MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

AS
MATHEMATICS
Paper 1

Wednesday 13 May 2020 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

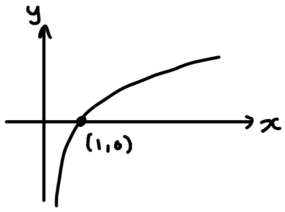
Answer **all** questions in the spaces provided.

1 At the point (1, 0) on the curve $y = \ln x$, which statement below is correct?

Tick (✓) **one** box.

[1 mark]

The gradient is negative and decreasing	☐
The gradient is negative and increasing	☐
The gradient is positive and decreasing	✓
The gradient is positive and increasing	☐



2 Given that $f(x) = 10$ when $x = 4$, which statement below must be correct?

Tick (✓) **one** box.

[1 mark]

$f(2x) = 5$ when $x = 4$	☐
$f(2x) = 10$ when $x = 2$	✓
$f(2x) = 10$ when $x = 8$	☐
$f(2x) = 20$ when $x = 4$	☐

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3 Jia has to solve the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

Jia's working is as follows:

$$2 - 2(1 - \cos^2\theta) = \cos\theta$$

$$2 - 2 + 2\cos^2\theta = \cos\theta$$

$$2\cos^2\theta = \cos\theta$$

$$2\cos\theta = 1$$

$$\cos\theta = 0.5$$

$$\theta = 60°$$

Jia's teacher tells her that her solution is incomplete.

3 (a) Explain the **two** errors that Jia has made.

[2 marks]

Between lines 3 and 4 Jia should not have cancelled by $\cos\theta$.

Jia has left out the other solution to the equation $\cos\theta = 0.5$.

3 (b) Write down all the values of θ that satisfy the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

[2 marks]

$2\cos^2\theta - \cos\theta = 0$

$\cos\theta(2\cos\theta - 1) = 0 \Rightarrow \cos\theta = 0$ and $\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \pm 90°$ and $\theta = \pm 60°$

Turn over ▶

Do not write outside the box

4 In the binomial expansion of $(\sqrt{3}+\sqrt{2})^4$ there are two irrational terms.

Find the difference between these two terms.

[3 marks]

$(\sqrt{3}+\sqrt{2})^4 = \binom{4}{0}(\sqrt{3})^4 + \binom{4}{1}(\sqrt{3})^3(\sqrt{2}) + \binom{4}{2}(\sqrt{3})^2(\sqrt{2})^2 + \binom{4}{3}(\sqrt{3})(\sqrt{2})^3 + \binom{4}{4}(\sqrt{2})^4$

Difference between irrational terms: $\binom{4}{1}(\sqrt{3})^3(\sqrt{2}) - \binom{4}{3}(\sqrt{3})(\sqrt{2})^3$

$= 4(3\sqrt{3})(\sqrt{2}) - 4\sqrt{3}(2\sqrt{2})$

$= 12\sqrt{6} - 8\sqrt{6}$

$= 4\sqrt{6}$

Do not write outside the box

5 Differentiate from first principles

$$y = 4x^2 + x$$

[4 marks]

$$\lim_{h \to 0} \frac{4(x+h)^2 + (x+h) - (4x^2 + x)}{h} = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + x + h - 4x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{8xh + 4h^2 + h}{h}$$

$$= \lim_{h \to 0} 8x + 4h + 1$$

$$= 8x + 1$$

Do not write outside the box

6 (a) It is given that

$$f(x) = x^3 - x^2 + x - 6$$

Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

[2 marks]

$f(2) = 2^3 - 2^2 + 2 - 6$

$\Rightarrow f(2) = 0$

This shows that $(x-2)$ is a factor.

6 (b) Find the quadratic factor of $f(x)$.

[1 mark]

$$\begin{array}{r} x^2 + x + 3 \\ x-2 \overline{) x^3 - x^2 + x - 6} \\ -(x^3 - 2x^2) \\ 0 \quad x^2 + x \\ -(x^2 - 2x) \\ 0 \quad 3x - 6 \\ -(3x-6) \\ 0 \end{array}$$

so $f(x) = (x-2)(x^2 + x + 3)$ and so the quadratic factor is $x^2 + x + 3$.

