MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

AS
MATHEMATICS

Paper 2

Wednesday 23 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Given that $\frac{dy}{dx} = \frac{1}{6x^2}$ find y.

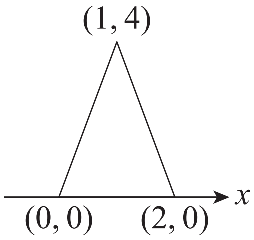
Circle your answer.

[1 mark]

$\frac{-1}{3x^3} + c$ $\frac{1}{2x^3} + c$ $\frac{-1}{6x} + c$ $\frac{-1}{3x} + c$

2 **Figure 1** shows $y = \mathrm{f}(x)$.

Figure 1



Which figure below shows $y = \mathrm{f}(2x)$?

Tick **one** box.

[1 mark]

Figure 2

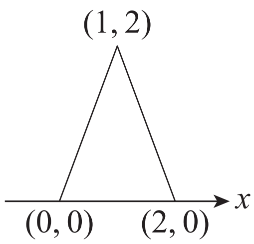


Figure 3

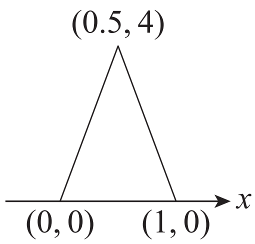


Figure 4

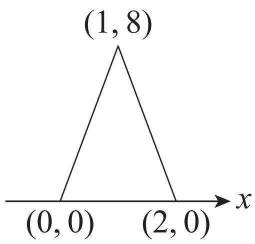


Figure 5

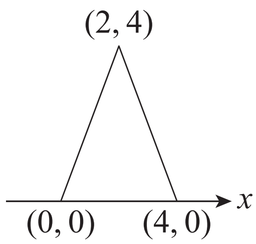


Figure 2 ☐

Figure 3 ☑

Figure 4 ☐

Figure 5 ☐

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3 Express as a single logarithm

$$2\log_a 6 - \log_a 3$$

[2 marks]

$2\log_a 6 - \log_a 3 = \log_a 6^2 - \log_a 3$

$= \log_a 36 - \log_a 3$

$= \log_a \left(\frac{36}{3}\right)$

$= \log_a 12$

4 Solve the equation $\tan^2 2\theta - 3 = 0$ giving all the solutions for $0° \leq \theta \leq 360°$

[4 marks]

$\tan^2 2\theta - 3 = 0$

$\tan^2 2\theta = 3$

$\tan 2\theta = \sqrt{3} \Rightarrow 2\theta = 60°,\ 60°+180°,\ 60°+360°,\ 60°+540°$

$2\theta = 60°,\ 240°,\ 420°,\ 600°$

$\theta = 30°,\ 120°,\ 210°,\ 300°$

$\tan 2\theta = -\sqrt{3} \Rightarrow 2\theta = 120°,\ 120°+180°,\ 120°+360°,\ 120°+540°$

$2\theta = 120°,\ 300°,\ 480°,\ 660°$

$\theta = 60°,\ 150°,\ 240°,\ 330°.$

$\therefore \theta = 30°,\ 60°,\ 120°,\ 150°,\ 210°,\ 240°,\ 300°,\ 330°$

5 $f'(x) = \left(2x - \dfrac{3}{x}\right)^2$ and $f(3) = 2$

Find $f(x)$.

[4 marks]

$f'(x) = 4x^2 - 12 + \dfrac{9}{x^2}$

$f(x) = \int 4x^2 - 12 + \dfrac{9}{x^2}\, dx$

$f(x) = \dfrac{4}{3}x^3 - 12x - \dfrac{9}{x} + c$

$f(3) = \dfrac{4}{3}(3)^3 - 12(3) - \dfrac{9}{3} + c = 2$

$36 - 36 - 3 + c = 2$

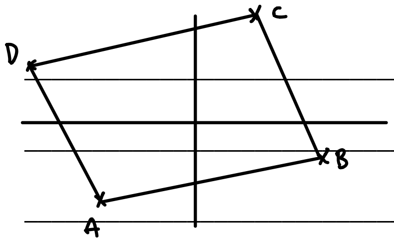
$c = 5$

$\therefore \quad f(x) = \dfrac{4}{3}x^3 - 12x - \dfrac{9}{x} + 5$

6 Points A $(-7, -7)$, B $(8, -1)$, C $(4, 9)$ and D $(-11, 3)$ are the vertices of a quadrilateral $ABCD$.

6 (a) Prove that $ABCD$ is a rectangle.

