MODEL ANSWERS

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

AS MATHEMATICS

Paper 1

Exam Date | Morning | Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 The curve $y=\sqrt{x}$ is translated onto the curve $y=\sqrt{x+4}$

The translation is described by a vector.

Find this vector.

Circle your answer.

[1 mark]

$\begin{bmatrix}4\\0\end{bmatrix}$ $\begin{bmatrix}-4\\0\end{bmatrix}$ $\begin{bmatrix}0\\4\end{bmatrix}$ $\begin{bmatrix}0\\-4\end{bmatrix}$

2 Consider the two statements, A and B, below.

A: $x^2-6x+8>0$

B: $x>4$

Choose the most appropriate option below.

Circle your answer.

[1 mark]

$A \Rightarrow B$ $A \Leftarrow B$ $A \Leftrightarrow B$ There is no connection between A and B

3 (a) Write down the value of p and the value of q given that:

3 (a) (i) $\sqrt{3} = 3^p$

[1 mark]

$p = \frac{1}{2}$

3 (a) (ii) $\frac{1}{9} = 3^q$

[1 mark]

$q = -2$

3 (b) Find the value of x for which $\sqrt{3} \times 3^x = \frac{1}{9}$

[2 marks]

$\sqrt{3} \times 3^x = 3^{\frac{1}{2}+x}$

$\frac{1}{9} = 3^{-2}$

compare powers: $\frac{1}{2} + x = -2$

$x = -2.5$

4 Show that $\frac{5\sqrt{2}+2}{3\sqrt{2}+4}$ can be expressed in the form $m+n\sqrt{2}$, where m and n are integers.

[3 marks]

$$\frac{5\sqrt{2}+2}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4}$$

$$= \frac{15(2) - 20\sqrt{2} + 6\sqrt{2} - 8}{9(2) - 16}$$

$$= \frac{22 - 14\sqrt{2}}{2}$$

$= 11 - 7\sqrt{2}$ (so $m = 11$, $n = -7$)

5 Jessica, a maths student, is asked by her teacher to solve the equation $\tan x = \sin x$, giving all solutions in the range $0° \le x \le 360°$

The steps of Jessica's working are shown below.

$$\tan x = \sin x$$

Step 1	$\Rightarrow$	$\dfrac{\sin x}{\cos x} = \sin x$	Write $\tan x$ as $\dfrac{\sin x}{\cos x}$
Step 2	$\Rightarrow$	$\sin x = \sin x \cos x$	Multiply by $\cos x$
Step 3	$\Rightarrow$	$1 = \cos x$	Cancel $\sin x$
	$\Rightarrow$	$x = 0°$ or $360°$	

The teacher tells Jessica that she has not found all the solutions because of a mistake.

Explain why Jessica's method is not correct.

[2 marks]

In step 3 she cancelled the sinx when sinx=0 will give you a solution.

Instead of cancelling you should factorise

$\sin x = \sin x \cos x$

$0 = \sin x(\cos x - 1)$

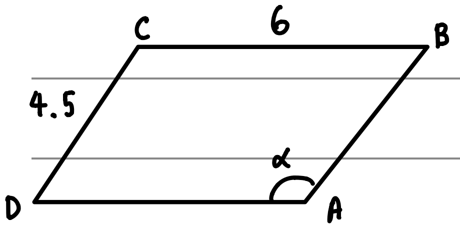
So either $\cos x - 1 = 0$ or $\sin x = 0$.

6 A parallelogram has sides of length 6 cm and 4.5 cm.

The larger interior angles of the parallelogram have size α

Given that the area of the parallelogram is 24 cm^2, find the exact value of $\tan\alpha$

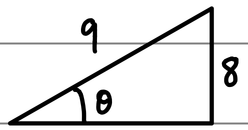
[4 marks]



Area of parallelogram = $AB \times AD \times \sin\alpha$

$\Rightarrow \quad 24 = 6 \times 4.5 \times \sin\alpha \quad \Rightarrow \quad \sin\alpha = \frac{8}{9}$

To work out $\tan\alpha$ from $\sin\alpha$ use right angled triangles



$\sin\theta = \frac{8}{9} = \frac{\text{opposite}}{\text{hypotenuse}}$

Adjacent side = $\sqrt{9^2 - 8^2} = \sqrt{17}$ by Pythagoras' theorem.

