MODEL ANSWERS

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

AS MATHEMATICS

Paper 2

Exam Date   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 $\mathrm{p}(x) = x^3 - 5x^2 + 3x + a$, where a is a constant.

Given that $x - 3$ is a factor of $\mathrm{p}(x)$, find the value of a

Circle your answer.

[1 mark]

–9 –3 3 9

If $x-3$ is a factor, $p(3)=0 \Rightarrow p(3) = 3^3 - 5(3)^2 + 3(3) + a$

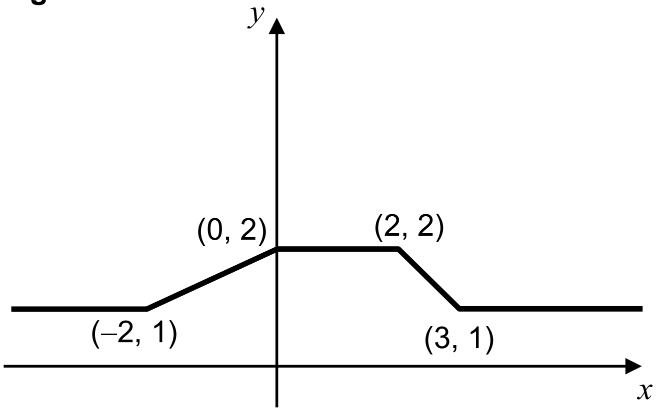
$0 = 27 - 45 + 9 + a$

$0 = -9 + a$

$a = 9$

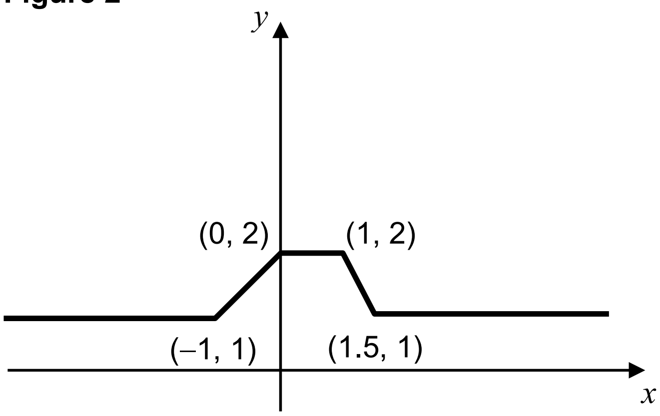
2 The graph of $y = \mathrm{f}(x)$ is shown in **Figure 1**.

Figure 1



State the equation of the graph shown in **Figure 2**.

Figure 2



Circle your answer.

[1 mark]

$y = \mathrm{f}(2x)$ $\qquad y = \mathrm{f}\left(\frac{x}{2}\right)$ $\qquad y = 2\mathrm{f}(x)$ $\qquad y = \frac{1}{2}\mathrm{f}(x)$

(The x-coordinates have been halved)

3 Find the value of $\log_a\left(a^3\right)+\log_a\left(\frac{1}{a}\right)$

[2 marks]

$\log_a a^3 + \log_a\left(\frac{1}{a}\right)$

$= \log_a a^3 + \log_a a^{-1}$

$= 3\log_a a - \log_a a$

$= 2\log_a a$ $\qquad \log_a a = 1$

$= 2$

4 Find the coordinates, in terms of a, of the minimum point on the curve $y = x^2 - 5x + a$, where a is a constant.

Fully justify your answer.

[3 marks]

$y = x^2 - 5x + a$

$\frac{dy}{dx} = 2x - 5$

Minimum point is when $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = 0 \Rightarrow 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

When $x = \frac{5}{2}$, $y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + a$

$y = \frac{25}{4} - \frac{25}{2} + a$

$y = a - \frac{25}{4}$

Coordinates are $\left(\frac{5}{2}, a - \frac{25}{4}\right)$

5 The quadratic equation $3x^2 + 4x + (2k - 1) = 0$ has real and distinct roots.

Find the possible values of the constant k

Fully justify your answer.

