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AQA

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Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

A-level
FURTHER MATHEMATICS
Paper 3 Statistics

MODEL ANSWERS

Thursday 13 June 2019 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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8	
TOTAL	

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has $\mathrm{Var}(X) = 5$

Find $\mathrm{Var}(4X - 3)$

$\mathrm{Var}(aX-b) = a^2\,\mathrm{Var}(X)$

Circle your answer.

$4^2 \times 5 = 16 \times 5 = 80$

[1 mark]

17 20 77 80

2 Amy takes a sample of size 50 from a normal distribution with mean μ and variance 16

She conducts a hypothesis test with hypotheses:

$H_0 : \mu = 52$

$H_1 : \mu > 52$

She rejects the null hypothesis if her sample has a mean greater than 53

The actual population mean is 53.5

Find the probability that Amy makes a Type II error.

false negative

Circle your answer.

$\mu \neq 52$ but $\bar{\mu} \leq 53$

[1 mark]

0.4% 3.9% 18.9% 45.0%

sample $\bar{\mu} \sim N(\mu, \frac{\sigma^2}{n})$

in this case, $\bar{\mu} \sim N(53.5, \frac{16}{50})$

$P(\bar{\mu} \leq 53) = 18.9\%$

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3 Alan's journey time to work can be modelled by a normal distribution with standard deviation 6 minutes.

Alan measures the journey time to work for a random sample of 5 journeys. The mean of the 5 journey times is 36 minutes.

3 (a) Construct a 95% confidence interval for Alan's mean journey time to work, giving your values to one decimal place.

[2 marks]

95% confidence interval $\Rightarrow z = 1.96$

$\frac{1+0.95}{2} = 0.975$

Inv. normal (0.975)
$= 1.96$

so $\bar{x} \pm z\sqrt{\frac{\sigma^2}{n}} = 36 \pm 1.96 \frac{6}{\sqrt{5}}$

$= (30.7, 41.3)$

3 (b) Alan claims that his mean journey time to work is 30 minutes.

State, with a reason, whether or not the confidence interval found in part **(a)** supports Alan's claim.

[1 mark]

30 minutes is outside the confidence interval, so Alan's claim isn't supported

3 (c) Suppose that the standard deviation is not known but a sample standard deviation is found from Alan's sample and calculated to be 6

Explain how the working in part **(a)** would change.

[1 mark]

a t distribution will be used, not a normal distribution

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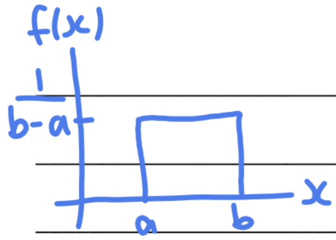
4 A random variable X has a rectangular distribution.

The mean of X is 3 and the variance of X is 3

4 (a) Determine the probability density function of X.

Fully justify your answer.

[5 marks]



for a rectangular distribution, $E(X) = \frac{1}{2}(a+b)$

so $\frac{1}{2}(a+b) = 3$.

$Var(X) = \frac{1}{12}(b-a)^2$ so $\frac{1}{12}(b-a)^2 = 3$

$a+b=6 \Rightarrow a = 6-b$

sub in: $\frac{1}{12}(b-6+b)^2 = 3$

$(2b-6)^2 = 36$

$4b^2 - 24b + 36 = 36$

$b(4b-24) = 0$

$\hookrightarrow b = 6$ or $a = 0$

$a+b=6$ so if $b=6$, $a=0$, & vice versa

$$\Rightarrow f(x) = \begin{cases} \frac{1}{6} & 0 \le x \le 6 \\ 0 & \text{else} \end{cases}$$

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4 (b) A 6 metre clothes line is connected between the point P on one building and the point Q on a second building.

Roy is concerned the clothes line may break. He uses the random variable X to model the distance in metres from P where the clothes line breaks.

