AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level
FURTHER MATHEMATICS

MODEL ANSWERS

Paper 3 - Discrete

Exam Date | Morning | Time allowed: 2 hours

Materials

For this paper you must have:

- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice

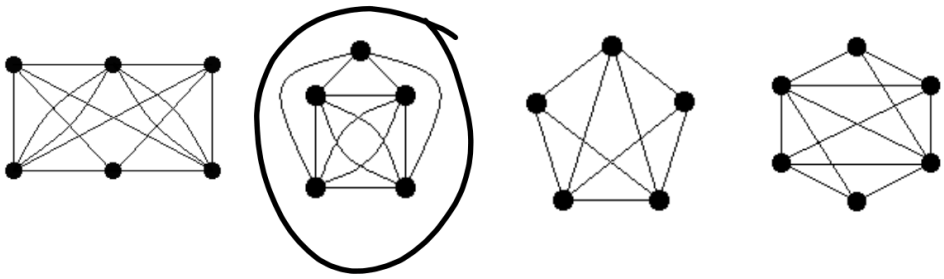
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 Which of the following graphs is **not** planar?

Circle your answer.

[1 mark]

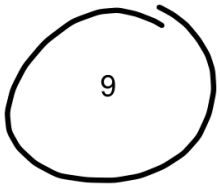


2 The set {1, 2, 4, 8, 9, 13, 15, 16} forms a group under the operation of multiplication modulo 17.

Which of the following is a generator of the group?

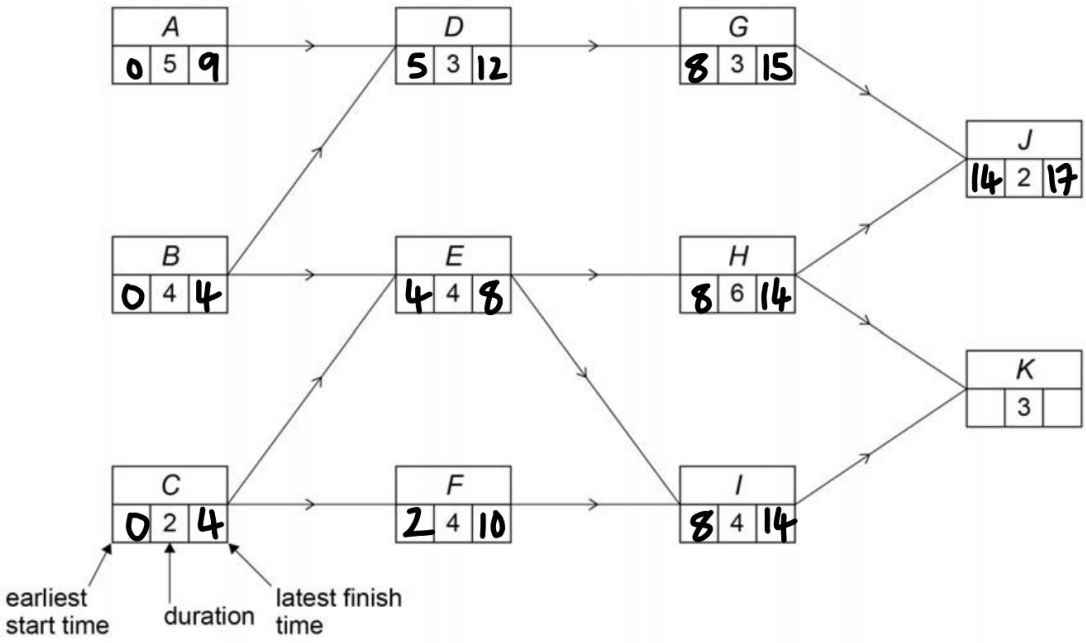
Circle your answer.

[1 mark]

4  9 13 16

3 Deva Construction Ltd undertakes a small building project. The activity network for this project is shown below in **Figure 1**, where each activity's duration is given in hours.

Figure 1



3 (a) Complete the activity network for the building project.

[2 marks]

3 (b) Deva Construction Ltd is able to reduce the duration of a single activity to 1 hour by using specialist equipment.

State, with a reason, which activity should have its duration reduced to 1 hour in order to minimise the completion time for the building project.

[3 marks]

reduce E to an hour, which will reduce the completion time to 17-3=14 hours. all other activities reduce the project completion time to 15 hours or more.

3 (c) State one limitation in the building project used by Deva Construction Ltd.

Explain how this limitation affects the project.

[2 marks]

it will take time to transition between tasks (e.g. travel between locations), which hasn't been taken into account. so the completion time will be greater than expected.

4 Optical fibre broadband cables are being installed between 5 neighbouring villages.

The distance between each pair of villages in metres is shown in the table.