So, $\tan\alpha = \pm\frac{8}{\sqrt{17}}$

α must be obtuse because it is one of the larger angles, so $\tan\alpha$ must be negative. So,

$$\tan\alpha = \frac{-8}{\sqrt{17}}.$$

7 Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

[4 marks]

$2x + 3y + 4 = 0$

$3y = -4 - 2x$

$y = -\frac{2}{3}x - \frac{4}{3}$ so gradient is $-\frac{2}{3}$.

For the line passing through (9,4) and (3,8):

gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{8-4}{3-9} = -\frac{4}{6} = -\frac{2}{3}$

The gradients are equal so the lines are parallel.

Turn over for the next question

8 (a) Find the first **three** terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$

[3 marks]

$1 + \binom{10}{1}(-2x)^1 + \binom{10}{2}(-2x)^2$

$= 1 + 10(-2x) + 45(4)x^2$

$= 1 - 20x + 180x^2$

8 (b) Carly has lost her calculator. She uses the first three terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$ to evaluate 0.998^{10}

Find Carly's value for 0.998^{10} and show that it is correct to **five** decimal places.

[3 marks]

$(1-2x)^{10} = 0.998^{10}$

$1-2x = 0.998$

Let $x = \frac{1-0.998}{2} = 0.001$

Substitute this into our formula from part (a):

$1-20(0.001)+180(0.001)^2$

$= 1 - 0.02 + 0.00018$

$= 0.98018$ so $0.998^{10} \approx 0.98018$.

The true value of 0.998^{10} is 0.980179 which is 0.98018 to 5.d.p.

So, Carly's answer is correct to 5.d.p.

9 (a) Given that $f(x) = x^2 - 4x + 2$, find $f(3+h)$

Express your answer in the form $h^2 + bh + c$, where b and $c \in \mathbb{Z}$.

[2 marks]

$f(x) = x^2 - 4x + 2$

$f(3+h) = (3+h)^2 - 4(3+h) + 2$

$= 9 + 6h + h^2 - 12 - 4h + 2$

$= h^2 + 2h - 1$ (so $b = 2$, $c = -1$)

9 (b) The curve with equation $y = x^2 - 4x + 2$ passes through the point $P(3, -1)$ and the point Q where $x = 3 + h$.

Using differentiation from first principles, find the gradient of the tangent to the curve at the point P.

[3 marks]

Gradient of line $= \dfrac{f(3+h) - f(3)}{h}$

$= \dfrac{h^2 + 2h - 1 + 1}{h}$ from (a)

$= h + 2$

As $h \to 0$, $h + 2 \to 2$ so the gradient is 2.

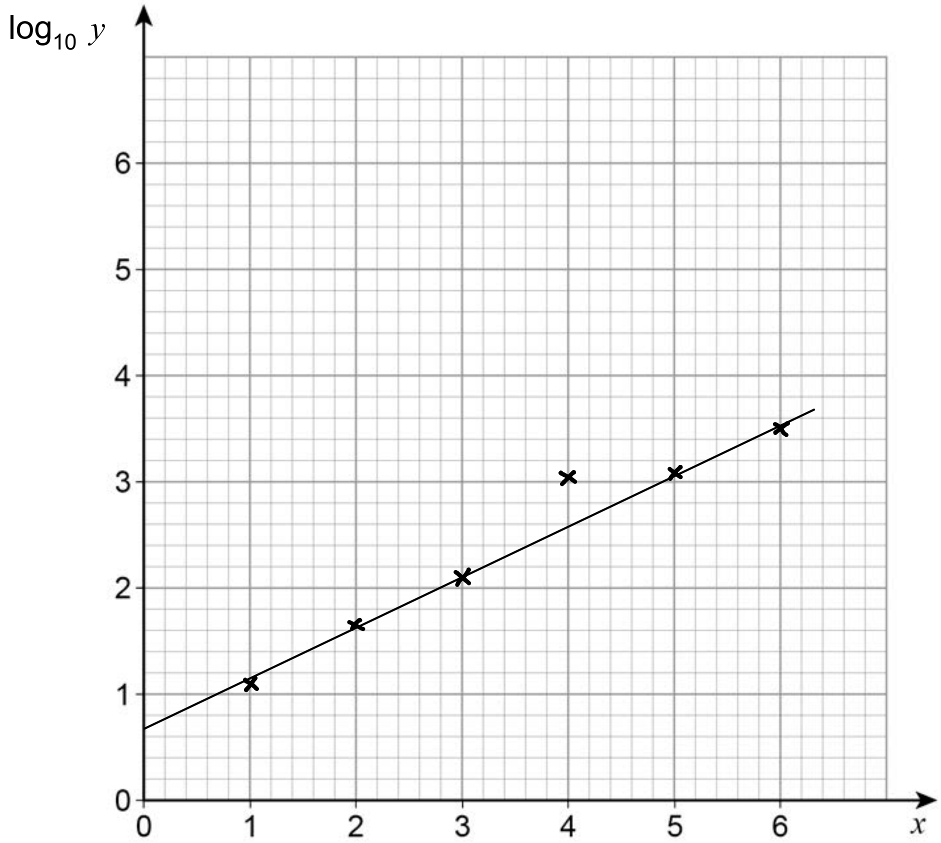
10 A student conducts an experiment and records the following data for two variables, x and y.