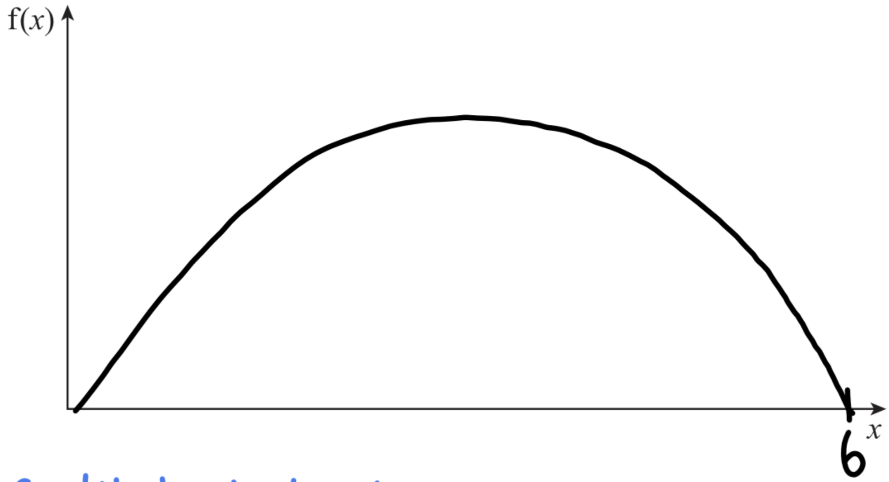
4 (b) (i) State a criticism of Roy's model.

[1 mark]

X is uniform along the length of the line, but the line is more likely to break in the middle.

4 (b) (ii) On the axes below, sketch the probability density function for an alternative model for the clothes line.

[1 mark]



More likely to break in middle ⇒ f(x) higher

Otherwise, pdf is symmetrical.

Turn over for the next question

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5 An insurance company models the claims it pays out in pounds (£) with a random variable X which has probability density function

$$f(x) = \begin{cases} \dfrac{k}{x} & 1 < x < a \\ 0 & \text{otherwise} \end{cases}$$

5 (a) The median claim is £200

Show that $k = \dfrac{1}{2 \ln 200}$

[3 marks]

median $\Rightarrow$ P $= 0.5$ $\displaystyle\int_1^{200} \frac{k}{x}\,dx = 0.5$

non-zero

$\rightarrow [k \ln x]_1^{200} = 0.5$

$k \ln 200 = 0.5$

$\Rightarrow k = \dfrac{0.5}{\ln 200} = \dfrac{1}{2 \ln 200}$

5 (b) Find $P(X < 2000)$, giving your answer to three significant figures.

[2 marks]

using value for k: $\dfrac{1}{2\ln 200} \displaystyle\int_1^{2000} \frac{1}{x}\,dx = \underline{0.717}$

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5 (c) The insurance company finds that the maximum possible claim is £2000 and they decide to refine their probability density function.

Suggest how this could be done.

[2 marks]

max claim is £2000, claim can't be <0, so non-zero range for f(x) is 1<x<2000. i.e., set a=2000. change k so that P(X<2000) = 1 (as it is impossible for claim to exceed 2000)

Turn over for the next question

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6 During August, 102 candidates took their driving test at centre *A* and 60 passed.

During the same month, 110 candidates took their driving test at centre *B* and 80 passed.

6 (a) Test whether the driving test result is independent of the driving test centre using the 5% level of significance.

[8 marks]

1. state hypotheses

H_0: there is no association between the driving test centre & the result

H_1: there is an association between the test result & the centre

2. write out observed table

O	Pass	Fail	total
A	60	42	102
B	80	30	110
total	140	72	212

3. calculate expected values using fractions of totals

E	Pass	Fail
A	$\frac{140}{212} \times \frac{102}{212} \times 212 = 67.4$	$\frac{72 \times 102}{212} = 34.6$
B	$\frac{140 \times 110}{212} = 72.6$	$\frac{72 \times 110}{212} = 37.4$

4. calculate χ^2-test statistic

modulus

Yates correction

$$\chi^2 = \frac{(|60-67.4|-0.5)^2}{67.4} + \frac{(|42-34.6|-0.5)^2}{34.6} + \frac{(|80-72.6|-0.5)^2}{72.6} + \frac{(|30-37.4|-0.5)^2}{37.4} = 4.0$$

5. calculate critical value & make conclusion

d.o.f. $= (2-1)(2-1) = 1 \Rightarrow \chi^2_{1,0.05} = 3.84$. $4.0 > 3.84 \therefore$ reject H_0 as there is sufficient evidence to suggest that driving test result & test centre are not independent.