6 (c) Hence, show that there is only one real solution to $f(x) = 0$

[3 marks]

$f(x) = (x-2)(x^2 + x + 3)$

There is clearly a real solution at $x = 2$.

For the quadratic: $x^2 + x + 3 = 0$

Discriminant $= b^2 - 4ac = 1^2 - 4(1)(3) = -11 < 0$.

Since the discriminant is negative, there are no real solutions to the quadratic.

Therefore $x = 2$ is the only real solution to $f(x) = 0$.

Do not write outside the box

6 (d) Find the exact value of x that solves

$$e^{3x} - e^{2x} + e^{x} - 6 = 0$$

[3 marks]

$e^{3x} - e^{2x} + e^{x} - 6 = 0$

Set $y = e^{x}$.

Then $y^3 - y^2 + y - 6 = 0$.

By question 6c, there is only the solution $y = 2$.

So, $2 = e^{x}$

$\Rightarrow x = \ln 2$

Turn over for the next question

Do not write outside the box

7 Curve *C* has equation $y = x^2$

C is translated by vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give curve C_1

Line *L* has equation $y = x$

L is stretched by scale factor 2 parallel to the x-axis to give line L_1

Find the exact distance between the two intersection points of C_1 and L_1

[6 marks]

Curve C: $y = x^2$

Curve C_1: $y = (x-3)^2$

Line L: $y = x$

Line L_1: $y = \frac{1}{2}x$

Intersection points of L_1 and C_1: $(x-3)^2 = \frac{1}{2}x$

$x^2 - 6x + 9 = \frac{1}{2}x$

$2x^2 - 13x + 18 = 0$

$(2x-9)(x-2) = 0$

$x = \frac{9}{2}$ or $x = 2$

When $x = \frac{9}{2}$, $y = \frac{9}{4}$: $(\frac{9}{2}, \frac{9}{4})$

When $x = 2$, $y = 1$: $(2, 1)$

Distance $= \sqrt{(\frac{9}{2} - 2)^2 + (\frac{9}{4} - 1)^2}$

$= \sqrt{\frac{25}{4} + \frac{25}{16}}$

$= \frac{5\sqrt{5}}{4}$

Turn over for the next question

Do not write outside the box

8 (a) Find the equation of the tangent to the curve $y = e^{4x}$ at the point (a, e^{4a}).

[3 marks]

$y = e^{4x}$

$\frac{dy}{dx} = 4e^{4x}$

Gradient $= 4e^{4a}$

Tangent equation at (a, e^{4a}): $y - e^{4a} = 4e^{4a}(x-a)$

$y = 4e^{4a}x - 4ae^{4a} + e^{4a}$

8 (b) Find the value of a for which this tangent passes through the origin.

[2 marks]

$y = 4e^{4a}x - 4ae^{4a} + e^{4a}$

At the origin $x=0$ and $y=0$: $-4ae^{4a} + e^{4a} = 0$

$e^{4a}(1-4a) = 0$

$(1-4a) = 0$ since $e^{4a} > 0$

$a = \frac{1}{4}$

Do not write outside the box

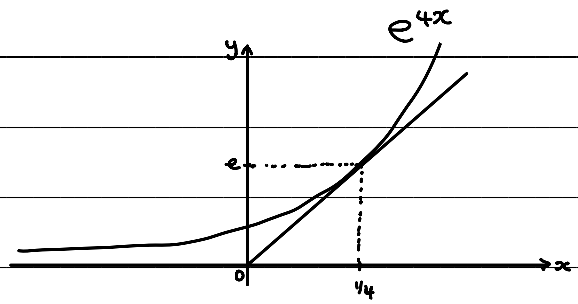
8 (c) Hence, find the set of values of m for which the equation

$$e^{4x} = mx$$

has no real solutions.

[3 marks]

$e^{4x} = mx$



As can be seen from the diagram, any negative gradient m will cut the curve and give solutions. So for no solutions there is lower bound: $0 \le m$

With $a = \frac{1}{4}$, contact point of tangent is $(\frac{1}{4}, e)$.

Gradient from $(0,0)$ to $(\frac{1}{4}, e)$: $\frac{e-0}{\frac{1}{4}-0} = 4e$

So $m < 4e$ in order for line to not touch or intersect e^{4x} in the positive quadrant.