[4 marks]

If the equation has two real distinct roots, the discriminant is greater than zero:

$$b^2 - 4ac > 0$$

$$4^2 - 4(3)(2k-1) > 0$$

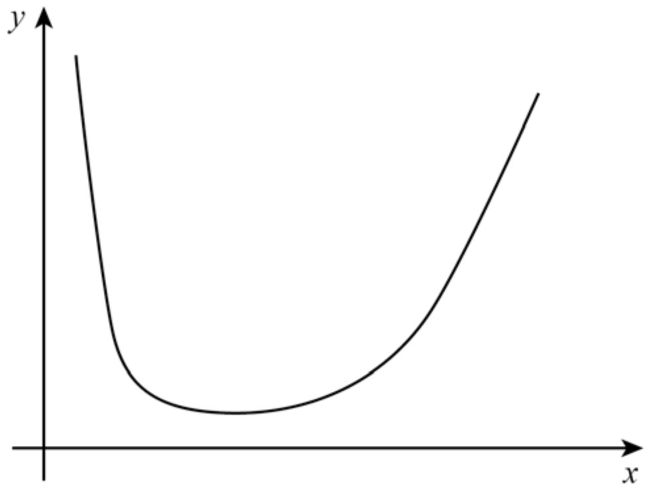
$$16 - 24k + 12 > 0$$

$$24k < 28$$

$$k < \frac{28}{24} = \frac{7}{6}$$

So, $k < \frac{7}{6}$.

6 A curve has equation $y = 6x^2 + \frac{8}{x^2}$ and is sketched below for $x > 0$



Find the area of the region bounded by the curve, the x-axis and the lines $x = a$ and $x = 2a$, where $a > 0$, giving your answer in terms of a

[4 marks]

$$\int_a^{2a} 6x^2 + \frac{8}{x^2}\,dx = \int_a^{2a} 6x^2 + 8x^{-2}\,dx$$

$$= \left[2x^3 - 8x^{-1}\right]_a^{2a}$$

$$= \left[2(2a)^3 - 8(2a)^{-1}\right] - \left[2(a)^3 - 8(a)^{-1}\right]$$

$$= 16a^3 - \frac{4}{a} - 2a^3 + \frac{8}{a}$$

$$= 16a^3 + \frac{4}{a}$$

7 Solve the equation

$$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta \quad \text{where} \quad \cos\theta \neq 0$$

Give **all** values of θ to the nearest degree in the interval $0° < \theta < 180°$

Fully justify your answer.

[5 marks]

$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta$

$\frac{\sin\theta\tan\theta}{\cos\theta} + \frac{2\sin\theta}{\cos\theta} = 3$

$\tan^2\theta + 2\tan\theta = 3$

$\tan^2\theta + 2\tan\theta - 3 = 0$

$(\tan\theta + 3)(\tan\theta - 1) = 0$

So, either $\tan\theta + 3 = 0 \Rightarrow \tan\theta = -3$

or $\tan\theta - 1 = 0 \Rightarrow \tan\theta = 1$

If $\tan\theta = -3$, $\theta = 108°$

If $\tan\theta = 1$, $\theta = 45°$

8 Prove that the function $\mathrm{f}(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

$f(x) = x^3 - 3x^2 + 15x - 1$

$f'(x) = 3x^2 - 6x + 15$

$= 3(x-1)^2 + 12$

$f'(x) > 0$ for all values of x, therefore $f(x)$ is an increasing function.

9 A curve has equation $y = e^{2x}$

Find the coordinates of the point on the curve where the gradient of the curve is $\frac{1}{2}$

Give your answer in an exact form.

[5 marks]

$y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$

When the gradient is $\frac{1}{2}$, $\frac{dy}{dx} = \frac{1}{2}$:

$2e^{2x} = \frac{1}{2}$

$e^{2x} = \frac{1}{4}$

$2x = \ln\left(\frac{1}{4}\right)$

$2x = \ln(2^{-2})$

$x = \frac{1}{2}\ln(2^{-2})$

$x = -\ln 2$

When $x = -\ln 2$, $y = e^{-2\ln 2} = e^{\ln\left(\frac{1}{4}\right)} = \frac{1}{4}$,

so the coordinates are $\left(-\ln 2, \frac{1}{4}\right)$.

