\sqrt{170} > 13$ so D lies outside the circle.

Turn over for the next question

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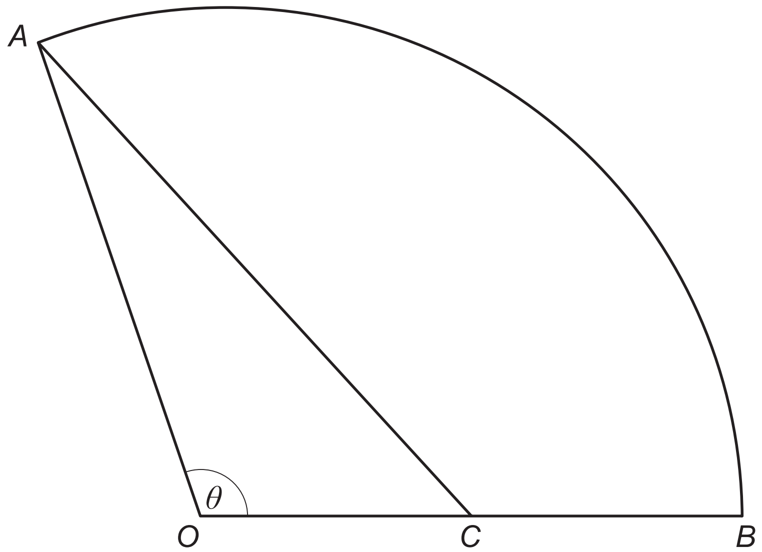
8 The diagram shows a sector of a circle *OAB*.

C is the midpoint of *OB*.

Angle *AOB* is θ radians.



8 (a) Given that the area of the triangle *OAC* is equal to one quarter of the area of the sector *OAB*, show that $\theta = 2\sin\theta$

[4 marks]

Area of sector $= \frac{1}{2}r^2\theta$

Area of triangle OCA $= \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}r\left(\frac{r}{2}\right)\sin\theta$

$= \frac{r^2}{4}\sin\theta$

$\frac{1}{2}r^2\theta \times \frac{1}{4} = \frac{r^2}{4}\sin\theta$

$\frac{1}{2}\theta = \sin\theta$

$\theta = 2\sin\theta$

8 (b) Use the Newton-Raphson method with $\theta_1 = \pi$, to find θ_3 as an approximation for θ.
Give your answer correct to five decimal places.

[3 marks]

Let $f(\theta) = 2\sin\theta - \theta$

Then $f'(\theta) = 2\cos\theta - 1$

$\theta_{n+1} = \theta_n - \dfrac{2\sin\theta_n - \theta_n}{2\cos\theta_n - 1}$

$\theta_1 = \pi \qquad \theta_2 = \pi - \dfrac{2\sin\pi - \pi}{2\cos\pi - 1} = \pi - \dfrac{-\pi}{-2-1} = \dfrac{2}{3}\pi = 2.0944\ldots$

$\theta_3 = 2.0944 - \dfrac{2\sin(2.0944) - 2.0944}{2\cos(2.0944) - 1}$

$= 1.91322\ldots = 1.91322 \quad (5.d.p)$

8 (c) Given that $\theta = 1.89549$ to five decimal places, find an estimate for the percentage error in the approximation found in part **(b)**.

[1 mark]

$\dfrac{1.91322 - 1.89549}{1.89549} \times 100 = 0.935\%$

Turn over for the next question

Do not write outside the box

9 An arithmetic sequence has first term a and common difference d.

The sum of the first 36 terms of the sequence is equal to the square of the sum of the first 6 terms.

9 (a) Show that $4a + 70d = 4a^2 + 20ad + 25d^2$

[4 marks]

$S_6 = 3(2a + 5d)$

$= 6a + 15d$

$S_{36} = 18(2a + 35d)$

$= 36a + 630d$

$36a + 630d = (6a + 15d)^2$

$36a + 630d = 36a^2 + 90ad + 90ad + 225d^2$

$36a + 630d = 36a^2 + 180ad + 225d^2$

$4a + 70d = 4a^2 + 20ad + 25d^2$

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9 (b) Given that the sixth term of the sequence is 25, find the smallest possible value of a.

[5 marks]

6^{th} term: $a + 5d = 25$

$a = 25 - 5d$

Substitute this into the equation from (a):

$4(25-5d) + 70d = 4(25-5d)^2 + 20d(25-5d) + 25d^2$

$100 - 20d + 70d = 4(625 - 250d + 25d^2) + 500d - 100d^2 + 25d^2$

$100 + 50d = 2500 - 1000d + 100d^2 + 500d - 100d^2 + 25d^2$

$0 = 2400 - 550d + 25d^2$

$0 = 96 - 22d + d^2$

$0 = (d-6)(d-16)$

So, $d=6$ or $d=16$.

