Model Answers

AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
FURTHER MATHEMATICS
Paper 3 - Mechanics

Exam Date | Morning | Time allowed: 2 hours

Materials

For this paper you must have:

- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A ball of mass 0.2 kg is travelling horizontally at 7 m s^{-1} when it hits a vertical wall.

It rebounds horizontally at 5 m s^{-1}

Find the magnitude of the impulse exerted on the ball by the wall.

Circle your answer.

[1 mark]

0.4 N s   1.4 N s   2 N s   2.4 N s

$I = m\Delta v$

$= 0.2(5 - -7)$

$= 0.2(12)$

$= 2.4$ Ns

2 In this question

a represents acceleration,

T represents time,

l represents length,

m represents mass,

v represents velocity,

F represents force.

One of these formulae is dimensionally consistent.

Circle your answer.

[1 mark]

$T = 2\pi\sqrt{\frac{a}{l}}$ $v^2 = \frac{2al}{T}$ $Fl = mv^2$ $FT = m\sqrt{a}$

$Fl = mv^2$

$ma \times l = mv^2$

$a \times l = v^2$

$$\frac{[L]}{[T]^2} \times [L] = \left(\frac{[L]}{[T]}\right)^2$$

So they are dimensionally consistent

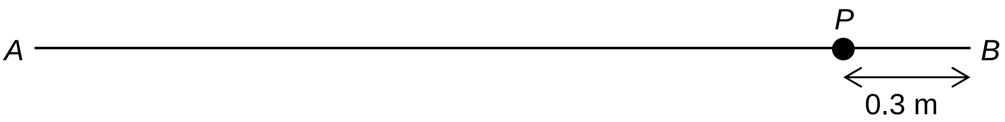
Turn over for the next question

3 **In this question use g = 9.8 m s^{-2}.**

A composite body consists of a uniform rod, *AB*, and a particle.

The rod has length 4 metres and mass 22.5 kilograms.

The particle, *P*, has mass 20 kilograms and is placed on the rod at a distance of 0.3 metres from *B*, as shown in the diagram.



3 (a) Find the distance of the centre of mass of the body from *A*.

[2 marks]

$$\bar{x} = \frac{\Sigma mx}{\Sigma m}$$

$$= \frac{3.7 \times 20 + 2 \times 22.5}{20 + 22.5}$$

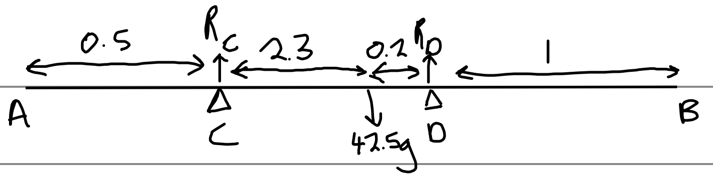
$$= 2.8 \text{ m}$$

3 (b) The body rests in equilibrium in a horizontal position on two supports, *C* and *D*.

The support *C* is 0.5 metres from *A* and the support *D* is 1 metre from *B*.

Find the magnitudes of the forces exerted on the body by the supports.

[4 marks]



$M(C): 2.3 \times 42.5g - 2.5R_D = 0$

$R_D = \frac{2.3 \times 42.5g}{2.5}$

$R_D = 383.2\text{ N}$ $R_D = 380\text{ N}$ (2sf)

$R(\uparrow): R_c + R_d - 42.5g = 0$

$R_c = 42.5g - R_D = 42.5g - 383.2$

$R_c = 33.3\text{N}$

$R_c = 33\text{N}$ (2sf)

Turn over for the next question

4 Two discs, A and B, have equal radii and masses 0.8 kg and 0.4 kg respectively. The discs are placed on a horizontal surface.

The discs are set in motion when they are 3 metres apart, so that they move directly towards each other, each travelling at a speed of 6 m s^{-1}. The discs collide directly with each other.

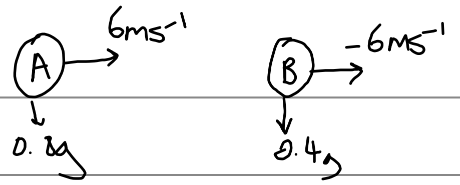
After the collision A moves in the opposite direction with a speed of 1.2 m s^{-1}

The coefficient of restitution between the two discs is e.