A D E H I

	Alvanley	**Dunham**	**Elton**	**Helsby**	**Ince**
Alvanley	-	2000	4000	750	5500
Dunham	2000	-	2500	2250	4000
Elton	4000	2500	-	3000	1250
Helsby	750	2250	3000	-	4250
Ince	5500	4000	1250	4250	-

The company installing the optical fibre broadband cables wishes to create a network connecting each of the 5 villages using the minimum possible length of cable.

Find the minimum length of cable required.

[3 marks]

start at A.
the shortest path to connect H is 750
ditto above → —— 11 —— D is a cable between D & A, which has length 2000.
you can then connect E, the shortest path being 2500 between D & E. finally connect I with a 1250-length path from E.
750+2000+2500+1250=6500 m
connected villages: A-H, D-A, D-E, E-I

5 The binary operation * is defined as

$$a * b = a + b + 4 \pmod 6$$

where $a, b \in \mathbb{Z}$.

5 (a) Show that the set {0, 1, 2, 3, 4, 5} forms a group *G* under *.

[5 marks]

the set is closed under * since all the results of a*b are in the set
identity element = 2 ∵ a*2 = a
2 and 5 are self-inverse elements
0 and 4 are inverses of each other
1 and 3 " "
a*(b*c) = a+(b+c+4)+4 = a+b+c+8
(a*b)*c = (a+b+4)+c+4 = a+b+c+8
so (a*b)*c = a*(b*c) ⇒ we have associativity
G satisfies all 4 axioms hence is a group under *

5 (b) Find the proper subgroups of the group *G* in part **(a)**.

[2 marks]

{2}, {0, 2, 4}, {2, 5}

5 (c) Determine whether or not the group G in part **(a)** is isomorphic to the group $K = (\langle 3 \rangle, \times_{14})$

[3 marks]

$G = (\langle 17 \rangle, *)$

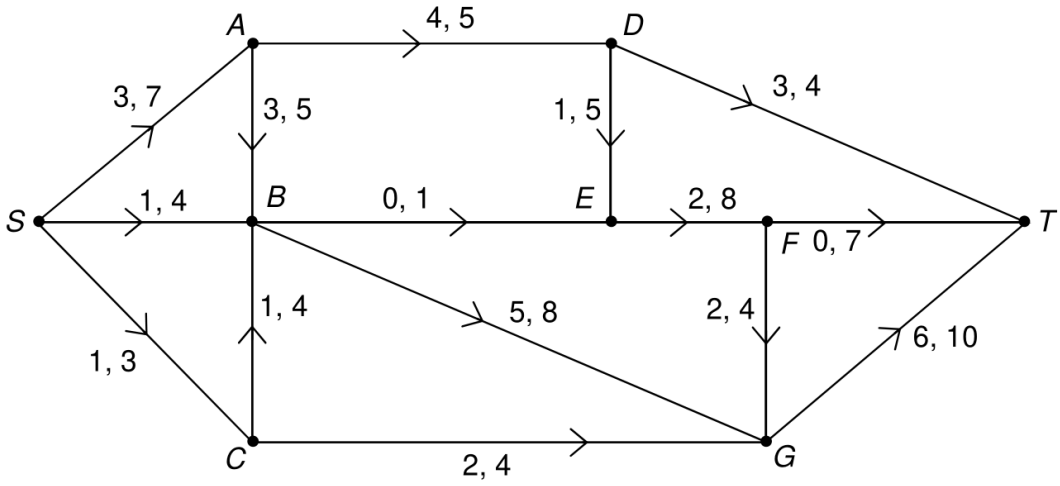
G	K
$1 \mapsto$	3
$0 \mapsto$	9
$5 \mapsto$	13
$4 \mapsto$	11
$3 \mapsto$	5
$2 \mapsto$	1

there is a one-to-one mapping between the elements of G & K, so they isomorphic

Turn over for the next question

6 The network shows a system of pipes, where S is the source and T is the sink.

The lower and upper capacities, in litres per second, of each pipe are shown on each arc.



6 (a) There is a feasible flow from S to T.

6 (a) (i) Explain why arc AD must be at its lower capacity.

[1 mark]

max flow into A = 7 & min flow out of A = 7
flow into A = flow out of A, so AB & AD must be at the lower capacity.

6 (a) (ii) Explain why arc BE must be at its upper capacity.

[1 mark]

DE = 1 & DT = 3 ∴ AD = 4. flow out of E ≥ 2, so need BE to be @ highest capacity of 1 so flow into E = 2 (DE = 1), matching flow out.

6 (b) Explain why a flow of 11 litres per second through the network is impossible.