If $d=6$, $a = 25 - 5(6) = -5$.

If $d=16$, $a = 25 - 5(16) = -55$.

So, the smallest value of a is -55.

Turn over for the next question

Do not write outside the box

10 A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using

$$m = m_0 e^{-kt}$$

where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

On average, it takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

10 (a) The scientist drinks two strong cups of coffee at 8 am. Use the model to estimate the mass of caffeine in the scientist's body at midday.

[4 marks]

At $t = 5.7$, $m = \frac{m_0}{2}$: $\frac{m_0}{2} = m_0 e^{-k(5.7)}$

$\frac{1}{2} = e^{-k(5.7)}$

$\ln(\frac{1}{2}) = -5.7k$

$k = -\frac{\ln 2}{-5.7} \Rightarrow k = 0.1216$

At $t=4$, when $m_0 = 200 \times 2 = 400$: $m = 400e^{-0.1216(4)}$

$m = 245.93...$

$m = 246$

Do not write outside the box

10 (b) The scientist wants the mass of caffeine in her body to stay below 480 mg

Use the model to find the earliest time that she could drink another cup of strong coffee.

Give your answer to the nearest minute.

[3 marks]

For her level to be below 480 she needs the amount from her morning coffee to be less than 280, because then drinking another cup will still keep it below 480:

$400e^{-0.1216t} \leq 280 \Rightarrow e^{-0.1216t} \leq 0.7$

$\Rightarrow -0.1216t \leq \ln(0.7) \Rightarrow t \geq \frac{\ln 0.7}{-0.1216} \Rightarrow t \geq 2.933$

So, she needs to drink coffee at least 2.933 hours after.
This is the same as 2 hours plus $0.933 \times 60 = 55.98 = 56$ minutes.
So, 2 hours 56 minutes after 8:00 am is 10:56 am.

10 (c) State a reason why the mass of caffeine remaining in the scientist's body predicted by the model may not be accurate.

[1 mark]

It will be different for different people. We have based the model on the average person but everybody has different rates.

Turn over for the next question

Do not write outside the box

11 The daily world production of oil can be modelled using

$$V = 10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4$$

where V is volume of oil in millions of barrels, and t is time in years since 1 January 1980.

11 (a) (i) The model is used to predict the time, T, when oil production will fall to zero.

Show that T satisfies the equation

$$T = \sqrt[3]{60T^2 + \frac{162\,000}{T}}$$

[3 marks]

$0 = 10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4$

$0 = 10 + 100\left(\frac{t^3}{27000}\right) - 50\left(\frac{t^4}{810000}\right)$

$0 = 10 + \frac{t^3}{270} - \frac{t^4}{16200}$

$0 = 162000 + 60t^3 - t^4$

$0 = \frac{162000}{t} + 60t^2 - t^3$

$t^3 = \frac{162000}{t} + 60t^2$

So T satisfies $T = \sqrt[3]{60T^2 + \frac{162000}{T}}$

11 (a) (ii) Use the iterative formula $T_{n+1} = \sqrt[3]{60{T_n}^2 + \frac{162\,000}{T_n}}$, with $T_0 = 38$, to find the values of T_1, T_2, and T_3, giving your answers to three decimal places.

[2 marks]

$T_0 = 38$

$T_1 = \sqrt[3]{60(38)^2 + \frac{162000}{38}} = 44.963$

$T_2 = \sqrt[3]{60(44.963)^2 + \frac{162000}{44.963}} = 49.987$

$T_3 = \sqrt[3]{60(49.987)^2 + \frac{162000}{49.987}} = 53.504$

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11 (a) (iii) Explain the relevance of using $T_0 = 38$

[1 mark]

38 represents 38 years after 1980, so 2018.

11 (b) From 1 January 1980 the daily use of oil by one technologically developing country can be modelled as

$$V = 4.5 \times 1.063^t$$

Use the models to show that the country's use of oil and the world production of oil will be equal during the year 2029.

[4 marks]

Set the two equations equal to each other and solve for t.

If you put $t = 49$ into them both you get

$V = 10 + 100\left(\frac{49}{30}\right)^3 - 50\left(\frac{49}{30}\right)^4 = 89.885$

and $V = 4.5 \times 1.063^{49} = 89.8137...$

These values are approximately equal.

$t = 49$ represents the year $1980 + 49 = 2029$.