[4 marks]



Gradient $AB = \frac{-1-(-7)}{8-(-7)} = \frac{6}{15} = \frac{2}{5}$

Gradient $CD = \frac{3-9}{-11-4} = \frac{-6}{-15} = \frac{2}{5}$

Gradient $AD = \frac{3-(-7)}{-11-(-7)} = \frac{10}{-4} = -\frac{5}{2}$

Gradient $BC = \frac{-1-9}{8-4} = \frac{-10}{4} = -\frac{5}{2}$

AB is parallel to CD and AD is parallel to BC so ABCD is a parallelogram.

Since $\frac{2}{5} \times -\frac{5}{2} = -1$, the adjoining sides are perpendicular.

All angles are 90° so ABCD is a rectangle.

6 (b) Find the area of $ABCD$.

[2 marks]

Length of $CD = \sqrt{(4-(-11))^2 + (9-3)^2} = \sqrt{15^2 + 6^2} = \sqrt{261} = 3\sqrt{29}$

Length of $AD = \sqrt{(7-11)^2 + (3-7)^2} = \sqrt{4^2 + 10^2} = \sqrt{116} = 2\sqrt{29}$

Area $= 3\sqrt{29} \times 2\sqrt{29} = 174$

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7 (a) Express $2x^2 - 5x + k$ in the form $a(x-b)^2 + c$

[3 marks]

$2x^2 - 5x + k = 2\left(x^2 - \frac{5}{2}x + \frac{k}{2}\right)$

$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{k}{2}\right]$

$= 2\left(x - \frac{5}{4}\right)^2 + k - \frac{25}{8}$ so $a = 2$, $b = \frac{5}{4}$, $c = k - \frac{25}{8}$

7 (b) Find the values of k for which the curve $y = 2x^2 - 5x + k$ does **not** intersect the line $y = 3$

[3 marks]

$k - \frac{25}{8}$ is the minimum point of the curve. We want the curve to not intersect $y=3$ so we need the minimum to be greater than 3. So,

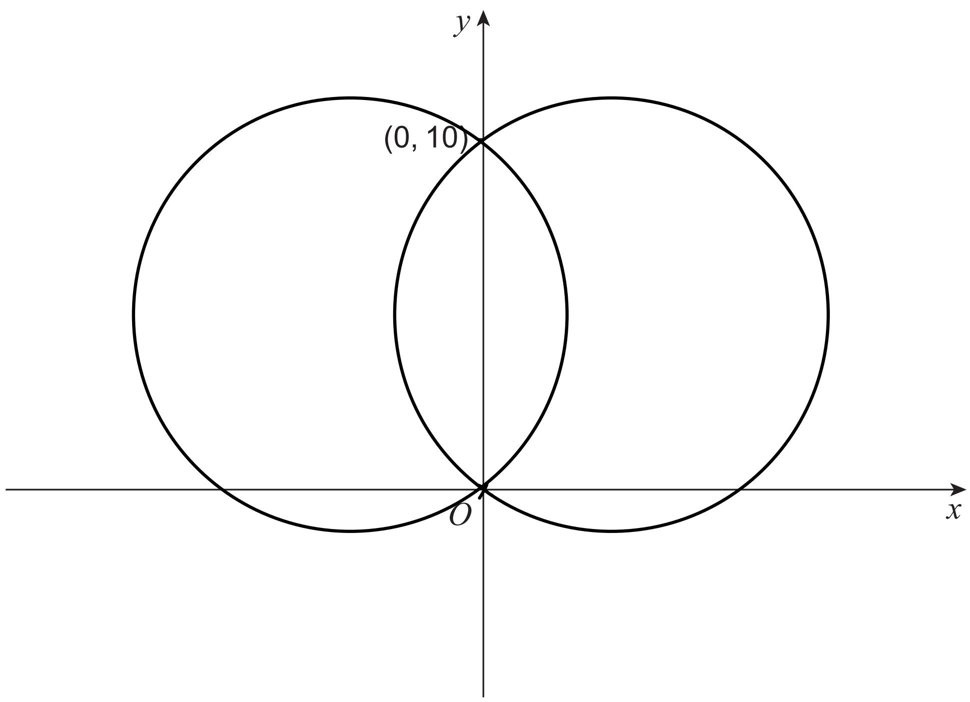
$k - \frac{25}{8} > 3$

$k > \frac{49}{8}$

8 A circle of radius 6 passes through the points (0, 0) and (0, 10).

8 (a) Sketch the two possible positions of the circle.

