Model Solutions PhysicsAndMathsTutor.com

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 2

Thursday 4 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Three of the four expressions below are equivalent to each other.

Which of the four expressions is **not** equivalent to any of the others?

Circle your answer.

[1 mark]

0+a×b	a×b+0	a×b+0	0 - a×b
a × (**a** + **b**)	(**a** + **b**) × **b**	(**a** − **b**) × **b**	**a** × (**a** − **b**)

2 Given that $\arg(a + bi) = \varphi$, where a and b are positive real numbers and $0 < \varphi < \frac{\pi}{2}$, three of the following four statements are correct.

Which statement is **not** correct?

Tick (✓) **one** box.

[1 mark]

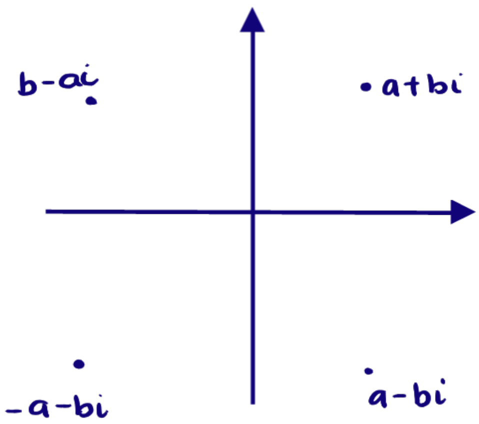
$\arg(-a-bi) = -(\pi+\varphi)$

$\arg(-a - bi) = \pi - \varphi$ ☑ ✓

$\arg(a - bi) = -\varphi$ ☐

$\arg(b + ai) = \frac{\pi}{2} - \varphi$ ☐

$\arg(b - ai) = \varphi - \frac{\pi}{2}$ ☐



Do not write outside the box

3 Find the gradient of the tangent to the curve

$$y = \sin^{-1} x$$

at the point where $x = \dfrac{1}{5}$

Circle your answer.

[1 mark]

$\dfrac{5\sqrt{6}}{12}$ $\quad$ $\dfrac{2\sqrt{6}}{5}$ $\quad$ $\dfrac{4\sqrt{3}}{25}$ $\quad$ $\dfrac{25}{24}$

$y = \sin^{-1} x$

$x = \sin y$

$\dfrac{dx}{dy} = \cos y$

$\dfrac{dy}{dx} = \sec y$

when $x = \dfrac{1}{5}$, $y = \sin^{-1}\left(\dfrac{1}{5}\right)$

when $y = \sin^{-1}\left(\dfrac{1}{5}\right)$

$\dfrac{dy}{dx} = \sec\left(\sin^{-1}\dfrac{1}{5}\right)$

$= \dfrac{5\sqrt{6}}{12}$

Turn over for the next question

Do not write outside the box

4 The matrices **A** and **B** are defined as follows:

$$\mathbf{A} = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

Show that there is a value of x for which $\mathbf{AB} = k\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix and k is an integer to be found.

[3 marks]

$$AB = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}\begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (x+1)(x-4) & (x+1)(x-2)-4 \\ (x+2)(x-4) & (x+2)(x-2)+6 \end{bmatrix}$$

$$= \begin{bmatrix} x^2-3x-4 & x^2-x-6 \\ x^2-2x-8 & x^2+2 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow x^2-x-6=0$

$(x+2)(x-3)=0$

$\underline{x=-2}$ or $x=3$

$\Rightarrow x^2-2x-8=0$

$\underline{x=-2}$ or $x=4$

$\therefore x=-2$

when $x=-2$

$(-2)^2-3(-2)-4$

$=6=6(1)$

$\therefore AB = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore k=6$

Do not write outside the box

5 Solve the inequality

$$\frac{2x+3}{x-1} \leq x+5$$

[5 marks]

Multiply by $(x-1)^2$ as we know squared numbers are positive numbers.