Turn over for the next question

12 $p(x) = 30x^3 - 7x^2 - 7x + 2$

12 (a) Prove that $(2x + 1)$ is a factor of $p(x)$

[2 marks]

$p(x) = 30x^3 - 7x^2 - 7x + 2$

If $(2x+1)$ is a factor then $p(-\frac{1}{2}) = 0$:

$p(-\frac{1}{2}) = 30(-\frac{1}{2})^3 - 7(-\frac{1}{2})^2 - 7(-\frac{1}{2}) + 2$

$= -\frac{30}{8} - \frac{7}{4} + \frac{7}{2} + 2 = 0.$

Hence, $(2x+1)$ is a factor.

12 (b) Factorise $p(x)$ completely.

[3 marks]

Divide $p(x)$ by $(2x+1)$:

$$
\begin{array}{r l}
 & 15x^2 - 11x + 2 \\
2x+1 & \overline{\big)\, 30x^3 - 7x^2 - 7x + 2} \\
 & -\underline{(30x^3 + 15x^2)} \\
 & 0 \quad -22x^2 - 7x + 2 \\
 & \quad -\underline{(-22x^2 - 11x)} \\
 & \qquad 4x + 2 \\
 & \qquad -\underline{(4x + 2)} \\
 & \qquad\quad 0
\end{array}
$$

So we get $p(x) = (2x+1)(15x^2 - 11x + 2)$

$= (2x+1)(5x-2)(3x-1)$

Do not write outside the box

12 (c) Prove that there are no real solutions to the equation

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

$30\sec^2 x + 2\cos x = 7\sec x + 7$

$\frac{30\sec^2 x}{\cos x} + 2 = \frac{7\sec x}{\cos x} + \frac{7}{\cos x}$

$30\sec^3 x + 2 = 7\sec^2 x + 7\sec x$

$30\sec^3 x - 7\sec^2 x - 7\sec x + 2 = 0$

$(2\sec x + 1)(5\sec x - 2)(3\sec x - 1) = 0$

So, $\sec x = -\frac{1}{2}, \frac{2}{5}$ or $\frac{1}{3}$

The range of sec is all numbers apart from those between -1 and 1. The three possibilities all lie between -1 and 1 so none of them are valid.

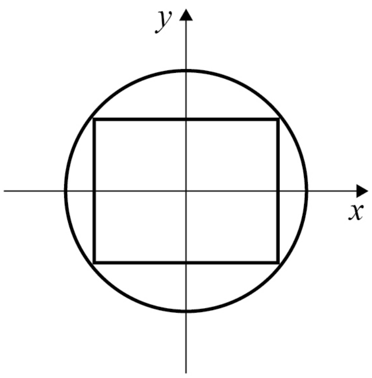
Hence, the equation has no solutions.

Turn over for the next question

Do not write outside the box

13 A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

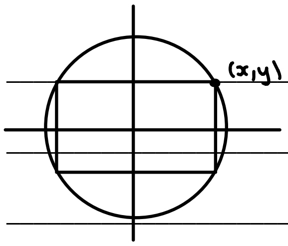
The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

[10 marks]



Height of rectangle $= 2y$

Length of rectangle $= 2x$

x and y satisfy $x^2 + y^2 = 16$ because the point (x,y) lies on the circle.

Area $= A = (2x)(2y) = 4xy = 4x\sqrt{16-x^2}$

$A = 4x\sqrt{16-x^2}$

We want to maximise A so want to find when $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 4x\left(\frac{1}{2}\right)(-2x)(16-x^2) + 4\sqrt{16-x^2}$

$\frac{dA}{dx} = \frac{-4x^2}{\sqrt{16-x^2}} + 4\sqrt{16-x^2}$

$0 = \frac{-4x^2 + 4(16-x^2)}{\sqrt{16-x^2}} \Rightarrow 0 = -4x^2 + 64 - 4x^2$

$\Rightarrow 8x^2 = 64 \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$.

When $x = 2\sqrt{2}$, $A = 4(2\sqrt{2})\sqrt{16-8} = 4(2\sqrt{2})(2\sqrt{2}) = 32$.