x	1	2	3	4	5	6
y	14	45	130	1100	1300	3400
$\log_{10} y$	1.15	1.65	2.11	3.04	3.11	3.53

The student is told that the relationship between x and y can be modelled by an equation of the form $y = kb^x$

10 (a) Plot values of $\log_{10} y$ against x on the grid below.

[2 marks]



10 (b) State, with a reason, which value of y is likely to have been recorded incorrectly.

[1 mark]

The point (4, 1100) does not lie on the line that would connect the other points. So $y = 1100$ may be incorrect.

10 (c) By drawing an appropriate straight line, find the values of k and b.

[4 marks]

$y = kb^x$

Take logs: $\log_{10} y = \log_{10}(kb^x)$

$\log_{10} y = \log_{10} k + \log_{10} b^x$

$\log_{10} y = \log_{10} k + x\log_{10} b$

From the graph the intercept is about 0.7 and the gradient is about 0.5.

$\log_{10} y = \log_{10} k + x\log_{10} b$ is in the form $y = mx + c$.

So, $\log_{10} k$ is the intercept and $\log_{10} b$ is the gradient:

$\log_{10} k = 0.7$

$k = 10^{0.7} = 5.0118\ldots$ so, $k = 5.0$ (2.s.f)

$\log_{10} b = 0.5$

$b = 10^{0.5} = 3.16227\ldots$ so, $b = 3.2$ (2.s.f)

Turn over for the next question

11 Chris claims that, "for any given value of x, the gradient of the curve $y = 2x^3 + 6x^2 - 12x + 3$ is always greater than the gradient of the curve $y = 1 + 60x - 6x^2$".

Show that Chris is wrong by finding all the values of x for which his claim is **not** true.

[7 marks]

For $y = 2x^3 + 6x^2 - 12x + 3$

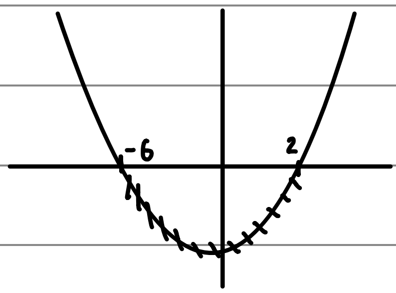
$\frac{dy}{dx} = 60 - 12x$

For $y = 1 + 60x - 6x^2$

$\frac{dy}{dx} = 60 - 12x$

So Chris is incorrect when:

$$60 - 12x \geq 6x^2 + 12x - 12$$
$$0 \geq 6x^2 + 24x - 72$$
$$0 \geq x^2 + 4x - 12$$
$$0 \geq (x+6)(x-2)$$



True when $-6 \leq x \leq 2$

His claim is wrong when $-6 \leq x \leq 2$ so he is incorrect.

12 A curve has equation $y = 6x\sqrt{x} + \frac{32}{x}$ for $x > 0$

12 (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

[4 marks]

$y = 6x^{\frac{3}{2}} + 32x^{-1}$

$\frac{dy}{dx} = 6\left(\frac{3}{2}\right)x^{\frac{1}{2}} + 32(-1)x^{-2}$

$= 9x^{\frac{1}{2}} - 32x^{-2}$

$= 9\sqrt{x} - \frac{32}{x^2}$

12 (b) The point *A* lies on the curve and has x-coordinate 4

Find the coordinates of the point where the tangent to the curve at *A* crosses the x-axis.