This means that if we multiple $(x-1)^2$ by $(x-5)$ the sign $(\leq)$ remains the same.

$$\frac{(x+1)^2(2x+3)}{(x+1)} \leq (x-1)^2(x+5)$$

$$(x-1)(2x+3) \leq (x-1)^2(x+5)$$

$$(x-1)(x-1)(x+5) - (x-1)(2x+3) \geq 0$$

$$(x-1)\big((x-1)(x+5) - (2x+3)\big) \geq 0$$

$$(x-1)(x^2+4x-5-2x-3) \geq 0$$

$$(x-1)(x^2+2x-8) \geq 0$$

$$(x-1)(x+4)(x-2) \geq 0$$

Considering cubic curve:

$x \geq 2$ or between -4 and 1

But $x \neq 1$

So $x \geq 2$ or $-4 \leq x < 1$

Turn over ▶

Do not write outside the box

6 Find the sum of all the integers from 1 to 999 inclusive that are not square or cube numbers.

[5 marks]

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{999} r = \frac{999 \times 1000}{2} = 499500$$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

$$\sum_{r=1}^{31} r^2 = \frac{31 \times 32 \times 63}{6} = 10416$$

$$\sum_{r=1}^{n} r^3 = \frac{n^2}{4}(n+1)^2$$

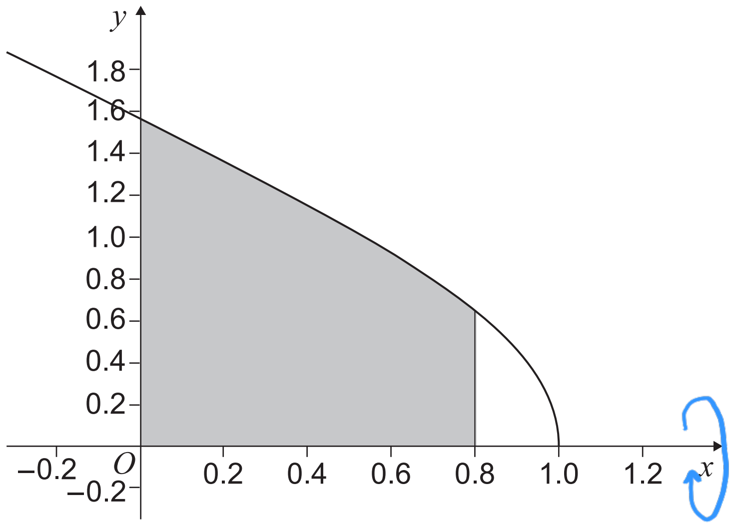
$$\sum_{r=1}^{9} r^3 = \frac{9^2 \times 10^2}{4} = 2025$$

Sixth Powers: $1 + 64 + 729 = 794$

Total: $499500 - 10416 - 2025 + 794$

$= 487853$

7 The diagram shows part of the graph of $y = \cos^{-1} x$



The finite region enclosed by the graph of $y = \cos^{-1} x$, the y-axis, the x-axis and the line $x = 0.8$ is rotated by 2π radians about the x-axis.

Use Simpson's rule with five ordinates to estimate the volume of the solid formed. Give your answer to four decimal places.

[5 marks]

$$V = \pi \int_0^{0.8} y^2 \, dx$$

Using Simpson's rule:

x	0	0.2	0.4	0.6	0.8
y^2	2.46740	1.87536	1.34393	0.85988	0.41409

$$\Rightarrow \pi \times \frac{0.2}{3} \times \left(2.46740 + 0.41409 + (4 \times 1.87536) + (4 \times 0.85988) + (2 \times 1.34393)\right)$$

$$= 3.4579$$

Do not write outside the box

8 (a) Factorise $\begin{vmatrix} 2a+b+x & x+b & x^2+b^2 \\ 0 & a & -a^2 \\ a+b & b & b^2 \end{vmatrix}$ as fully as possible.

[6 marks]

Extract a :

$$a\begin{vmatrix} 2 & x+b & x^2+b^2 \\ -1 & a & -a^2 \\ 1 & b & b^2 \end{vmatrix}$$

$$a\begin{vmatrix} 0 & x-b & x^2-b^2 \\ -1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$$

Extract factors:

$$\Rightarrow a(x-b)\begin{vmatrix} 0 & 1 & x+b \\ -1 & a & -a^2 \\ 1 & b & b^2 \end{vmatrix}$$

$$= a(x-b)(x+a)\begin{vmatrix} 0 & 1 & x+b \\ 0 & 1 & b-a \\ 1 & b & b^2 \end{vmatrix}$$

Do not write outside the box

Find the determinant in fully factorised form

$\Rightarrow a(x-b)(a+b)((b-a)-(x+b)) - a(x-b)(a+b)(x+a)$

8 (b) The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 13+x & x+3 & x^2+9 \\ 0 & 5 & -25 \\ 8 & 3 & 9 \end{bmatrix}$$

Under the transformation represented by **M**, a solid of volume $0.625\,\text{m}^3$ becomes a solid of volume $300\,\text{m}^3$

Use your answer to part **(a)** to find the possible values of x.

