MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 3 Mechanics

Thursday 11 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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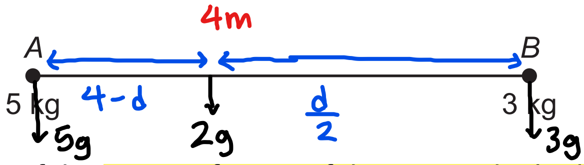
Answer **all** questions in the spaces provided.

1 A uniform rigid rod, *AB*, has mass 2 kg and length 4 metres.

Two particles of masses 5 kg and 3 kg are fixed to *A* and *B* respectively to create a composite body, as shown in the diagram.



Find the distance of the centre of mass of the composite body from *B*.

Circle your answer.

[1 mark]

1.5 metres   1.6 metres   2.4 metres   2.5 metres

$M(B): \frac{5(4) + 2(\frac{4}{2})}{2+3+5} = 2.4\text{ m}$

2 The tension, T newtons, in a spring is given by $T = 20e$, where e metres is the extension of the spring.

Calculate the work done when the extension is increased from 0.2 metres to 0.4 metres.

Circle your answer.

[1 mark]

0.4 J   0.9 J   1.2 J   1.6 J

$T = \frac{\lambda x}{l} = 20e$

$EPE = \frac{\lambda x^2}{2l} = \frac{\lambda x}{l} \cdot \frac{x}{2} = \frac{Tx}{2} = \frac{20e^2}{2} = 10e^2$

$\text{Work Done} = \Delta EPE = 10(0.4)^2 - 10(0.2)^2 = 1.6 - 0.4 = 1.2\text{ J}$

$\therefore WD = 1.2\text{ J}$

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3 The speed, v, of a particle moving in a horizontal circle is given by the formula $v = r\omega$ where:

$v =$ speed

$r =$ radius

$\omega =$ angular speed.

Show that the dimensions of angular speed are T^{-1}

[2 marks]

$v = r\omega$

$LT^{-1} = L\omega$

$\therefore \omega = T^{-1}$

Turn over for the next question

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4 A car has mass 1000 kg and travels on a straight horizontal road.

The maximum speed of the car on this road is $48\,\text{m s}^{-1}$

In a simple model, it is assumed that the car experiences a resistance force that is proportional to its speed.

When the car travels at $20\,\text{m s}^{-1}$, the magnitude of the resistance force is 600 newtons.

4 (a) Show that the maximum power of the car is 69 120 W

[2 marks]

$R = kv$

$600 = k \cdot 20$

$\therefore k = \frac{600}{20} = 30$

$\therefore R = 30v$

$P = Fv = 30v^2 = 30(48)^2 = 69120\text{ W}$

$\therefore P = 69120\text{ W}$

4 (b) Find the maximum acceleration of the car when it is travelling at $25\,\text{m s}^{-1}$

[3 marks]

$\Delta F = ma \rightarrow \frac{P}{v} - 30v = ma$

$\frac{69120}{25} - 30(25) = 1000a$

$\frac{10074}{5} = 1000a$

$a = \frac{10074}{5} \div 1000$

$\therefore a = 2.01\text{ ms}^{-1}$ (3sf)

Do not write outside the box

4 (c) Find the maximum acceleration of the car when it is travelling at $3\,\text{m}\,\text{s}^{-1}$

[1 mark]

$\frac{69120}{3} - 30(3) = 1000a$

$22950 = 1000a$

$\therefore a = 22.95 \approx 23.0\ \text{ms}^{-1}$ (3sf)

4 (d) Comment on the validity of the model in the context of your answers to parts **(b)** and **(c)**.

[2 marks]

Acceleration is very high when speed is 3ms^{-1}.
∴ Model only seems valid for higher speeds.

Turn over for the next question

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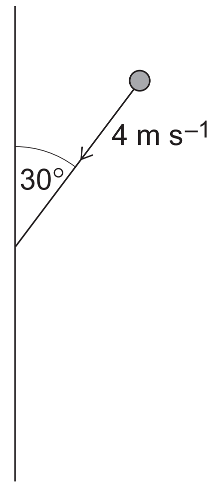
5 A ball, of mass 0.3 kg, is moving on a smooth horizontal surface.

The ball collides with a smooth fixed vertical wall and rebounds.

Before the ball hits the wall, the ball is moving at 4 m s^{-1} at an angle of 30° to the wall as shown in the diagram.



The magnitude of the force, F newtons, exerted on the ball by the wall at time t seconds is modelled by

$$F = kt^2(0.1 - t)^2 \quad \text{for} \quad 0 \le t \le 0.1$$

where k is a constant.

The ball is in contact with the wall for 0.1 seconds.

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5 (a) Show that the impulse exerted on the ball by the wall while they are in contact has magnitude $\frac{k}{3\,000\,000}$

Fully justify your answer.