10 David has been investigating the population of rabbits on an island during a three-year period.

Based on data that he has collected, David decides to model the population of rabbits, R, by the formula

$$R = 50e^{0.5t}$$

where t is the time in years after 1 January 2016.

10 (a) Using David's model:

10 (a) (i) state the population of rabbits on the island on 1 January 2016;

[1 mark]

When $t=0$, $R = 50e^{0} = 50$

10 (a) (ii) predict the population of rabbits on 1 January 2021.

[1 mark]

2021 is 5 years after 2016 so $t=5$:

When $t=5$, $R = 50e^{0.5(5)} = 609.12\ldots = 609$ (3. s.f)

10 (b) Use David's model to find the value of t when $R = 150$, giving your answer to three significant figures.

[2 marks]

$150 = 50e^{0.5t}$

$3 = e^{0.5t}$

$\ln 3 = 0.5t$

$t = 2\ln 3$

$t = 2.20$ to 3 significant figures

10 (c) Give **one** reason why David's model may **not** be appropriate.

[1 mark]

The number of rabbits will continue to increase so will go to infinity, this is unrealistic.

10 (d) On the same island, the population of crickets, C, can be modelled by the formula

$$C = 1000e^{0.1t}$$

where t is the time in years after 1 January 2016.

Using the two models, find the year during which the population of rabbits first exceeds the population of crickets.

[3 marks]

The populations will be equal when

$$50e^{0.5t} = 1000e^{0.1t}$$

$$e^{0.4t} = 20$$

$$0.4t = \ln 20$$

$$t = \frac{\ln 20}{0.4} = 7.49$$

So $t = 7.49$ is in the year 2023.

11 The circle with equation $(x-7)^2 + (y+2)^2 = 5$ has centre C.

11 (a) (i) Write down the radius of the circle.

[1 mark]

$\sqrt{5}$

11 (a) (ii) Write down the coordinates of C.

[1 mark]

$(7, -2)$

11 (b) The point P (5, –1) lies on the circle.

Find the equation of the tangent to the circle at P, giving your answer in the form $y = mx + c$

[4 marks]

Gradient of the line CP is : $\frac{-1-(-2)}{5-7} = -\frac{1}{2}$

The line CP is perpendicular to the tangent at P. So the tangent at P has gradient 2.

The equation of the tangent is :

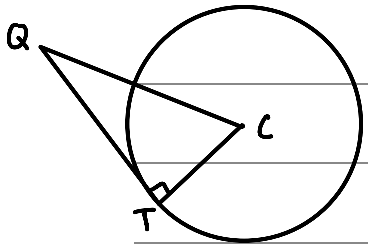
$y-(-1) = 2(x-5)$

$y+1 = 2x-10$

$y = 2x-11$

11 (c) The point Q (3, 3) lies outside the circle and the point T lies on the circle such that QT is a tangent to the circle. Find the length of QT.

[4 marks]



QTC is a right-angled triangle

$QC = \sqrt{(7-3)^2 + (-2-3)^2}$

$= \sqrt{4^2 + 5^2}$

$\therefore \quad QC^2 = QT^2 + TC^2$

$4^2 + 5^2 = QT^2 + (\sqrt{5})^2$

$16 + 25 = QT^2 + 5$

$QT^2 = 36$

$QT = 6$

12 (a) Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12

[3 marks]

$9n^2 + 6 = 3(3n+2)$

Let $n = 2m$ because n is even.

$3n(n+2) = 3(2m)(3(2m)+2)$

$= 6m(6m+2)$

$= 12m(3m+1)$

So 12 is a factor of $9n^2 + 6n$.

12 (b) Determine if $9n^2 + 6n$ has a factor of 12 for any integer n .

[1 mark]

We can show it is false using a counter example:

e.g. if $n=1$: $9n^2 + 6n = 9 + 6 = 15$

15 does not have a factor of 12, so it is not true for all n.