[5 marks]

When $x = 4$, $y = 6(4)(2) + \frac{32}{4}$

$= 48 + 8$

$= 56$

and $\frac{dy}{dx} = 9(2) - \frac{32}{16} = 18 - 2 = 16.$

So the equation of the tangent is: $y - 56 = 16(x - 4)$

$y = 16x - 8$

The line crosses the x axis when $y=0$: $y = 0 = 16x - 8 \Rightarrow x = \frac{1}{2}$

So, the coordinates are $\left(\frac{1}{2}, 0\right)$.

END OF SECTION A
TURN OVER FOR SECTION B

Section B

Answer **all** questions in the spaces provided.

13 (a) The unit vectors **i** and **j** are perpendicular.

Find the magnitude of the vector $-20\mathbf{i} + 21\mathbf{j}$

Circle your answer.

[1 mark]

-1 $\quad$ 1 $\quad$ $\sqrt{41}$ $\quad$ 29

$\sqrt{20^2 + 21^2} = 29$

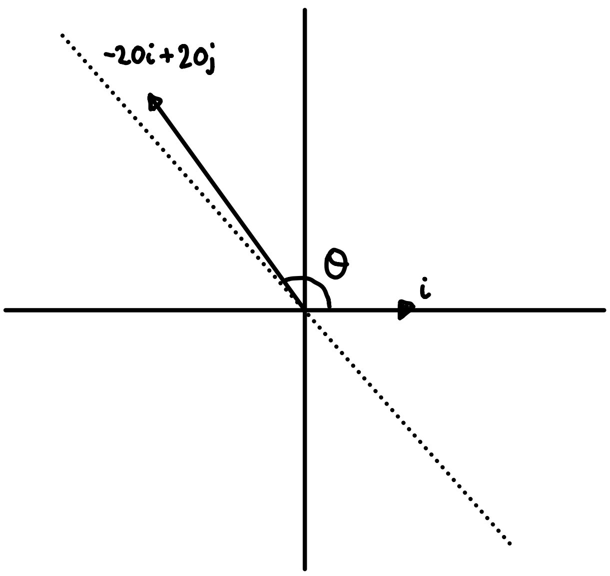
13 (b) The angle between the vector **i** and the vector $-20\mathbf{i} + 21\mathbf{j}$ is θ

Which statement about θ is true?

Circle your answer.

[1 mark]

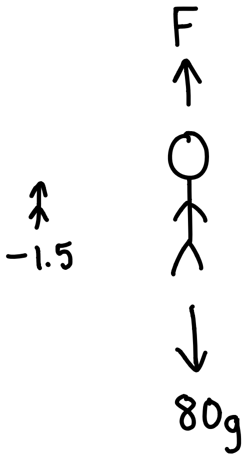
$0° < \theta < 45°$ $\quad$ $45° < \theta < 90°$ $\quad$ $90° < \theta < 135°$ $\quad$ $135° < \theta < 180°$



14 **In this question use $g = 10 \text{ m s}^{-2}$.**

A man of mass 80 kg is travelling in a lift.

The lift is rising vertically.



The lift decelerates at a rate of 1.5 m s^{-2}

Find the magnitude of the force exerted on the man by the lift.

[3 marks]

Let F be the force exerted on the man by the lift.