[1 mark]

min flow {S, A, B, C} / {D, E, G, T}
= AD + BE + min(BG) + min(CG) = 4 + 1 + 5 + 2 = 12.
12 is the minimum. 11 < 12 ∴ 11 not possible.

6 (c) The network in **Figure 2** shows a second system of pipes, where *S* is the source and *T* is the sink.

The lower and upper capacities, in litres per second, of each pipe are shown on each edge.

Figure 2

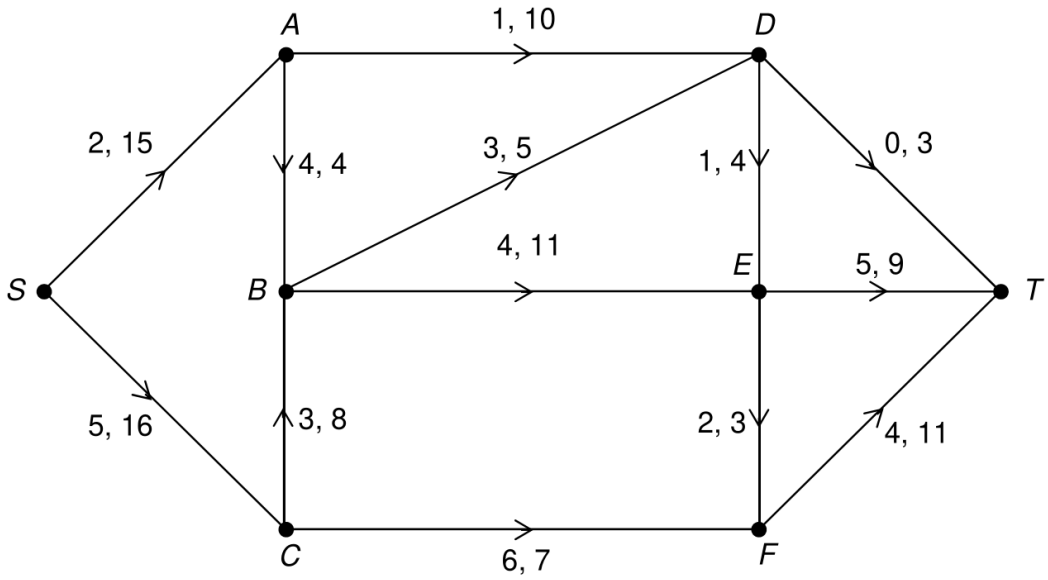
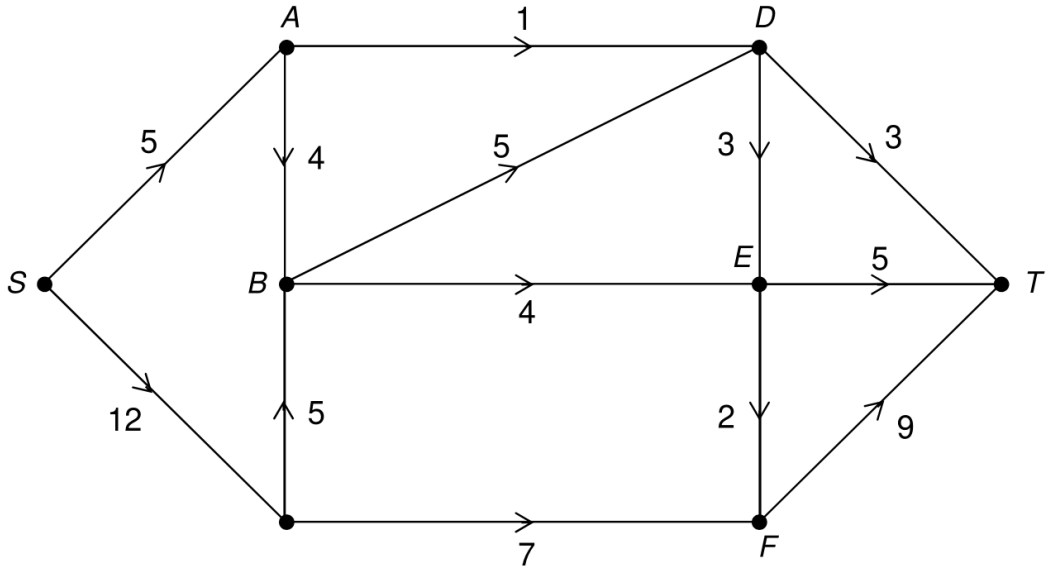


Figure 3 shows a feasible flow of 17 litres per second through the system of pipes.