END OF SECTION A

Section B

Answer **all** questions in the spaces provided.

13 The number of pots of yoghurt, X, consumed per week by adults in Milton is a discrete random variable with probability distribution given by

x	0	1	2	3	4	5	6	7 or more
P($X=x$)	0.30	0.10	0.05	0.07	0.03	0.16	0.09	0.20

Find $P(3 \leq X < 6)$

Circle the correct answer.

[1 mark]

0.26   0.31   0.35   0.40

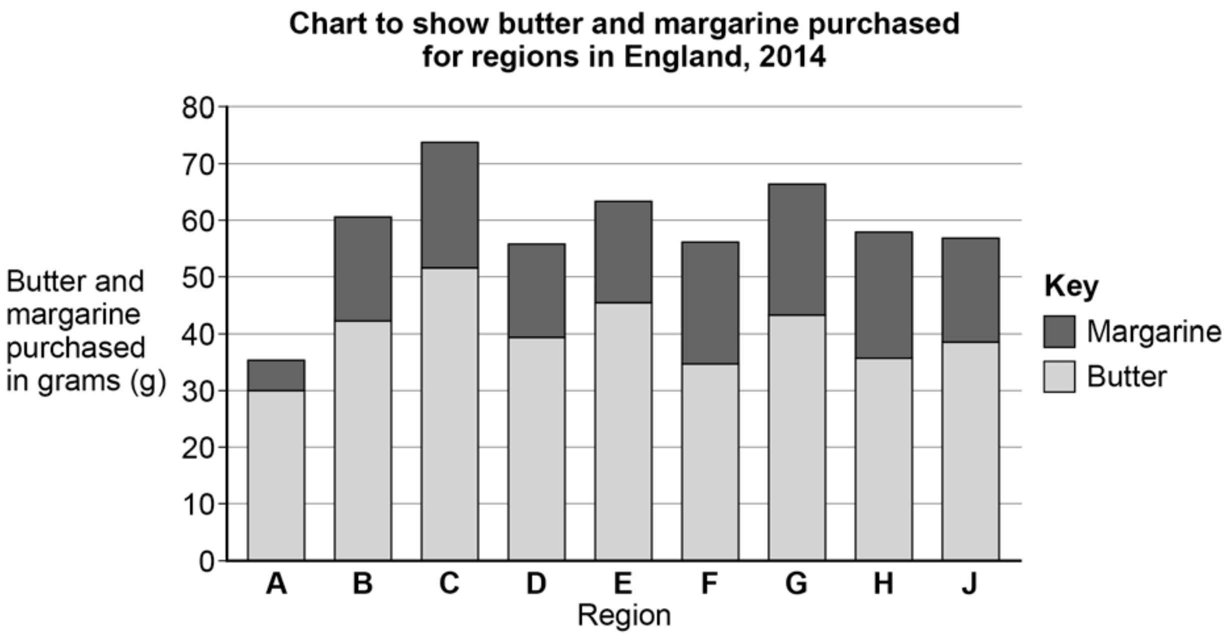
$P(3 \leq X < 6) = P(X=3) + P(X=4) + P(X=5)$

$= 0.07 + 0.03 + 0.16$

$= 0.26$

14 The chart below illustrates the butter and margarine purchases in the regions of England in 2014.

This data is taken from the Large Data Set.



Name the region labelled A on the graph.

Circle your answer.

[1 mark]

North West London South East East Midlands

15 A school took 225 children on a trip to a theme park.

After the trip the children had to write about their favourite ride at the park from a choice of three.

The table shows the number of children who wrote about each ride.

		Ride written about			
		The Drop	The Beanstalk	The Giant	**Total**
Year group	Year 7	24	45	23	**92**
	Year 8	36	17	22	**75**
	Year 9	20	13	25	**58**
	Total	**80**	**75**	**70**	**225**

Three children were randomly selected from those who went on the trip.

Calculate the probability that one wrote about ‘The Drop’, one wrote about ‘The Beanstalk’ and one wrote about The Giant’.