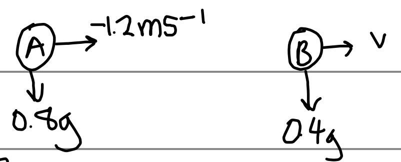
4 (a) Assuming that the surface is smooth, show that $e = 0.8$

[5 marks]

Before:



After:



$e = \frac{v - -1.2}{6 - -6} = \frac{v+1.2}{12}$

Conservation of momentum: $6(0.8) - 6(0.4) = -1.2(0.8) + 0.4v$

$4.8 - 2.4 = -0.96 + 0.4v$

$v = \frac{1}{0.4}(4.8 - 2.4 + 0.96)$

$v = 8.4 \text{ ms}^{-1}$

$e = \frac{1.2+v}{12} = \frac{1.2+8.4}{12} = \frac{9.6}{12} = 0.8$

4 (b) Describe one way in which the model you have used could be refined.

[1 mark]

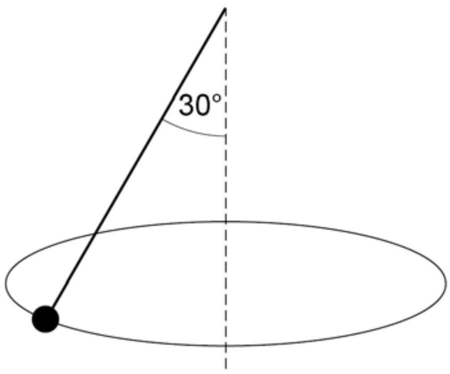
Take friction into account as the speeds of the particles would decrease as the particles approach each other

Turn over for the next question

5 **In this question use g = 9.8 m s^{-2}.**

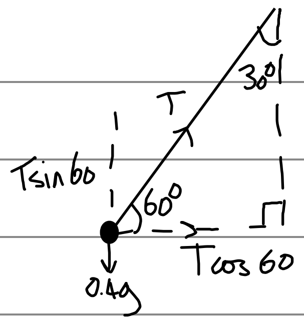
A conical pendulum consists of a string of length 60 cm and a particle of mass 400 g.

The string is at an angle of 30° to the vertical, as shown in the diagram.



5 (a) Show that the tension in the string is 4.5 N.

[2 marks]



$R(\uparrow)$: $T\sin 60 - 0.4g = 0$

$T = \frac{0.4g}{\sin 60}$

$= \frac{0.4(9.8)}{\sqrt{3}/2} = 4.5\text{ N (2sf)}$

$T = 4.5\text{ N}$

5 (b) Find the angular speed of the particle.

[3 marks]

$l = 0.6 \Rightarrow r = 0.6\cos 60 = 0.3\text{m}$

$T\cos 60 = m\omega^2 r$

$= 0.4(0.3)\omega^2$

$\Rightarrow \sqrt{\frac{4.5\cos 60}{0.4(0.3)}} = \omega$

$\omega = 4.33 \text{ rad s}^{-1}$

5 (c) State **two** assumptions that you have made about the string.

[1 mark]

That it's light and inextensible

Turn over for the next question

6 A uniform solid is formed by rotating the region enclosed by the positive x-axis, the line $x = 2$ and the curve $y = \frac{1}{2}x^2$ through 360° around the x-axis.

6 (a) Find the centre of mass of this solid.

[5 marks]

$V\bar{x} = \int_0^a \pi x y^2 \, dx$ where $V = \int_0^a \pi y^2 \, dx$

$V = \int_0^2 \pi \left(\frac{1}{2}x^2\right)^2 dx = \frac{\pi}{4}\int_0^2 x^4 \, dx = \frac{\pi}{4}\left[\frac{x^5}{5}\right]_0^2 = \frac{\pi}{20}(2)^5$

$\int_0^2 \pi x y^2 \, dx = \int_0^2 \pi x \left(\frac{1}{2}x^2\right)^2 dx = \frac{\pi}{4}\int_0^2 x^5 \, dx = \frac{\pi}{4}\left[\frac{x^6}{6}\right]_0^2 = \frac{\pi}{24}(2)^6$

$V\bar{x} = \int_0^2 \pi x y^2 \, dx \Rightarrow \frac{\pi}{20}(2)^5 \bar{x} = \frac{\pi}{24}(2)^6$

$\Rightarrow \frac{1}{5}(2)^5 \bar{x} = \frac{1}{6}(2)^6$

$\Rightarrow \frac{1}{5}\bar{x} = \frac{1}{6}(2)$

$\Rightarrow \bar{x} = \frac{5}{6}(2) = \frac{5}{3}$

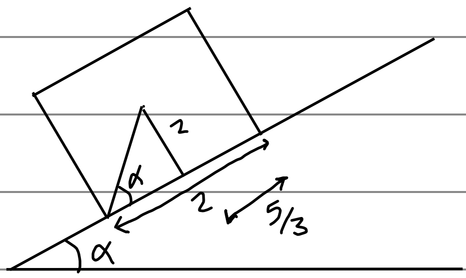
6 (b) The solid is placed with its plane face on a rough inclined plane and does not slide.