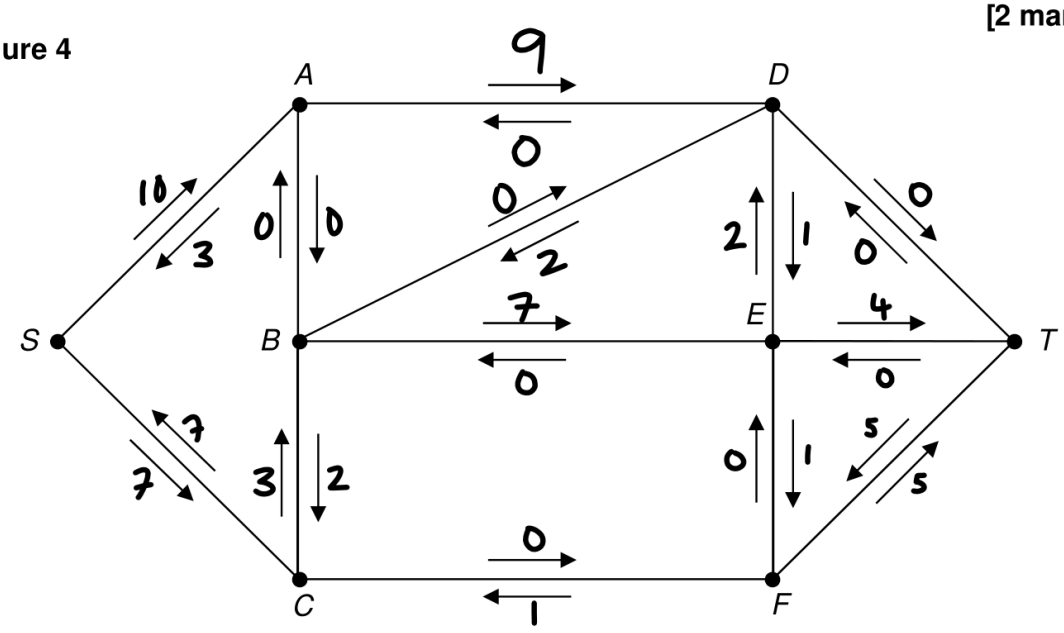
Figure 3



6 (c) (i) Using **Figures 2** and **3**, indicate on **Figure 4** potential increases and decreases in the flow along each arc.

[2 marks]

Figure 4



6 (c) (ii) Use flow augmentation on **Figure 4** to find the maximum flow from *S* to *T*.

You should indicate any flow augmenting paths clearly in the table below and modify the potential increases and decreases of the flow on **Figure 4**.

[3 marks]

Augmenting Path	Flow
SADEFT	1
SCBET	3
SADBET	1

Max flow through network = 22 litres/second

6 (c) (iii) Prove the flow found in part **(d) (ii)** is maximum.

[1 mark]

{S, A, B, C, D, E}/{F, T} = 7+3+9+3 = 22

the flow = the cut, so by the max flow-min. cut theorem, 22 is the max. flow

6 (c) (iv) Due to maintenance work, the flow through node *E* is restricted to 9 litres per second.

Interpret the impact of this restriction on the maximum flow through the system of pipes.

[2 marks]

DT & CF are already saturated so flow can only increase by 2 litres/second. Max flow = 17+2=19L/s when flow through E is restricted.

Turn over for the next question

7 A company repairs and sells computer hardware, including monitors, hard drives and keyboards.

Each monitor takes 3 hours to repair and the cost of components is £40.

Each hard drive takes 2 hours to repair and the cost of components is £20.

Each keyboard takes 1 hour to repair and the cost of components is £5.

Each month, the business has 360 hours available for repairs and £2500 available to buy components.

Each month, the company sells all of its repaired hardware to a local computer shop.

Each monitor, hard drive and keyboard sold gives the company a profit of £80, £35 and £15 respectively.

The company repairs and sells x monitors, y hard drives and z keyboards each month.

The company wishes to maximise its total profit.

7 (a) Find **five** inequalities involving x, y and z for the company's problem.

[3 marks]

7 (b) (i) Find how many of each type of computer hardware the company should repair and sell each month.

[6 marks]

7 (b) (ii) Explain how you know that you had reached the optimal solution in part **(b) (i)**.

[1 mark]

7 (b) (iii) The local computer shop complains that they are not receiving one of the types of computer hardware that the company repairs and sells.

Using your answer to part **(b) (i)**, suggest a way in which the company's problem can be modified to address the complaint.

[1 mark]

8 John and Danielle play a zero-sum game which does not have a stable solution.

The game is represented by the following pay-off matrix for John.

		Danielle		
	Strategy	***X***	***Y***	***Z***
John	***A***	2	1	–1
	B	–3	–2	2
	C	–3	–4	1

Find the optimal mixed strategy for John.

[6 marks]

END OF QUESTIONS