[1 mark]



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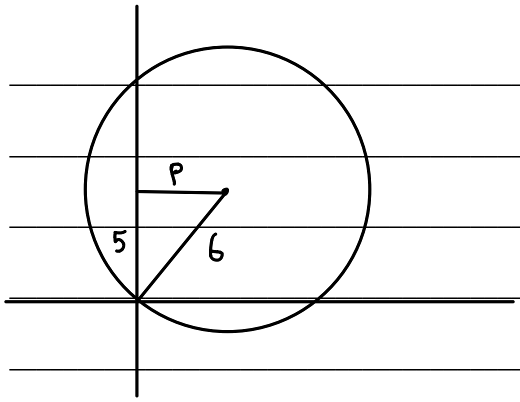
8 (b) Find the equations of the two circles.

[3 marks]



$p = \sqrt{6^2 - 5^2} = \sqrt{36-25} = \sqrt{11}$.

The centre of the circle has y coordinate 5.

It has x coordinate $\pm\sqrt{11}$ because it can be either side of the y axis.

$(x-\sqrt{11})^2 + (y-5)^2 = 36$

or

$(x+\sqrt{11})^2 + (y-5)^2 = 36$

Turn over for the next question

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9 It is given that $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ and $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

Show that $\tan^2 15°$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

Fully justify your answer.

[3 marks]

$\tan^2 15 = \left(\frac{\sin 15}{\cos 15}\right)^2 = \left(\frac{\frac{1}{2}\sqrt{2-\sqrt{3}}}{\frac{1}{2}\sqrt{2+\sqrt{3}}}\right)^2$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{4-4\sqrt{3}+3}{4-3}$

$= 7 - 4\sqrt{3}$ $(a = 7,\ b = -4)$

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10 In the binomial expansion of $(1+x)^n$, where $n \geq 4$, the coefficient of x^4 is $1\frac{1}{2}$ times the sum of the coefficients of x^2 and x^3

Find the value of n.

[5 marks]

$(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \ldots$

$= 1 + nx + \frac{n!}{2!(n-2)!}x^2 + \frac{n!}{3!(n-3)!}x^3 + \frac{n!}{4!(n-4)!}x^4 + \ldots$

$= 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \frac{n(n-1)(n-2)(n-3)}{24}x^4 + \ldots$

$\frac{3}{2}\left(\frac{\cancel{n}(\cancel{n-1})}{2} + \frac{\cancel{n}(\cancel{n-1})(n-2)}{6}\right) = \frac{\cancel{n}(\cancel{n-1})(n-2)(n-3)}{24}$

$\times 4$

$3 + n - 2 = \frac{(n-2)(n-3)}{6}$

$18 + 6n - 12 = n^2 - 5n + 6$

$n^2 - 11n = 0$

$n(n-11) = 0$

$n \neq 0$ so $n = 11.$

Turn over for the next question

Turn over ▶

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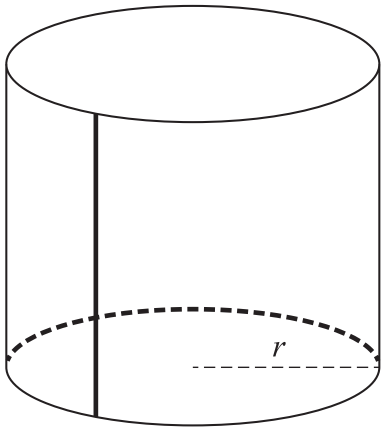
11 Rakti makes open-topped cylindrical planters out of thin sheets of galvanised steel.

She bends a rectangle of steel to make an open cylinder and welds the joint. She then welds this cylinder to the circumference of a circular base.



The planter must have a capacity of 8000 cm^3

Welding is time consuming, so Rakti wants the total length of weld to be a minimum.

Calculate the radius, r, and height, h, of a planter which requires the minimum total length of weld.

Fully justify your answers, giving them to an appropriate degree of accuracy.

[9 marks]

Volume = 8000 = $\pi r^2 h$

Total length welded = $h + 2r\pi = w$

We want to minimise w so we want $\frac{dw}{dr} = 0$

$w = h + 2\pi r$

$w = \frac{8000}{\pi r^2} + 2\pi r$

$\frac{dw}{dr} = \frac{-2(8000)}{\pi r^3} + 2\pi = 0$

$2\pi = \frac{16000}{\pi r^3}$

$r3 = \frac{8000}{\pi^2}$

$r = \sqrt[3]{\frac{8000}{\pi^2}} \approx 9.324$

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So w is at a minimum when $r = 9.324$.

$h = \frac{8000}{\pi(9.324)^2} = 29.292$

We need to check that this value of r gives a minimum and not a maximum.