The angle between the inclined plane and the horizontal is gradually increased.

When the angle between the inclined plane and the horizontal is α, the solid is on the point of toppling.

Find α, giving your answer to the nearest 0.1°

[2 marks]



$\tan\alpha = \frac{2}{2 - 5/3}$

$\tan\alpha = 6$

$\alpha = 80.5°$

7 **In this question use $g = 9.8$ m s^{-2}.**

When a car, of mass 1200 kg, travels at a speed of v m s^{-1} it experiences a total resistive force which can be modelled as being of magnitude $36v$ newtons.

The maximum power of the car is 90 kilowatts.

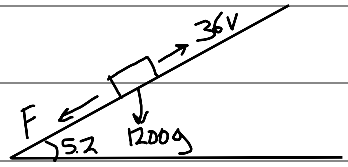
The car starts to descend a hill, inclined at 5.2° to the horizontal, along a straight road.

Find the maximum speed of the car down this hill.

[5 marks]

$P = Fv \Rightarrow F = \frac{P}{v}$

$\Rightarrow F = \frac{90,000}{v}$



$\Sigma F = ma$

$\frac{90,000}{v} + 1200g\sin(5.2) - 36v = 0$ (a = 0 when the speed is its greatest)

$90,000 + (1200g\sin(5.2))v - 36v^2 = 0$

Using a scientific calculator to solve the quadratic:

$v = -37.342$ or $v = 66.449$

$v > 0 \Rightarrow v = 67$ ms^{-1} (2sf)

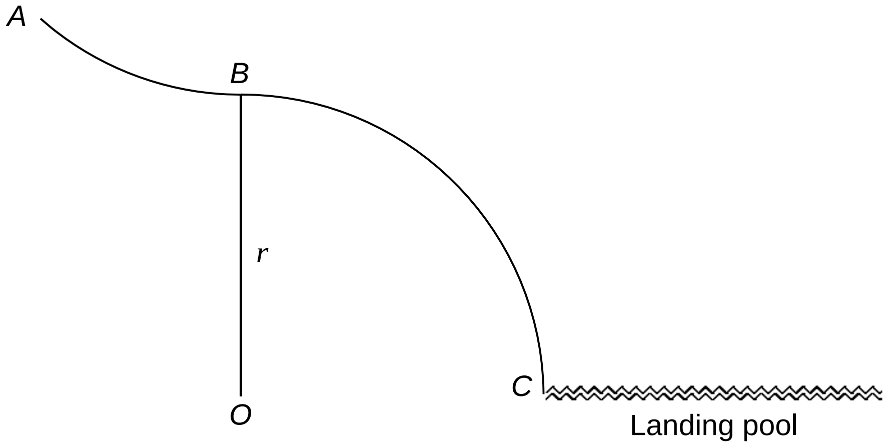
8 The diagram shows part of a water park slide, *ABC*.

The slide is in the shape of two circular arcs, *AB* and *BC*, each of radius *r*.

The point *A* is at a height of $\frac{r}{4}$ above *B*.

The circular arc *BC* has centre *O* and *B* is vertically above *O*.

These points are joined as shown in the diagram.



A child starts from rest at *A*, moves along the slide past the point *B* and then loses contact with the slide at a point *D*.

The angle between the vertical, *OB*, and *OD* is θ

Assume that the slide is smooth.

8 (a) Show that the speed v of the child at *D* is given by $v = \sqrt{\frac{gr}{2}(5 - 4\cos\theta)}$, where g is the acceleration due to gravity.

[3 marks]

At A: $GPE + KE = (r + \frac{1}{4}r)mg + 0$

At B: $GPE + KE = mg(r\cos\theta) + \frac{1}{2}mv^2$

Conservation of energy:

$mg(r + \frac{1}{4}r) = mg(r\cos\theta) + \frac{1}{2}mv^2$

$mg(\frac{5}{4}r - r\cos\theta) = \frac{1}{2}mv^2$

$\frac{gr}{4}(5 - 4\cos\theta) = \frac{1}{2}v^2$

$\frac{gr}{2}(5 - 4\cos\theta) = v^2$

$v = \sqrt{\frac{gr}{2}(5 - 4\cos\theta)}$

8 (b) Find θ, giving your answer to the nearest degree.