[2 marks]

If you select 3 children in order, there are 6 ways that each wrote about a different story.

The probability of one of the combinations is

$$\frac{80}{225} \times \frac{75}{224} \times \frac{70}{223}.$$

But there are 6 options so you get

$$\frac{80}{225} \times \frac{75}{224} \times \frac{70}{223} \times 6 = 0.224$$

16 The table contains an extract from the Large Data Set.

	Units	2005-06	2007	2009	2011
Confectionery	**g**	**122**	**126**	**131**	**130**
Chocolate bars - solid	g	31	31	30	31
Chocolate bars - filled	g	53	55	58	56
Chewing gum	g	2	3	2	2
Mints and boiled sweets	g	33	35	37	37
Mints	g	4	4	4	3
Boiled sweets	g	28	30	33	34
Fudges, toffees, caramels	g	4	3	4	3
Takeaway confectionery	g	0	0	0	0

16 (a) Bilal states that there is an error in the Large Data Set because the figures for Mints and boiled sweets in the 2007 column do not total to 35.

Give a reason why Bilal's statement may be incorrect.

[1 mark]

The numbers given are rounded to the nearest integer. The unrounded values may total to 35 but the rounded values may not.

16 (b) Maria claims that there is no need to collect Takeaway confectionery data because the table shows that nobody purchases any of that category of confectionery.

State, with a reason, whether you agree or disagree with Maria's claim. Use your knowledge of the Large Data Set to support your answer.

[1 mark]

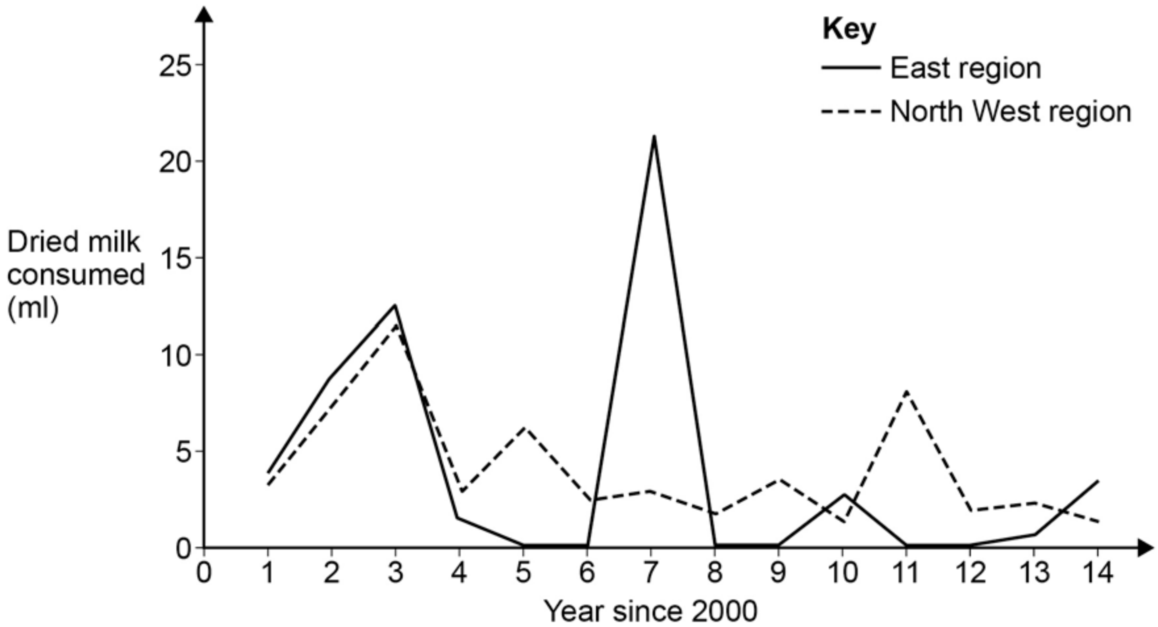
Just because they are values of zero in this data set doesn't mean they will always be zero. She is incorrect.