To check that this is a maximum we need to find $\frac{d^2A}{dx^2}$:

Do not write outside the box

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{64-8x^2}{\sqrt{16-x^2}}\right) = \left(\frac{(16-x^2)^{\frac{1}{2}}(-16x) - (64-8x^2)(\frac{1}{2})(-2x)(16-x^2)^{-\frac{1}{2}}}{16-x^2}\right)$$

$$= \frac{-16x\sqrt{16-x^2} + 8x(8-x^2)(16-x^2)^{-\frac{1}{2}}}{16-x^2}$$

At $x = 2\sqrt{2}$: $\frac{d^2A}{dx^2} = \frac{-16(2\sqrt{2})\sqrt{16-8} + 8(2\sqrt{2})(8-8)(16-8)^{-1/2}}{16-8}$

$$= \frac{-16(2\sqrt{2})(2\sqrt{2})}{8} = \frac{-16(8)}{8} = -16.$$

Since $-16 < 0$ it is a maximum. Therefore the maximum area is 32.

Turn over for the next question

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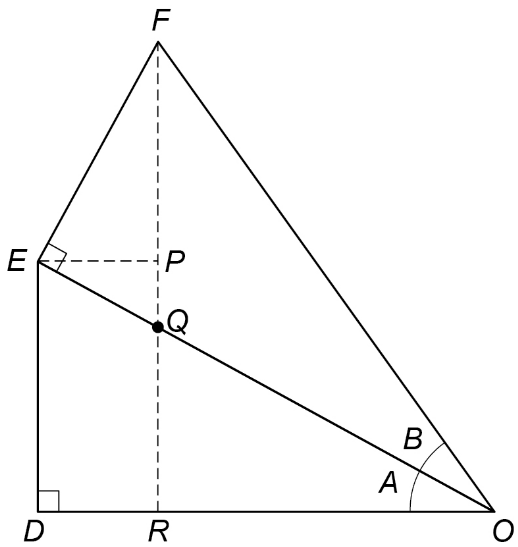
14 Some students are trying to prove an identity for $\sin(A+B)$.

They start by drawing two right-angled triangles *ODE* and *OEF*, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle *OFR*,

Line 1 $\sin(A+B) = \dfrac{RF}{OF}$

Line 2 $= \dfrac{RP+PF}{OF}$

Line 3 $= \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $= \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= \text{..............} + \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

$\angle EQF$ and $\angle OQR$ are opposite angles so are equal.

This means that $\triangle EFQ$ and $\triangle OQR$ are similar triangles, so

$\angle EFQ = \angle ROQ = A$. Since $\angle EFQ = A$, $\dfrac{PF}{EF} = \cos A$.

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From ΔOEF you can see $\frac{EF}{OF} = \sin B$.

So, $\frac{PF}{EF} \times \frac{EF}{OF} = \cos A \sin B$.

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $= \frac{DE}{OE} \times \frac{OE}{OF} + \frac{PF}{EF} \times \frac{EF}{OF}$

Line 5 $=$ $\sin A \cos B$ $+ \cos A \sin B$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when *A* and *B* are acute angles.

[1 mark]

By using right angled triangles in the proof we assumed A and B are acute. Therefore, the proof will only work for acute angles.

14 (d) Another student claims that by replacing *B* with $-B$ in the identity for $\sin(A + B)$ it is possible to find an identity for $\sin(A - B)$.

Assuming the identity for $\sin(A + B)$ is correct for all values of *A* and *B*, prove a similar result for $\sin(A - B)$.

[3 marks]

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(A+(-B)) = \sin A \cos(-B) + \sin(-B)\cos A$

Using $\sin(-\alpha) = -\sin\alpha$ and $\cos(-\alpha) = \cos\alpha$,

$\sin(A-B) = \sin(A+(-B)) = \sin A \cos B - \sin B \cos A$.

Turn over ▶

Do not write outside the box

15 A curve has equation $y = x^3 - 48x$

The point *A* on the curve has x coordinate -4

The point *B* on the curve has x coordinate $-4 + h$

15 (a) Show that the gradient of the line *AB* is $h^2 - 12h$

[4 marks]

$y = x^3 - 48x$

When $x = -4$, $y = (-4)^3 - 48(-4)$

$= -64 + 192$

$= 128$

When $x = -4+h$, $y = (-4+h)^3 - 48(-4+h)$

$y = (-4+h)(16 - 8h + h^2) + 192 - 48h$

$y = -64 + 32h - 4h^2 + 16h - 8h^2 + h^3 + 192 - 48h$

$y = h^3 - 12h^2 + 128$

Gradient: $\dfrac{h^3 - 12h^2 + 128 - 128}{-4 + h - (-4)} = \dfrac{h^3 - 12h^2}{h}$

$= h^2 - 12h$

15 (b) Explain how the result of part **(a)** can be used to show that *A* is a stationary point on the curve.

[2 marks]

The gradient of the curve at A is equal to

$\lim\limits_{h \to 0} h^2 - 12h = 0$

so the gradient is zero, meaning it is a stationary point.

END OF QUESTIONS

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