Therefore, the values of m for which the equation has no solutions are:

$0 \le m < 4e$

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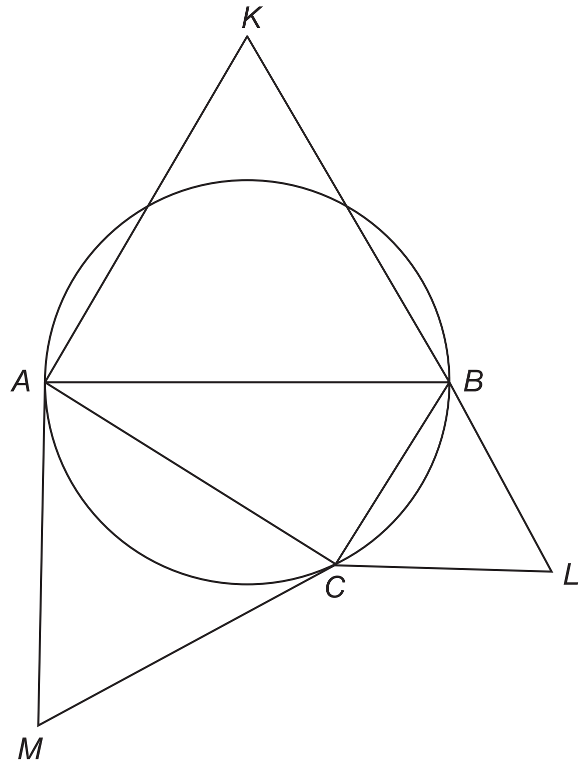
9 The diagram below shows a circle and four triangles.



AB is a diameter of the circle. *C* is a point on the circumference of the circle.

Triangles *ABK*, *BCL* and *CAM* are equilateral.

Prove that the area of triangle *ABK* is equal to the sum of the areas of triangle *BCL* and triangle *CAM*.

[5 marks]

Angle ACB is the angle in a semicircle so angle $ACB = 90°$.

By pythagoras' theorem: $AB^2 = AC^2 + BC^2$

Angles in ABK, BCL and CAM are all 60° since the triangles are equilateral.

Area of $BCL = \frac{1}{2} \times BC \times BC \times \sin 60 = \frac{1}{2} BC^2 \sin 60$

Area of $CAM = \frac{1}{2} \times AC \times AC \times \sin 60 = \frac{1}{2} AC^2 \sin 60$

Do not write outside the box

Area of $ABK = \frac{1}{2} \times AB \times AB \times \sin 60 = \frac{1}{2} AB^2 \sin 60°$

So,

(Area of BCL) + (Area of CAM) $= \frac{1}{2} (BC)^2 \sin 60 + \frac{1}{2} (AC)^2 \sin 60$

$= \frac{1}{2} (BC^2 + AC^2) \sin 60$

$= \frac{1}{2} AB^2 \sin 60$

= Area of ABK

Turn over for the next question

Do not write outside the box

10 Raj is investigating how the price, P pounds, of a brilliant-cut diamond ring is related to the weight, C carats, of the diamond.

He believes that they are connected by a formula

$$P = aC^n$$

where a and n are constants.

10 (a) Express $\ln P$ in terms of $\ln C$.

[2 marks]

$P = aC^n$

$\ln P = \ln(aC^n)$

$\ln P = \ln a + \ln C^n$

$\ln P = \ln a + n \ln C$

10 (b) Raj researches the price of three brilliant-cut diamond rings on a website with the following results.

