AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

A-level
FURTHER MATHEMATICS
Paper 3 - Statistics

Exam Date | Morning | Time allowed: 2 hours

Materials
For this paper you must have:
- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Mechanics **or** Discrete). You will have 2 hours to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A χ^2-test for association is carried out on frequency data given in a 5×3 contingency table using the 5% level of significance. All expected frequencies are greater than 5

State the number of degrees of freedom for this test.

Circle your answer.

[1 mark]

6 8 14 15

$v = (5-1) \times (3-1)$
$= 4 \times 2$
$= 8$

2 The continuous random variable Y has cumulative distribution function defined by

$$F(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^2}{36} & 0 \le y \le 6 \\ 1 & y > 6 \end{cases}$$

Find the value of $P(Y > 4)$

Circle your answer.

[1 mark]

$\frac{4}{9}$ $\quad$ $\frac{5}{9}$ $\quad$ $\frac{16}{27}$ $\quad$ $\frac{11}{27}$

$P(X>4) = 1 - F(4) = 1 - \frac{4^2}{36} = 1 - \frac{16}{36} = \frac{20}{36} = \frac{5}{9}$

3 The continuous random variable R follows a rectangular distribution with probability density function given by

$$f(r) = \begin{cases} k & -a \le r \le b \\ 0 & \text{otherwise} \end{cases}$$

Prove, using integration, that $E(R) = \frac{1}{2}(b - a)$

[4 marks]

$\int_{-a}^{b} k\, dr = [kr]_{-a}^{b} = kb - (-ka) = k(b+a) = 1$

$= k = \frac{1}{a+b}$.

$E(R) = \int_{-a}^{b} r f(r)\, dr = \int_{-a}^{b} kr\, dr = \left[\frac{1}{2}kr^2\right]_{-a}^{b} = \frac{1}{2}k(b^2 - (-a)^2)$

$= \frac{1}{2}\left(\frac{1}{a+b}\right)(b^2 - a^2)$

$E(R) = \frac{1}{2(a+b)}(a+b)(a-b) = \frac{1}{2}(a-b)$

4 David, a zoologist, is investigating a particular species of monitor lizard. He measures the lengths, in centimetres, of a random sample of this particular species of lizard. His measured lengths are

53.2 57.8 55.3 58.9 59.0 60.2 61.8 62.3 65.4 66.5

The lengths may be assumed to be normally distributed.

David correctly constructed a 90% confidence interval for the mean length of lizard using the measured lengths given and the formula $\bar{x} \pm \left(b \times \frac{s}{\sqrt{n}} \right)$

This interval had limits of 57.63 and 62.45, correct to two decimal places.

4 (a) State the value for b used in David's formula.

[1 mark]

T-distribution with 10 - 1 = 9 degrees of freedom

At 90%, t-value = 1.83

4 (b) David interprets his interval and states,

"My confidence interval indicates that exactly 90% of the population of lizard lengths for this particular species lies between 57.63 cm and 62.45 cm".

Do you think David's statement is true? Explain your reasoning.

[2 marks]

David's statement may not be true because the interval is centred on the sample mean not the population mean

4 (c) David's assistant, Amina, correctly constructs a β% confidence interval from David's random sample of measured lengths.

Amina informs David that the width of her confidence interval is 8.54.

Find the value of β.

[3 marks]

Confidence interval width $= 2\frac{bs}{\sqrt{n}}$

$62.45 - 57.63 = 4.82$

$\frac{1}{2}(4.82) = \frac{bs}{\sqrt{n}} \Rightarrow s = \frac{2.41\sqrt{n}}{b} = \frac{2.41\sqrt{10}}{1.83} = 4.1645$

Using β as the confidence interval:

$2 \times \frac{\alpha \times s}{\sqrt{n}} = 8.54$

$\Rightarrow \alpha = \frac{\sqrt{10}(8.54)}{2(4.1654)} = 3.242$ and from t-tables $\beta = 99$

Turn over for the next question

5 Students at a science department of a university are offered the opportunity to study an optional language module, either German or Mandarin, during their second year of study.

From a sample of 50 students who opted to study a language module, 31 were female.

Of those who opted to study Mandarin, 8 were female and 12 were male.

Test, using the 5% level of significance, whether choice of language is independent of gender.

The sample of students may be regarded as random.

[8 marks]

H_0: Language choice is independent of gender

H_1: Language choice is not independent of gender

	German		Mandarin		
	O	E	O	E	
M	7	11.4	12	7.6	19
F	23	18.6	8	12.4	31
	30		20		50

General Rule: $E_i = \frac{(\text{row total})(\text{column total})}{\text{table total}}$

$$\chi^2 = \sum \frac{(|O-E|-0.5)^2}{E} = \frac{(11.4-7-0.5)^2}{11.4} + \frac{(23-18.6-0.5)^2}{18.6} + \frac{(12-7.6-0.5)^2}{7.6} + \frac{(12.4-8-0.5)^2}{12.4} = 5.38$$

$v = (2-1)\times(2-1) = 1 \Rightarrow$ critical value $= 3.84$

$5.38 > 3.84$ so the result is significant

Therefore we reject H_0 so there is evidence to suggest that language choice is not independent of gender

Turn over for the next question

6 The random variable T can take the value $T = -2$ or any value in the range $0 \leq T < 12$

The distribution of T is given by $P(T = -2) = c$, $P(0 \leq T \leq t) = 225k - k(15 - t)^2$

6 (a) (i) Show that $1 - c = 216k$

[3 marks]

Sum of probabilities $= c + 225k - k(15-12)^2 = 1$

$\Rightarrow c + 225k - 9k = 1$

$\Rightarrow c + 216k = 1$

$\Rightarrow 1 - c = 216k$

6 (a) (ii) Given that $c = 0.1$, find the value of $\mathrm{E}(T)$

[3 marks]