[3 marks]

$a=5,\ b=3$

Volume scale factor $= \dfrac{300}{0.625} = \pm 480$

Volume scale factor = det M

$\pm 480 = -5 \times 8 \times (x-3)(x+5)$

$\pm 12 = (x-3)(x+5)$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=1 \quad x=-3$

$x^2+2x-27=0$

$\dfrac{-2 \pm \sqrt{4-4\times1\times-27}}{2}$

$x=-1+2\sqrt{7}$

$x=-1-2\sqrt{7}$

Do not write outside the box

9 The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where a and b are positive real numbers,

and $\mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Use $\mathbf{C}$ to show that $\cos\frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where m and n are integers.

[7 marks]

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2-b^2 & -2ab \\ 2ab & a^2-b^2 \end{bmatrix}$$

$$\begin{bmatrix} a^2-b^2 & -2ab \\ 2ab & a^2-b^2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$a^2-b^2 = \frac{\sqrt{3}}{2}$ — 1 $\qquad 2ab = \frac{1}{2}$

$b = \frac{1}{4a}$

Sub $b = \frac{1}{4a}$ into − 1

$$a^2 - \frac{1}{16a^2} = \frac{\sqrt{3}}{2}$$

$$16a^4 - 8\sqrt{3}\,a^2 - 1 = 0$$

$$a^2 = \frac{\sqrt{3}+2}{4}$$

$a = \frac{\sqrt{\sqrt{3}+2}}{2}$ $\quad$ (a > 0 so not negative)

Do not write outside the box

C^2 represents a rotation of $\frac{\pi}{6}$

$\therefore$ C represents a rotation of $\frac{\pi}{12}$

So if $a = \cos\frac{\pi}{12}$

$\Rightarrow$ $\cos\frac{\pi}{12} = \frac{\sqrt{\sqrt{3}+2}}{2}$

Turn over for the next question

Do not write outside the box

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 0 \qquad u_{n+1} = \frac{5}{6 - u_n}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{5^n - 5}{5^n - 1}$$

[6 marks]

Let $n = 1$

$$u_1 = \frac{5^1 - 5}{5^1 - 1} = 0$$

$\therefore$ result for $n=1$ is true.

Assume $n = k$ is true

$$u_k = \frac{5^k - 5}{5^k - 1} \text{ is true}$$

when $n = k+1$

$$u_{k+1} = \frac{5}{6 - \left(\frac{5^k - 5}{5^k - 1}\right)}$$

$$\Rightarrow \quad 6 - \left(\frac{5^k - 5}{5^k - 1}\right) = \frac{6(5^k - 1) - (5^k - 5)}{5^k - 1}$$

$$= \frac{5 \times 5^k - 1}{5^k - 1} = \frac{5^{k+1} - 1}{5^k - 1}$$

Do not write outside the box

$$u_{k+1} = 5 \times \frac{5^k - 1}{5^{k+1} - 1} = \frac{5^{k+1} - 5}{5^{k+1} - 1}$$

$\therefore$ the result is also true for $n = k+1$

Conclusion

The formula for u_n is true for $n=1$, if true for $n=k$, then it's also true for $n=k+1$ and hence by induction

$$u_n = \frac{5^n - 5}{5^n - 1} \text{ for } n \geq 1.$$

Turn over for the next question

Do not write outside the box

11 (a) Starting from the series given in the formulae booklet, show that the general term of the Maclaurin series for

$$\frac{\sin x}{x} - \cos x$$

is

$$(-1)^{r+1}\frac{2r}{(2r+1)!}x^{2r}$$

[4 marks]

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} + \ldots + \frac{(-1)^r x^{2r}}{(2r+1)!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!}$$