$\frac{d^2w}{dr^2} = \frac{6(8000)}{\pi r^4}$

At $r = 9.324$, $\frac{d^2w}{dr^2} = \frac{6(8000)}{\pi(9.324)^4} = 2.02$

Since $2.02 > 0$, at $r = 9.324$ we have a minimum.

Therefore, $r = 9.32$ m (3. s.f)

$h = 29.3$ m (3. s.f)

Turn over ▶

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12 Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_A, can be modelled by the formula

$$n_A = ae^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_B, can be modelled by the formula

$$n_B = be^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

12 (a) Show that $b = 90$, to the nearest integer, and find the value of a.

[3 marks]

$n_A + n_B = ae^{0.1t} + be^{0.2t}$

In 2017, $t=0$: $290 = a+b$ ①

In 2018, $t=1$: $331 = ae^{0.1} + be^{0.2}$ ②

From ①: $a = 290 - b$

Substitute into ②: $331 = (290-b)e^{0.1} + be^{0.2}$

$331 = 290e^{0.1} + b(e^{0.2} - e^{0.1})$

$b = \dfrac{331 - 290e^{0.1}}{e^{0.2} - e^{0.1}}$

$b = 90.3\ldots = 90$

$a = 290 - b = 200$

Do not write outside the box

12 (b) Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020.

[1 mark]

In 2020, $t=3$:

$n_A + n_B = 200e^{0.1(3)} + 90e^{0.2(3)}$

$= 433.96... = 434$ trees

12 (c) Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A.

[3 marks]

$90e^{0.2t} > 200e^{0.1t}$

$e^{0.1t} > \frac{200}{90}$

$0.1t > \ln\left(\frac{20}{9}\right)$

$t > 10\ln\left(\frac{20}{9}\right)$

$t > 7.985$

So disease B exceeds disease A during the 7^{th} year after 2017, so in 2024.

12 (d) Comment on the long-term accuracy of the model.

[1 mark]

n_A and n_B will tend to infinity which is unrealistic - all the trees will eventually die.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

13 The table below shows the probability distribution for a discrete random variable X.

x	0	1	2	3	4 or more
P$(X = x)$	0.35	0.25	k	0.14	0.1

Find the value of k.

$0.35 + 0.25 + k + 0.14 + 0.1 = 1$
$k = 0.16$

Circle your answer.

[1 mark]

0.14   0.16   0.18   1

14 Given that $\sum x = 364$, $\sum x^2 = 19412$, $n = 10$, find σ, the standard deviation of X.

Circle your answer.

[1 mark]

24.8   44.1   616.2   1941.2

$\sqrt{\frac{19412}{10} - \left(\frac{364}{10}\right)^2} = 24.8$

15 Nicola, a darts player, is practising hitting the bullseye. She knows from previous experience that she has a probability of 0.3 of hitting the bullseye with each dart.

Nicola throws eight practice darts.

15 (a) Using a binomial distribution, calculate the probability that she will hit the bullseye three or more times.

[2 marks]

Let X be the number of bullseye hit

$X \sim B(8, 0.3)$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.5518$

$= 0.4482$

15 (b) Nicola throws eight practice darts on three different occasions. Calculate the probability that she will hit the bullseye three or more times on all three occasions.

[2 marks]

$(0.4482)^3 = 0.090$

15 (c) State two assumptions that are necessary for the distribution you have used in part **(a)** to be valid.

[2 marks]

Hitting the bullseye is independent of whether or not the other darts hit the bullseye.

The probability of hitting the bullseye remains fixed at 0.3.

Turn over ▶

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16 Kevin is the Principal of a college.

He wishes to investigate types of transport used by students to travel to college.

There are 3200 students in the college and Kevin decides to survey 60 of them.

Describe how he could obtain a simple random sample of size 60 from the 3200 students.

[4 marks]

Number each student 1 to 3200.

Generate random four digit numbers.

Select the students corresponding to these numbers. If a random number is repeated, ignore the repeats. Ignore any numbers outside the range.

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17 The table below is an extract from the Large Data Set, showing the purchased quantities of fats and oils for the South East of England in 2014.

Description	**Purchased quantity**
Butter	42
Soft margarine	16
Olive oil	17
Other vegetable and salad oils	28

Kim claims that more olive oil was purchased in the South East than soft margarine.

Explain why Kim may be incorrect.

[2 marks]

The oils are liquids so could be measured in a different way and with different units to margarine and butter.
Since there are no units given in the table, the quantities could each refer to different units.