17 The data in the table gives the average amount, in millilitres, consumed per person per week of dried milk products for the East (E) and for the North West (NW) regions.

The line graph illustrates these data.

	Year since 2000													
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
E	4	9	12	1	0	0	21	0	0	3	0	0	1	0
NW	3	7	12	3	6	2	3	2	3	1	8	2	2	1

Line graph to illustrate average dried milk consumption for E and NW regions



17 (a) (i) Identify, by year and region, a likely outlier.

[1 mark]

East region 2007

17 (a) (ii) A value in a data set can be identified as an outlier if it exceeds $Q_3 + 1.5 \times \text{IQR}$ where IQR is the Interquartile range.

Finding any necessary statistics from the given data, confirm that the suggested outlier in part **(a)(i)** can be confirmed as an outlier.

[3 marks]

Order the data points for East region :

0 0 0 0 0 0 0 | 1 1 3 4 9 12 21

$Q_1 = 0$ $Q_2 = 0.5$ $Q_3 = 4$

$Q_1 = 0$, $Q_3 = 4$ } $IQR = Q_3 - Q_1 = 4 - 0 = 4$

$Q_3 + 1.5 \times IQR = 4 + (1.5 \times 4) = 10$

$21 > 10$ so is an outlier.

17 (b) For the region with the confirmed outlier, explain why the mean is **not** an appropriate measure of average to use.

[1 mark]

The mean is affected by large values, so the outlier would have a big impact on it. It would not represent the other data points.

17 (c) Suggest a reason for the occurrence of the identified outlier.

[1 mark]

Error in data entry.

18 Neesha wants to open an Indian restaurant in her town.

Her cousin, Ranji, has an Indian restaurant in a neighbouring town. To help Neesha plan her menu, she wants to investigate the dishes chosen by a sample of Ranji's customers.

Ranji has a list of the 750 customers who dined at his restaurant during the past month and the dish that each customer chose.

Describe how Neesha could obtain a simple random sample of size 50 from Ranji's customers.

[4 marks]

Number each customer from 1-750

Use random numbers to select 50 people who correspond to these numbers.

If a number is repeated, ignore it.

19 Ellie, a statistics student, read a newspaper article that stated that 20 per cent of students eat at least five portions of fruit and vegetables every day.

Ellie suggests that the number of people who eat at least five portions of fruit and vegetables every day, in a sample of size n, can be modelled by the binomial distribution B(n , 0.20).

19 (a) There are 10 students in Ellie's statistics class.

Using the distributional model suggested by Ellie, find the probability that, of the students in her class:

19 (a) (i) two or fewer eat at least five portions of fruit and vegetables every day;

[1 mark]

$X \sim B(10, 0.2)$, X = number of students eating at least 5 portions fruit and veg

$P(X \leq 2) = 0.678$

19 (a) (ii) at least one but fewer than four eat at least five portions of fruit and vegetables every day;

[2 marks]

$P(1 \leq X \leq 3) = P(X \leq 3) - P(X \leq 1)$

$= 0.8791 - 0.1074$

$= 0.772$

19 (b) Ellie's teacher, Declan, believes that more than 20 per cent of students eat at least five portions of fruit and vegetables every day. Declan asks the 25 students in his other statistics classes and 8 of them say that they eat at least five portions of fruit and vegetables every day.

19 (b) (i) Name the sampling method used by Declan.

[1 mark]

Opportunity sampling

19 (b) (ii) Describe **one** weakness of this sampling method.

[1 mark]

The students all come from the same school so will live in the same area. This may not be representative of the whole country.

19 (b) (iii) Assuming that these 25 students may be considered to be a random sample, carry out a hypothesis test at the 5% significance level to investigate whether Declan's belief is supported by this evidence.

[6 marks]

$H_0 : p = 0.2$

$H_1 : p > 0.2$

Using $X \sim B(25, 0.2)$ we get that

$$P(X \geq 8) = 0.109$$

$0.109 > 0.05$ so accept H_0, insufficient evidence to suggest that more than 20% of students eat at least 5 a day.

END OF QUESTIONS