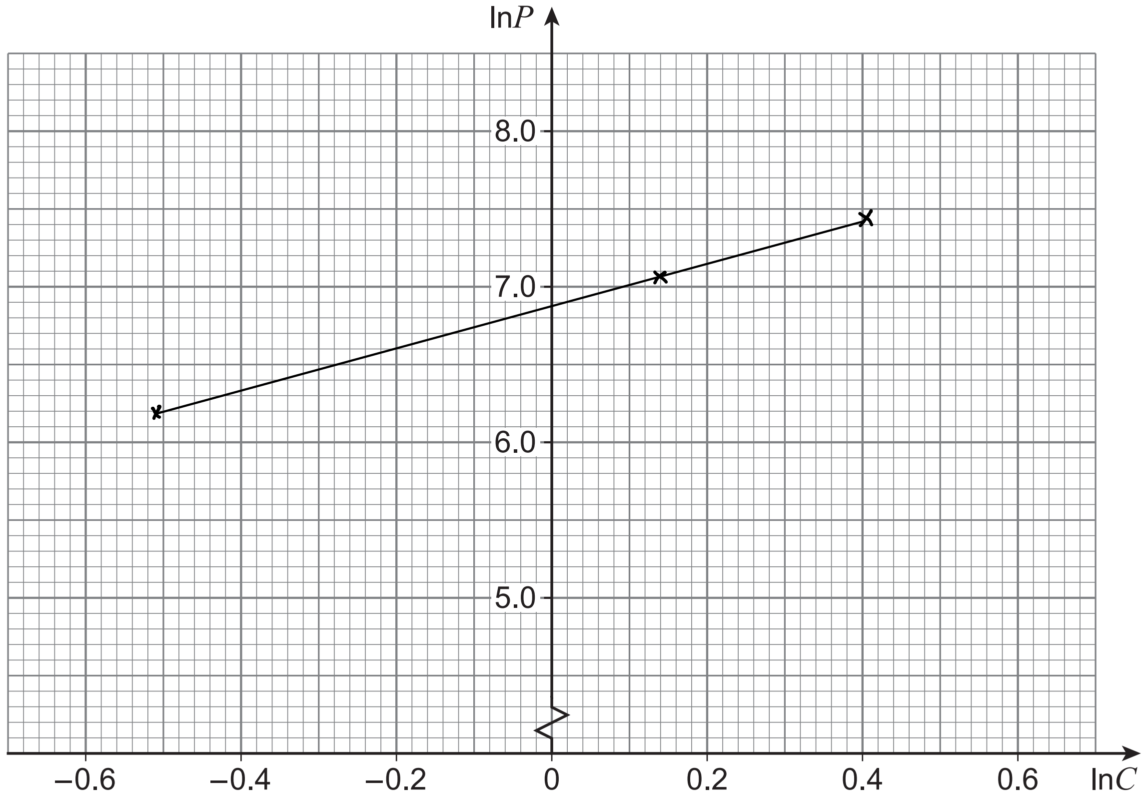
C	0.60	1.15	1.50
P	495	1200	1720

Do not write outside the box

10 (b) (i) Plot $\ln P$ against $\ln C$ for the three rings on the grid below.

[2 marks]

$\ln C$	-0.51	0.140	0.405
$\ln P$	6.20	7.09	7.45



10 (b) (ii) Explain which feature of the plot suggests that Raj's belief may be correct.

[1 mark]

The points all lie on a straight line.

Question 10 continues on the next page

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10 (b) (iii) Using the graph on page 15, estimate the value of a and the value of n.

[4 marks]

$\ln P = \ln a + n \ln C$

$\ln a$ is the intercept so: $\ln a = 6.9$

$a = e^{6.9}$

$a = 992$

n is the gradient so: $n = \dfrac{(0.405 - 0.140)}{7.45 - 7.09} = 1.36$

So $a = 992$, $n = 1.36$

10 (c) Explain the significance of a in this context.

[1 mark]

The line crosses the y axis when $\ln C = 0$.

$\ln C = 0$ when $C = 1$ so a is the price for a 1 carat diamond.

Do not write outside the box

10 (d) Raj wants to buy a ring with a brilliant-cut diamond of weight 2 carats.

Estimate the price of such a ring.

[2 marks]

$P = aC^n$

$P = 992 \times 2^{1.36}$

$P = £2546$

Turn over for the next question

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Section B

Answer **all** questions in the spaces provided.

11 A go-kart and driver, of combined mass 55 kg, move forward in a straight line with a constant acceleration of $0.2\,\text{m s}^{-2}$

The total driving force is 14 N

Find the total resistance force acting on the go-kart and driver.

Circle your answer.

[1 mark]

0 N　　3 N　　11 N　　14 N

$F = ma$
$14 - R = 55(0.2)$
$14 - R = 11 \Rightarrow R = 3$

12 One of the following is an expression for the distance between the points represented by position vectors $5\mathbf{i} - 3\mathbf{j}$ and $18\mathbf{i} + 7\mathbf{j}$

Identify the correct expression.

Tick (✓) **one** box.