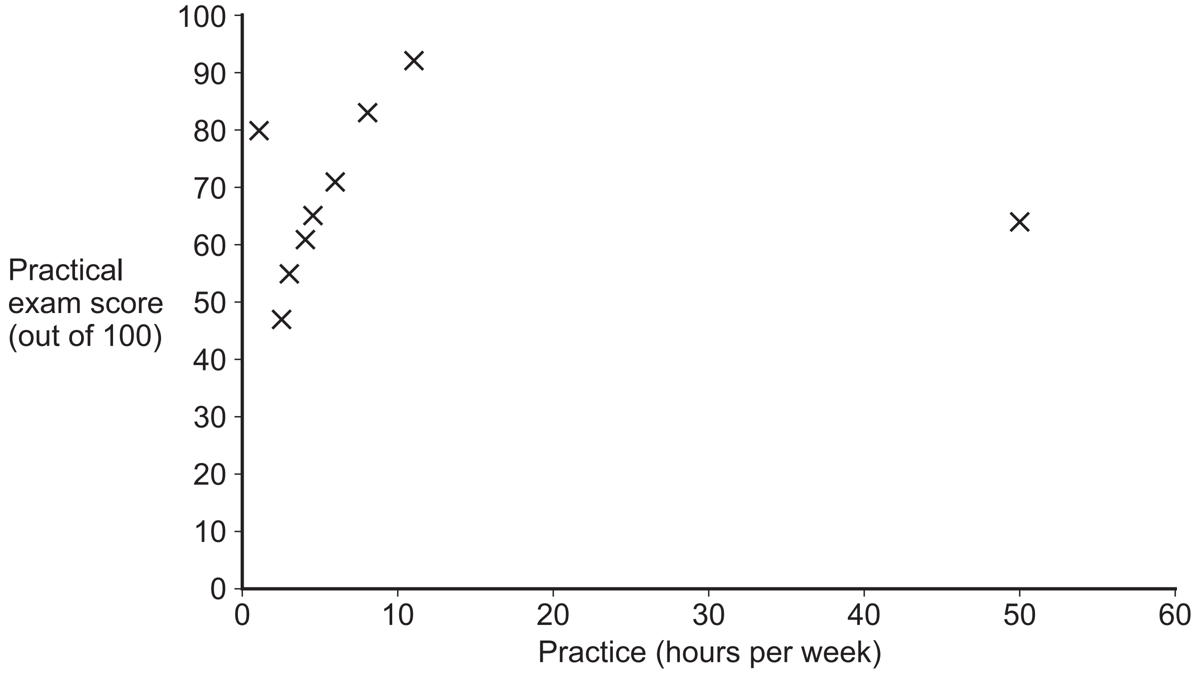
Turn over for the next question

18 Jennie is a piano teacher who teaches nine pupils.

She records how many hours per week they practice the piano along with their most recent practical exam score.

Student	**Practice (hours per week)**	**Practical exam score (out of 100)**
Donovan	50	64
Vazquez	6	71
Higgins	3	55
Begum	2.5	47
Collins	1	80
Coldbridge	4	61
Nedbalek	4.5	65
Carter	8	83
White	11	92

She plots a scatter diagram of this data, as shown below.



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18 (a) Identify two possible outliers by name, giving a possible explanation for the position on the scatter diagram of each outlier.

[4 marks]

First outlier Collins

Possible reason Very good student

Second outlier Donovan

Possible reason Data entered incorrectly

18 (b) Jennie discards the two outliers.

18 (b) (i) Describe the correlation shown by the scatter diagram for the remaining points.

[1 mark]

Strong positive correlation

18 (b) (ii) Interpret this correlation in the context of the question.

[1 mark]

Students who practice more achieve higher exam scores.

Turn over for the next question

19 Martin grows cucumbers from seed.

In the past, he has found that 70% of all seeds successfully germinate and grow into cucumber plants.

He decides to try out a new brand of seed.

The producer of this brand claims that these seeds are more likely to successfully germinate than other brands of seeds.

Martin sows 20 of this new brand of seed and 18 successfully germinate.

Carry out a hypothesis test at the 5% level of significance to investigate the producer's claim.

[7 marks]

$H_0 : p = 0.7$

$H_1 : p > 0.7$

Let X be the number of seeds that germinate.

$X \sim B(20, 0.7)$

$P(X \geq 18) = 1 - P(X \leq 17)$

$= 1 - 0.9645$

$= 0.0355$

$0.0355 < 0.05$ so reject H_0.

There is sufficient evidence to suggest that the new seeds germinate better.

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END OF QUESTIONS

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