$1 - c = 216k \Rightarrow 0.9 = 216k \Rightarrow k = \frac{0.9}{216} = \frac{1}{240}$

$E(T) = -2(c) + \int_0^{12} t f(t)\, dt$ $\qquad F(t) = 225k - k(15-t)^2$

$= -0.2 + \int_0^{12} 2k(15t - t^2)\, dt$ $\qquad$ So $f(t) = \frac{d}{dt}F(t)$

$= \frac{d}{dt}(225k - k(15-t)^2)$

$= -0.2 + 2k\int_0^{12} 15t - t^2\, dt$ $\qquad = 2k(15-t)$

$= -0.2 + 2\left(\frac{1}{240}\right)\left[\frac{15}{2}t^2 - \frac{1}{3}t^3\right]_0^{12}$

$= -0.2 + \frac{1}{120}\left(7.5(12)^2 - \frac{1}{3}(12)^3\right) = -0.2 + 4.2 = 4 \Rightarrow E(T) = 4$

6 (b) Show that $\mathrm{E}(\sqrt{|T|}\ ) = \dfrac{5\sqrt{2} + 52\sqrt{3}}{50}$

[3 marks]

$E(\sqrt{|T|}) = 0.1(\sqrt{|-2|}) + \int_0^{12} 2k\sqrt{t}(15 - t)\, dt$

$= \frac{\sqrt{2}}{10} + 2k\int_0^{12} 15t^{\frac{1}{2}} - t^{3/2}\, dt = \frac{\sqrt{2}}{10} + \frac{1}{120}\left[10t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^{12}$

$= \frac{\sqrt{2}}{10} + \frac{1}{120}\left(10(12)^{3/2} - \frac{2}{5}(12)^{5/2}\right) = \frac{\sqrt{2}}{10} + \frac{1}{120}(12)^{3/2}\left(10 - \frac{2}{5}(12)\right)$

$= \frac{\sqrt{2}}{10} + \frac{24\sqrt{3}}{120}\left(10 - \frac{24}{5}\right) = \frac{\sqrt{2}}{10} + \frac{24\sqrt{3}}{120}\left(\frac{26}{5}\right) = \frac{\sqrt{2}}{10} + \frac{26\sqrt{3}}{25}$

$= \frac{5\sqrt{2} + 52\sqrt{3}}{50}$

$E(|T|) = \frac{5\sqrt{2} + 52\sqrt{3}}{50}$

7 Petroxide Industries produces a chemical used in the production of mobile phone covers for a mobile phone company.

The chemical becomes less effective when the mean level of impurity is greater than 3 per cent.

Sunita is the Quality Control manager at Petroxide Industries. After a complaint from the mobile phone company, Sunita obtains a random sample of this chemical from 9 batches.

She measures the level of impurity, X per cent, in each sample.

The summarised results are as follows.

$$\sum x = 28.8 \qquad \sum (x - \bar{x})^2 = 0.6$$

7 (a) (i) Investigate using the 5% level of significance whether the mean level of impurity in the chemical is greater than 3 per cent.

[7 marks]

$H_0: \mu = 3 \quad H_1: \mu > 3$

$\bar{x} = \frac{28.8}{9} = 3.2 \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{0.6}{8}$

$= 0.075$

$T = \frac{X - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.2 - 3}{\sqrt{\frac{0.075}{3}}} = 2.19$

Degrees of freedom: $9 - 1 = 8$ Critical value at 5% $= 1.8595$

$2.19 > 1.8595$ so the result is significant

Reject H_0. There is evidence to suggest that the impurity level is too high

7 (a) (ii) State the assumption that it was necessary for you to make in order for the test in part **(a)(i)** to be valid.

[1 mark]

The level of impurity has a normal distribution

7 (b) State the changes that would be required to your test in part **(a)** if you were told that the standard deviation of the level of impurity is known to be 0.25 per cent.

[2 marks]

We could use the normal distribution rather than the t-distribution since σ is known. We'd use $\sigma = 0.25$ instead of s

Turn over for the next question

8 The time in hours to failure of a component may be modelled by an exponential distribution with parameter $\lambda = 0.025$

In a manufacturing process, the machine involved uses one of these components continuously until it fails.

The component is then immediately replaced.

8 (a) Write down the mean time to failure for a component.

[1 mark]

Mean $= \frac{1}{0.025} = 40$ hours

8 (b) Find the probability that a component will fail during a 12-hour shift.

[1 mark]

$P(T < 12) = 1 - e^{-12(0.025)} = 1 - e^{-0.3}$

$= 0.2592$ (4dp)

8 (c) A component has not failed for 30 hours. Find the probability that this component lasts for at least another 30 hours.

[2 marks]

The exponential distribution has a memoryless property so the answer is $P(T > 30) = e^{-30(0.025)} = e^{-0.75}$

$= 0.472$

8 (d) Find the probability that a component does **not** fail during 4 consecutive 12-hour shifts.

[3 marks]

Four consecutive 12 hour shifts is 48 hours
so the new mean is 48
$P(T > 48) = e^{-0.025(48)} = e^{-1.2} = 0.301$

8 (e) (i) State the distribution that can be used to model the number of components that fail during one hour of the manufacturing process.

[2 marks]

$Po(0.025)$ (Poisson distribution)

8 (e) (ii) Hence, or otherwise, find the probability that no components fail during 5 consecutive 12-hour shifts.

[2 marks]

5 consecutive 12 hour shifts is 60 hours
$60 \times 0.025 = 1.5$
Let $X \sim Po(1.5)$
$P(X = 0) = e^{-1.5} = 0.223$

END OF QUESTIONS