[1 mark]

Distance $= \sqrt{(18-5)^2 + (7--3)^2}$
$= \sqrt{13^2 + 10^2}$

$\sqrt{13^2 + 4^2}$ ☐

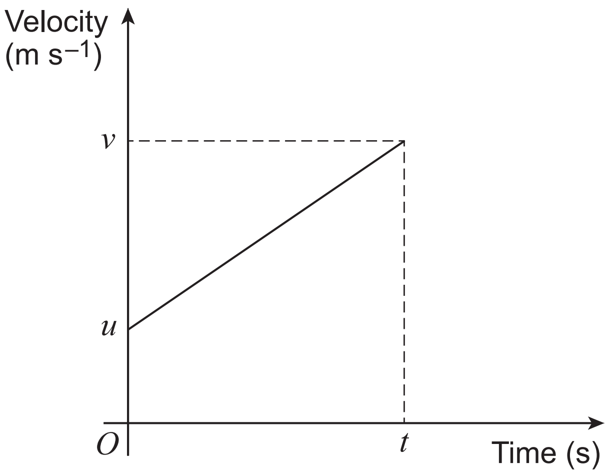
$\sqrt{13^2 + 10^2}$ ☑

$\sqrt{23^2 + 4^2}$ ☐

$\sqrt{23^2 + 10^2}$ ☐

13 An object is moving in a straight line, with constant acceleration $a\,\text{m s}^{-2}$, over a time period of t seconds.

It has an initial velocity u and final velocity v as shown in the graph below.



Use the graph to show that

$$v = u + at$$

[3 marks]

Acceleration is given by the gradient of the velocity-time graph:

$$a = \frac{v-u}{t}$$

$$\Rightarrow at = v - u$$

$$\Rightarrow v = u + at$$

Turn over for the next question

Do not write outside the box

14 A particle of mass 0.1 kg is initially stationary.

A single force **F** acts on this particle in a direction parallel to the vector $7\mathbf{i} + 24\mathbf{j}$

As a result, the particle accelerates in a straight line, reaching a speed of $4\,\text{m s}^{-1}$ after travelling a distance of 3.2 m

Find **F**.

[5 marks]

$S = 3.2 \qquad v^2 = u^2 + 2as$

$u = 0 \qquad \Rightarrow 2as = v^2 - u^2$

$v = 4 \qquad \Rightarrow a = \dfrac{v^2 - u^2}{2s} \Rightarrow a = \dfrac{4^2 - 0^2}{2 \times 3.2} = 2.5\,\text{ms}^{-2}$

$a =$

$t =$

$F = ma$

$F = 0.1 \times 2.5$

$F = 0.25\,\text{N}$

$\underline{F}$ is parralel to $7i + 24j$. Magnitude of $7i + 24j$ is $\sqrt{7^2 + 24^2} = 25$.

The magnitude of $\underline{F}$ is 0.25 so as $0.25 = \frac{1}{100} \times 25$, we get:

$$\underline{F} = \frac{1}{100}(7i + 24j)$$

Turn over for the next question

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15 A particle, P, is moving in a straight line with acceleration $a\,\mathrm{m\,s^{-2}}$ at time t seconds, where

$$a = 4 - 3t^2$$

15 (a) Initially P is stationary.

Find an expression for the velocity of P in terms of t.

[2 marks]

$a = 4 - 3t^2$

$v = \int a \, dt$

$v = \int 4 - 3t^2 \, dt$

$v = 4t - t^3 + c$

When $t=0$, $v=0$ so $c=0$.

Therefore, $v = 4t - t^3$.

Do not write outside the box

15 (b) When $t = 2$, the displacement of P from a fixed point, O, is 39 metres.

Find the time at which P passes through O, giving your answer to three significant figures.

Fully justify your answer.

[5 marks]

$s = \int v \, dt$

$s = \int 4t - t^3 \, dt$

$s = 2t^2 - \frac{t^4}{4} + c$

When $t=2$, $s=39$: $39 = 2(2)^2 - \frac{(2)^4}{4} + c$

$c = 35$

$s = 2t^2 - \frac{t^4}{4} + 35$

When P passes through the origin: $2t^2 - \frac{t^4}{4} + 35 = 0$

$\frac{t^4}{4} - 2t^2 - 35 = 0$

$t^4 - 8t^2 - 140 = 0$

Let $y = t^2$: $y^2 - 8y - 140 = 0$

$y = \frac{8 \pm \sqrt{64+140}}{2}$

$y = 16.49$ or $y = -8.49$

Since $y = t^2$, $y > 0$ so $y = 16.49$

So $t^2 = 16.49$

$\Rightarrow t = \pm\sqrt{16.49} \Rightarrow t = \sqrt{16.49}$ as $t > 0$.