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5. Calculate critical value & make conclusion

d.o.f. $= (2-1)(2-1) = 1 \Rightarrow \chi^2_{1,0.05} = 3.84$. $4.0 > 3.84$ $\therefore$ reject H_0 as there is sufficient evidence to suggest that driving test result & test centre are not independent.

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6 (b) Rebecca claims that if the result of the test in part **(a)** is to reject the null hypothesis then it is easier to pass a driving test at centre *B* than centre *A*.

State, with a reason, whether or not you agree with Rebecca's claim.

[1 mark]

there still isn't evidence to support this claim, as there may be another, underlying cause for the higher pass rate at B.

Turn over for the next question

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7 A shopkeeper sells chocolate bars which are described by the manufacturer as having an average mass of 45 grams.

The shopkeeper claims that the mass of the chocolate bars, X grams, is getting smaller on average.

A random sample of 6 chocolate bars is taken and their masses in grams are measured. The results are

$$\sum x = 246 \quad \text{and} \quad \sum x^2 = 10\,198$$

Investigate the shopkeeper's claim using the 5% level of significance.

State any assumptions that you make.

[9 marks]

1. state hypotheses $H_0: \mu = 45$; $H_1: \mu < 45$

2. state assumption for a t-test

we assume X can be modelled by a normal distribution

3. calculate sample values for t-test

$$\bar{x} = \frac{\Sigma x}{n} = \frac{246}{6} = 41$$

$$s^2 = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right) = \frac{1}{6-1}\left(10198 - \frac{246^2}{6}\right)$$

$$= 22.4$$

4. calculate t-test statistic

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}} = \frac{41 - 45}{\sqrt{\frac{22.4}{6}}} = -2.07$$

5. find critical value & make conclusion

t_5 @ 95% $= 2.015$

confidence level

$-2.07 < -2.015$ $\therefore$ reject H_0 as there is sufficient evidence to support the claim that the mass of chocolate bars is getting lighter

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Turn over for the next question

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8 The number of telephone calls received by an office can be modelled by a Poisson distribution with mean 3 calls per 10 minutes.

8 (a) Find the probability that:

8 (a) (i) the office receives exactly 2 calls in 10 minutes;

[1 mark]

$X \sim Po(3)$

$P(X=2) = 0.224$

8 (a) (ii) the office receives more than 30 calls in an hour.

[3 marks]

3 calls / 10 mins ⇒ x6 ⇒ 18 calls / hour

$Y \sim Po(18)$

$P(Y>30) = P(Y \geq 31)$

$= 1 - P(Y \leq 30)$

$= 0.0033$

8 (b) The office manager splits an hour into 6 periods of 10 minutes and records the number of telephone calls received in each of the 10 minute periods.

Find the probability that the office receives exactly 2 calls in a 10 minute period exactly twice within an hour.

[3 marks]

constant probability in each 10min period ⇒ can model as binomial distribution, using value in (a)(i)

$C \sim B(6, 0.224)$

10 x 6 → 1 hour

$P(C=2) = \binom{6}{2} \times 0.224^2 \times (1-0.224)^4$

$= 0.273$

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8 (c) The office has just received a call.

8 (c) (i) Find the probability that the next call is received more than 10 minutes later.

[3 marks]

exponential distribution $\Rightarrow T \sim \exp(3)$

$P(T > 1) = e^{-3 \times 1}$

$= 0.0498$

8 (c) (ii) Mahah arrives at the office 5 minutes after the last call was received.

State the probability that the next call received by the office is received more than 10 minutes later.

Explain your answer.

[2 marks]

P = 0.0498 from (c)(ii). the distribution is memoryless, so the probability calculated is unaffected by the time passed since the last call.

END OF QUESTIONS

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