Subtracts general terms of $\frac{\sin x}{x}$ and $\cos x$:

$$\frac{1}{(2r+1)!} - \frac{1}{(2r)!} = \frac{1-(2r+1)}{(2r+1)!} = \frac{-2r}{(2r+1)!}$$

$$\therefore \frac{(-1)^r x^{2r}}{(2r-1)!} - \frac{(-1)^r x^{2r}}{(2r)!} = \frac{(-1)^r x^{2r}(-2r)}{(2r+1)!}$$

$$= (-1)^{r+1} \times \frac{2r}{(2r+1)!} \times x^{2r}$$

(as required)

Do not write outside the box

11 (b) Show that

$$\lim_{x \to 0} \left[\frac{\frac{\sin x}{x} - \cos x}{1 - \cos x} \right] = \frac{2}{3}$$

[4 marks]

First non-zero terms of series expansion of $\frac{\sin x}{x} - \cos x$ are :

$\frac{x^2}{3}$ and $\frac{-x^4}{30}$

First non-zero terms of series expansion of $1 - \cos x$ are:

$\frac{x^2}{2}$ and $\frac{-x^4}{24}$

$$\lim_{x \to 0} \left[\frac{\frac{x^2}{3} - \frac{x^4}{30} + \ldots}{\frac{x^2}{2} - \frac{x^4}{24} + \ldots} \right] = \lim_{x \to 0} \left[\frac{\frac{1}{3} - \frac{x^2}{30} + \ldots}{\frac{1}{2} - \frac{x^2}{24} + \ldots} \right]$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$ (as required)

Turn over ▶

Do not write outside the box

12 (a) Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that

$$I = \left[q e^{2t} \sin t + r e^{2t} \cos t \right]_a^b$$

where q and r are rational numbers to be found.

[6 marks]

$I = \int_a^b e^{2t} \sin t \, dt$

Using integration by parts:

$u = \sin t \qquad v' = e^{2t} \qquad I = uv - \int u'v \, dt$

$u' = \cos t \qquad v = \frac{1}{2} e^{2t}$

$= \left[\frac{1}{2} e^{2t} \sin t \right]_a^b - \frac{1}{2} \int_a^b e^{2t} \cos t \, dt$

$I = \left[\frac{1}{2} e^{2t} \sin t \right]_a^b - \frac{1}{2} \left\{ \left[\frac{1}{2} e^{2t} \cos t \right]_a^b + \frac{1}{2} \int_a^b e^{2t} \sin t \, dt \right\}$

$I = \left[\frac{1}{2} e^{2t} \sin t - \frac{1}{4} e^{2t} \cos t \right]_a^b - \frac{1}{4} I$

$\frac{5}{4} I = \left[\frac{1}{2} e^{2t} \sin t - \frac{1}{4} e^{2t} \cos t \right]_a^b$

$I = \left[\frac{2}{5} e^{2t} \sin t - \frac{1}{5} e^{2t} \cos t \right]_a^b$ (as required)

$q = \frac{2}{5} \qquad r = -\frac{1}{5}$

Do not write outside the box

12 (b) A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{dv}{dt} + v = 5e^t \sin t$$

where v is the velocity at time t.

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

[6 marks]

$\frac{dv}{dt} + v = 5e^t \sin t$

Integrating Factor:

$e^{\int 1 dt} = e^t$

Multiply both sides by e^t:

$e^t \frac{dv}{dt} + ve^t = 5e^{2t} \sin t$

$\frac{d}{dt}(ve^t) = 5e^{2t} \sin t$

$ve^t = \int 5e^{2t} \sin t \, dt$

$ve^t = 2e^{2t} \sin t - e^{2t} \cos t + C$

when $t=0$, $v=0$

$0 = 0 - 1 + C \quad \Rightarrow C = 1$

$ve^t = 2e^{2t} \sin t - e^{2t} \cos t + 1$

$ve^{2\pi} = 2e^{4\pi} \sin 2\pi - e^{4\pi} \cos 2\pi + 1$

$= -e^{4\pi} + 1$

$v = -e^{2\pi} + e^{-2\pi}$

Speed $= (e^{-2\pi} - e^{2\pi})$

Do not write outside the box

13 Charlotte is trying to solve this mathematical problem:

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 10e^{-2x}$$

Charlotte's solution starts as follows:

Particular integral: $y = \lambda e^{-2x}$

so

$$\frac{dy}{dx} = -2\lambda e^{-2x}$$

and

$$\frac{d^2y}{dx^2} = 4\lambda e^{-2x}$$

13 (a) Show that Charlotte's method will fail to find a particular integral for the differential equation.