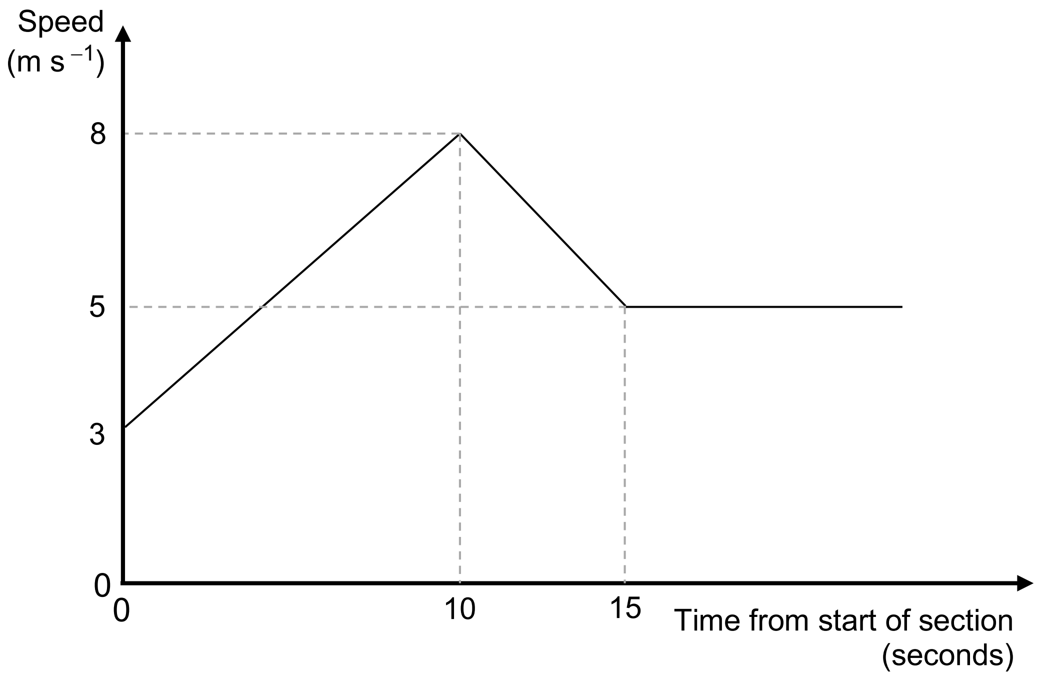
Using $F = ma$: $F - 80g = 8(-1.5)$

$F = -120 + 80(10)$

$F = 680$

$F = 700$ N (1 s.f)

15 The graph shows how the speed of a cyclist varies during a timed section of length 120 metres along a straight track.



15 (a) Find the acceleration of the cyclist during the first 10 seconds.

[1 mark]

$$\text{acceleration} = \frac{\text{change in speed}}{\text{time}}$$

$$= \frac{8-3}{10}$$

$$= 0.5 \text{ ms}^{-2}$$

15 (b) After the first 15 seconds, the cyclist travels at a constant speed of 5 m s^{-1} for a further T seconds to complete the 120-metre section.

Calculate the value of T.

[4 marks]

The distance travelled is equal to the area under the curve.

Total area = $\frac{5 \times 10}{2} + (3 \times 10) + (5 \times 5) + \frac{5 \times 3}{2} + 5T$

$= 25 + 30 + 25 + 7.5 + 5T$

$= 87.5 + 5T$

We are told that the total distance is 120m:

$120 = 87.5 + 5T$

$5T = 32.5$

$T = 6.5$

Turn over for the next question

16 A particle, of mass 400 grams, is initially at rest at the point O.

The particle starts to move in a straight line so that its velocity, v m s^{-1}, at time t seconds is given by

$$v = 6t^2 - 12t^3 \text{ for } t > 0$$

16 (a) Find an expression, in terms of t, for the force acting on the particle.

[3 marks]

$400g = 0.4 \text{ kg}$

$v = 6t^2 - 12t^3$

$a = \frac{dv}{dt} = 12t - 36t^2$

$F = ma$: $F = 0.4(12t - 36t^2)$

$F = 4.8t - 14.4t^2$

16 (b) Find the time when the particle next passes through *O*.

[5 marks]

position $= r = \int v$

$r = \int 6t^2 - 12t^3 \, dt$

$r = 2t^3 - 3t^4 + c$

Using initial conditions, $t = 0$, $r = 0$:

$0 = 2(0) - 3(0) + c$

$\Rightarrow c = 0$

So, $r = 2t^3 - 3t^4$

$r = t^3(2 - 3t)$

So, when $r = 0$, $t = 0$ or $t = \frac{2}{3}$.