[4 marks]

$$I = \int_0^{0.1} kt^2(0.1-t)^2 \, dt$$

$$= k\int_0^{0.1} t^2(t^2 - 0.2t + 0.01) \, dt$$

$$= k\int_0^{0.1} t^4 - 0.2t^3 + 0.01t^2 \, dt$$

$$= k\left[\frac{t^5}{5} - \frac{0.2t^4}{4} + \frac{0.01t^3}{3}\right]_0^{0.1}$$

$$= k\left[\frac{0.1^5}{5} - \frac{0.1^4}{20} + \frac{0.1^3}{300} - 0\right]$$

$$= \frac{k}{3000000}$$

$$\therefore I = \frac{k}{3000000}$$

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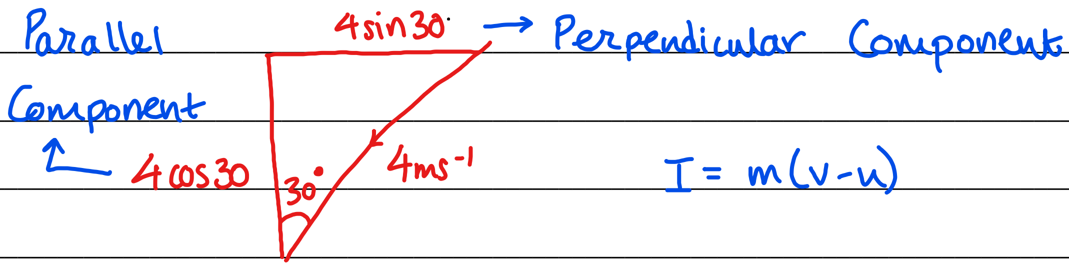
5 (b) Explain why $1\,800\,000 < k \leq 3\,600\,000$

Fully justify your answer.

[5 marks]



$I = m(v-u)$

$I_{max} = 2(\text{Perpendicular Velocity} \cdot \text{Mass})$
$= 2(0.3 \cdot 4\sin 30)$
$= 1.2$

$I_{min} = 0.3 \cdot 4\sin 30 = 0.6$

$I_{min} < \frac{k}{3\,000\,000} \leq I_{max}$

$0.6 \cdot 3000000 < k \leq 1.2 \cdot 3000000$

$\therefore 1800\,000 < k \leq 3\,600\,000$

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5 (c) Given that $k = 2\,400\,000$

Find the speed of the ball after the collision with the wall.

[4 marks]

$I = \frac{2400000}{3000000} = 0.8$

$I = m(v-u)$

$0.8 = 0.3(v - -4\sin 30)$

$\frac{8}{3} = v + 2$

$\therefore v = \frac{8}{3} - 2 = \frac{2}{3}$

$\text{Speed} = \sqrt{\left(\frac{2}{3}\right)^2 + (4\cos 30)^2}$

$\therefore \text{Speed} = \frac{4\sqrt{7}}{3} \approx 3.53 \text{ ms}^{-1}$ (3sf)

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6 A particle moves with constant speed on a circular path of radius 2 metres.

The centre of the circle has position vector 2**j** metres.

At time $t = 0$, the particle is at the origin and is moving in the positive **i** direction.

The particle returns to the origin every 4 seconds.

The unit vectors **i** and **j** are perpendicular.

6 (a) Calculate the angular speed of the particle.

[2 marks]

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore \omega = \frac{\pi}{2}$ rad s^{-1}

6 (b) Write down an expression for the position vector of the particle at time t seconds.

[2 marks]

$r = 2\sin/\cos(\omega t)\,\underline{i} + (2\sin/\cos(\omega t) + c)\,\underline{j}$

$\therefore r = 2\sin\left(\frac{\pi t}{2}\right)\underline{i} + \left(2 - 2\cos\left(\frac{\pi t}{2}\right)\right)\underline{j}$ metres

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6 (c) Find an expression for the acceleration of the particle at time t seconds.

[3 marks]

$v = \frac{dr}{dt} = \frac{\pi}{2}(2)\cos\left(\frac{\pi t}{2}\right)i + \frac{\pi}{2}(-2)\left(-\sin\left(\frac{\pi t}{2}\right)\right)j$

$\therefore v = \pi\cos\left(\frac{\pi t}{2}\right)i + \pi\sin\left(\frac{\pi t}{2}\right)j$

$\therefore a = \frac{dv}{dt} = -\frac{\pi^2}{2}\sin\left(\frac{\pi t}{2}\right)i + \frac{\pi^2}{2}\cos\left(\frac{\pi t}{2}\right)j$

6 (d) State the magnitude of the acceleration of the particle.

[1 mark]

At $t = 4$, $\sin\left(\frac{\pi t}{2}\right) = 0$ & $\cos\left(\frac{\pi t}{2}\right) = 1$.

$\therefore a = \frac{\pi^2}{2}\ ms^{-2}$

6 (e) State the time when the acceleration is first directed towards the origin.