$\Rightarrow t = 4.06$ seconds

Turn over ▶

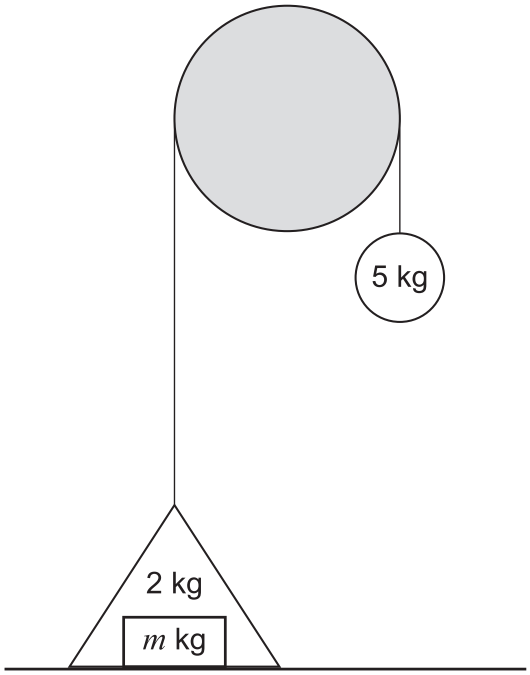
16 A simple lifting mechanism comprises a light inextensible wire which is passed over a smooth fixed pulley.

One end of the wire is attached to a rigid triangular container of mass 2 kg, which rests on horizontal ground.

A load of m kg is placed in the container.

The other end of the wire is attached to a particle of mass 5 kg, which hangs vertically downwards.

The mechanism is initially held at rest as shown in the diagram below.



The mechanism is released from rest, and the container begins to move upwards with acceleration a m s^{-2}

The wire remains taut throughout the motion.

Do not write outside the box

16 (a) Show that

$$a = \left(\frac{3-m}{m+7}\right) g$$

[4 marks]

Force = mass x acceleration

For the container: $T - mg - 2g = (m+2)a \Rightarrow T = mg + 2g + (m+2)a$

For the particle: $5g - T = 5a \Rightarrow T = 5g - 5a$

Equating T: $mg + 2g + (m+2)a = 5g - 5a$

$a(5 + m + 2) = 5g - 2g - mg$

$a(m+7) = (3-m)g$

$a = \left(\frac{3-m}{m+7}\right) g$

16 (b) State the range of possible values of m.

[1 mark]

Container begins to move upwards with acceleration a ms^{-2} so $a > 0$.

By $a = \left(\frac{3-m}{m+7}\right) g$, m must be in the region $0 < m < 3$.

Question 16 continues on the next page

Do not write outside the box

16 (c) **In this question use $g = 9.8\,\text{m s}^{-2}$**

The load reaches a height of 2 metres above the ground 1 second after it is released.

Find the mass of the load.

[4 marks]

$S = 2$ $\quad s = ut + \frac{1}{2}at^2$

$u = 0$ $\quad \frac{1}{2}at^2 = S - ut$

$V =$ $\quad a = \frac{2(s - ut)}{t^2}$

$a =$

$t = 1$ $\quad$ So, $a = \frac{2(2-0)}{1} = 4\ \text{ms}^{-2}$.

$a = \left(\frac{3-m}{m+7}\right)g$

$7a + ma = 3g - mg$

$mg + ma = 3g - 7a$

$m(g + a) = 3g - 7a$

$m = \frac{3g - 7a}{g + a}$

$m = \frac{3(9.8) - 7(4)}{9.8 + 4} = 0.101449\ldots$

So $\quad m = 0.10\ \text{kg}$

Do not write outside the box

16 (d) Ignoring air resistance, describe **one** assumption you have made in your model.

[1 mark]

Assumed that the particle is at least 2m above the ground.

END OF QUESTIONS

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