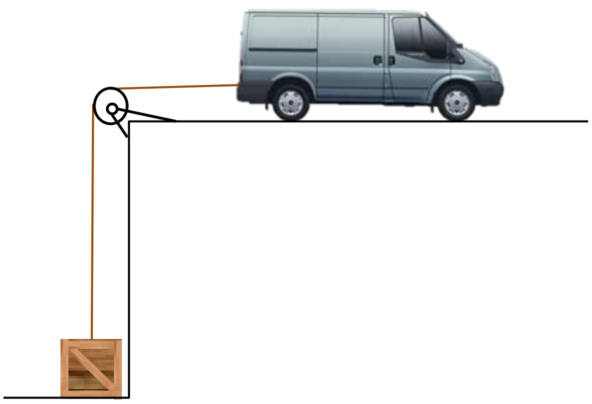
so, the particle passes through O at $\frac{2}{3}$ seconds.

17 **In this question use $g = 9.8$ m s^{-2}.**

A van of mass 1300 kg and a crate of mass 300 kg are connected by a light inextensible rope.

The rope passes over a light smooth pulley, as shown in the diagram.

The rope between the pulley and the van is horizontal.



Initially, the van is at rest and the crate rests on the lower level. The rope is taut.

The van moves away from the pulley to lift the crate from the lower level.

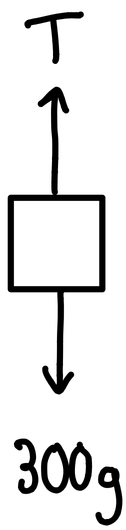
The van's engine produces a constant driving force of 5000 N.

A constant resistance force of magnitude 780 N acts on the van.

Assume there is no resistance force acting on the crate.

17 (a) (i) Draw a diagram to show the forces acting on the crate while it is being lifted.

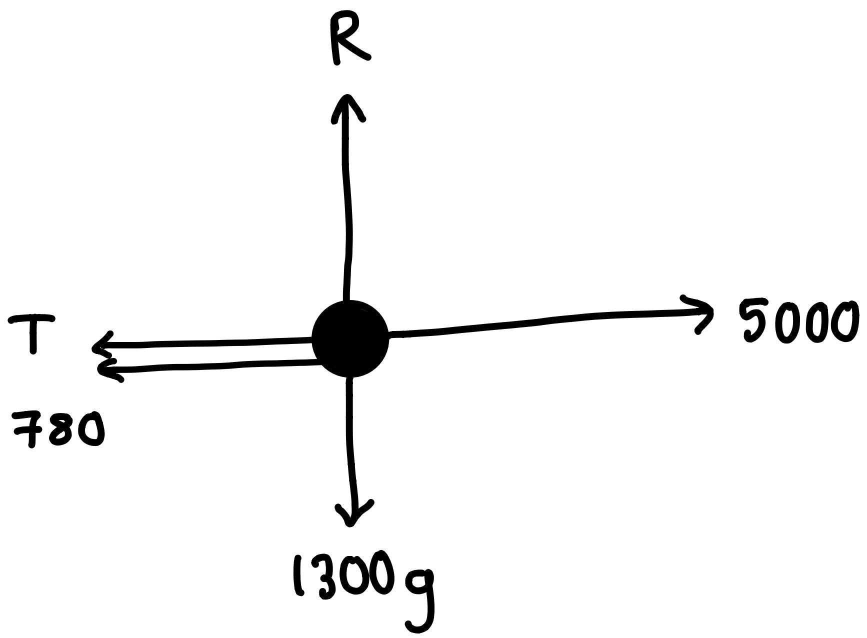
[1 mark]



where T is the tension

17 (a) (ii) Draw a diagram to show the forces acting on the van while the crate is being lifted.

[1 mark]



17 (b) Show that the acceleration of the van is 0.80 m s^{-2}

[4 marks]

Use Newton's second law to create equations of motion for both the crate and van.

Crate: $T - 300g = 300a$

$T - 300(9.8) = 300a$

① $T - 2940 = 300a$

Van: $5000 - 780 - T = 1300a$

② $4220 - T = 1300a$

Solve simultaneously to find a: ① + ②: $4220 - 2940 = 1600a$

$1280 = 1600a$

$a = \frac{1280}{1600}$

$a = 0.80$ ms^{-2}

17 (c) Find the tension in the rope.

[2 marks]

Substitute value for a into ① to find T:

$T - 2940 = 300(0.8)$

$T = 2940 + 240$

$T = 3180$

$T = 3200$ N (2.s.f)

17 (d) Suggest how the assumption of a constant resistance force could be refined to produce a better model.

[1 mark]

Resistance increases with speed.

END OF QUESTIONS