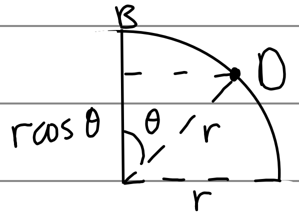
[2 marks]

Resolving radially:

$mg\cos\theta = \frac{mv^2}{r}$

$g\cos\theta = \frac{1}{r}\left(\frac{gr}{2}(5-4\cos\theta)\right)$

$= \frac{g}{2}(5-4\cos\theta)$



$\cos\theta = \frac{5}{2} - 2\cos\theta$

$3\cos\theta = \frac{5}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{6}\right) = 34°$ (2sf)

8 (c) A refined model takes into account air resistance. Explain how taking air resistance into account would affect your answer to part **(b)**.

[2 marks]

Air resistance would slow down the child so v would decrease and therefore so would θ

8 (d) In reality the slide is not smooth. It has a surface with the same coefficient of friction between the slide and the child for its entire length.

Explain why the frictional force experienced by the child is not constant.

[1 mark]

$F = \mu R$

As the reaction force R the child experiences, the force F will also vary

9 **In this question use g = 9.8 m s^{-2}.**

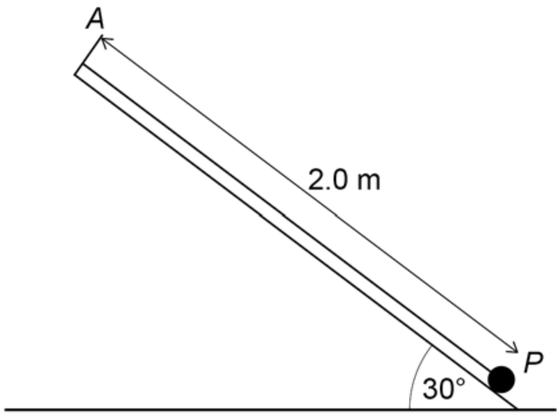
A light elastic string has one end attached to a fixed point, *A*, on a rough plane inclined at 30° to the horizontal.

The other end of the string is attached to a particle, *P*, of mass 2 kg.

The elastic string has natural length 1.3 metres and modulus of elasticity 65 N.

The particle is pulled down the plane in the direction of the line of greatest slope through *A*.

The particle is released from rest when it is 2 metres from *A*, as shown in the diagram.



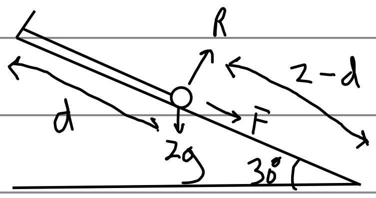
The coefficient of friction between the particle and the plane is 0.6

After the particle is released it moves up the plane.

The particle comes to rest at a point *B*, which is a distance, d metres, from *A*.

9 (a) Show that the value of d is 1.4.

[7 marks]



F = Friction

$EPE = \frac{\lambda x^2}{2l} = \frac{65(0.7)^2}{2(1.3)} = 12.25J$

Using work-energy principle:

$$EPE = F(2-d) + (mg\sin 30)(2-d) + \frac{65(d-1.3)^2}{2(1.3)}$$

$$12.25 = (\mu R + mg\sin 30)(2-d) + 25(d-1.3)^2$$

$$12.25 = (10.1845 + 9.8)(2-d) + 25(d^2 - 2.6d + 1.69)$$

$$12.25 = 20.369 - 10.1845d + 19.6 - 9.8d + 25d^2 - 65d + 42.25$$

$$0 = 25d^2 - 84.9845d + 69.969$$

Using a scientific calculator: $d = 1.399$ or $d = 2$

$d \neq 2m$ as that is where the particle starts therfor $d = 1.4m$

9 (b) Determine what happens after *P* reaches the point *B*.

Fully justify your answer.

[3 marks]

Weight of the ball parallel to the plane: $2g\sin 30$
$= g = 9.8N$

$F_{max} = \mu R = 10.1845$ (From before)

$10.1845 > 9.8$ so the particle does not slide back down the plane

END OF QUESTIONS