[1 mark]

At $t = 2$ seconds

7 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A box, of mass 8 kg, is on a rough horizontal surface.

A string attached to the box is used to pull it along the surface.

The string is inclined at an angle of 40° above the horizontal.

The tension in the string is 50 newtons.

As the box moves a distance of x metres, its speed increases from $2\,\text{m s}^{-1}$ to $5\,\text{m s}^{-1}$

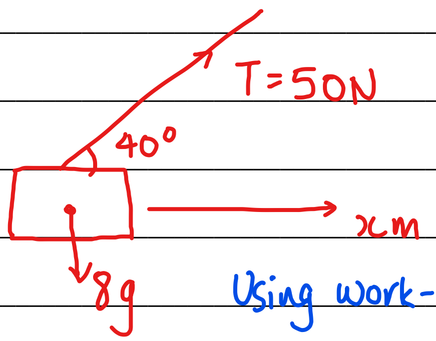
The coefficient of friction between the box and the surface is 0.4

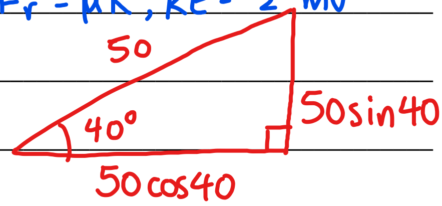
↳ $\mu = 0.4$

7 (a) By using an energy method, find x.

[6 marks]

$WD = fd,\ F_r = \mu R,\ KE = \frac{1}{2}mv^2$





Using work-energy principle:

WD by Tension + KE = WD by Friction + KE

WD by Tension $= 50\cos40\,x$

WD by Friction $= 0.4\,(8(9.8) - 50\sin40)\,x$

$= 31.36x - 20x\sin40$

$50\cos40\,x + \frac{1}{2}(8)(2)^2 = 31.36x - 20x\sin40 + \frac{1}{2}(8)(5)^2$

$50\cos40\,x - 31.36x + 20x\sin40 = 100 - 16$

$19.797\ldots x = 84$

$x = \dfrac{84}{19.797\ldots}$

$\therefore x = 4.242\ldots \approx 4.24\text{m (3sf)}$

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7 (b) Describe how the model could be refined to obtain a more realistic value of x and use an energy argument to explain whether this would increase or decrease the value of x.

[2 marks]

Model could be refined by including air resistance, which would increase the value of x because mo work would need to be done to reach the required speed.

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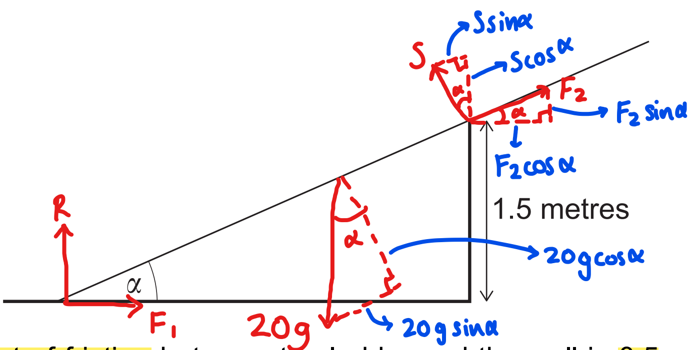
8 A ladder has length 4 metres and mass 20 kg

The ladder rests in equilibrium with one end on a horizontal surface and the ladder resting on the top of a vertical wall.

In this position the ladder is on the point of slipping.

The top of the wall is 1.5 metres above the horizontal surface.

The angle between the ladder and the horizontal surface is α, as shown in the diagram.



The coefficient of friction between the ladder and the wall is 0.5

The coefficient of friction between the ladder and the ground is also 0.5

Show that

$$\cos\alpha \sin^2\alpha = \frac{3}{10}$$

stating clearly any assumptions you make.

[8 marks]

Resolving Horizontally: $F_2\cos\alpha + F_1 = S\sin\alpha$

Resolving Vertically: $R + S\cos\alpha + F_2\sin\alpha = 20g$

Taking Moments about the base of ladder:

$20g(2\cos\alpha) = S\left(\frac{1.5}{\sin\alpha}\right) \rightarrow 40g\cos\alpha = \frac{1.5S}{\sin\alpha}$

$R = 20g - S\cos\alpha - \frac{1}{2}S\sin\alpha$

$Fr = \mu R$

$\frac{S}{2}\cos\alpha + 10g - \frac{S}{2}\cos\alpha - \frac{S}{4}\sin\alpha = S\sin\alpha$

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$$S = \frac{8g}{\sin\alpha}$$

$$40g\cos\alpha = \frac{12g}{\sin^2\alpha}$$

$$\therefore \cos\alpha \sin^2\alpha = \frac{12g}{40g} = \frac{3}{10}$$

END OF QUESTIONS

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