[2 marks]

Continuing the method:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4\lambda e^{-2x} - 2\lambda e^{-2x} - 2\lambda e^{-2x}$$

$$= 0$$

This would make $10e^{-2x} = 0$, which is impossible, so this method fails.

Do not write outside the box

13 (b) Explain how Charlotte should have started her solution differently and find the general solution of the differential equation.

[8 marks]

$m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = -2 \quad m = 1$

Complementary Function: $y = Ae^{x} + Be^{-2x}$

Particular Integral: $y = \lambda x e^{-2x}$

$\frac{dy}{dx} = \lambda e^{-2x}(-2x+1)$

$\frac{d^2y}{dx^2} = \lambda e^{-2x}(4x - 4)$

$\lambda e^{-2x}(4x - 4 - 2x + 1 - 2x) = 10e^{-2x}$

$-3\lambda \cancel{e^{-2x}} = 10\cancel{e^{-2x}}$

$\lambda = -\frac{10}{3}$

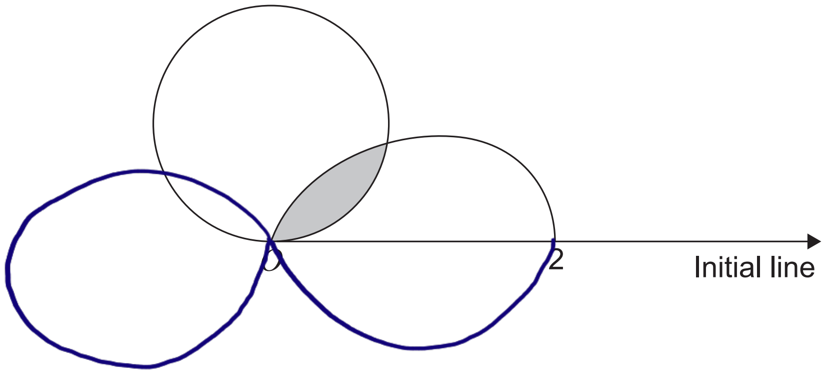
General Solution:

$y = Ae^{x} + Be^{-2x} - \frac{10}{3}xe^{-2x}$

Do not write outside the box

14 The diagram shows the polar curve C_1 with equation $r = 2\sin\theta$

The diagram also shows part of the polar curve C_2 with equation $r = 1 + \cos 2\theta$



14 (a) On the diagram above, complete the sketch of C_2

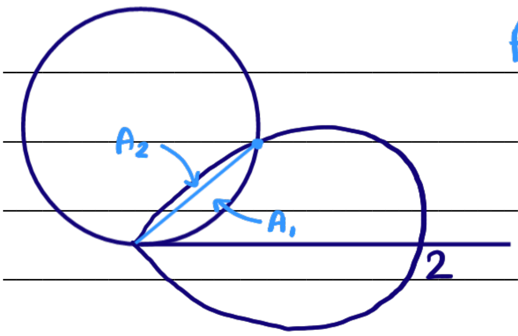
[2 marks]

14 (b) Show that the area of the region shaded in the diagram is equal to

$$k\pi + m\alpha - \sin 2\alpha + q\sin 4\alpha$$

where $\alpha = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$, and k, m and q are rational numbers.

[9 marks]



Point of intersection :

$1 + \cos 2\theta = 2\sin\theta$

$2\cos^2\theta = 2\sin\theta$

$\cancel{2}(1-\sin^2\theta) = \cancel{2}\sin\theta$

$\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{5}}{2}$

Do not write outside the box

θ is acute so $\theta = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right) = \alpha$

$A_1 = \frac{1}{2}\int_0^{\alpha} (2\sin\theta)^2 \, d\theta$

$= \int_0^{\alpha} 1 - \cos 2\theta \, d\theta$

$= \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\alpha}$

$A_1 = \left(\alpha - \frac{1}{2}\sin 2\alpha\right) - 0$

$A_2 = \frac{1}{2}\int_{\alpha}^{\frac{\pi}{2}} (1 + \cos 2\theta)^2 \, d\theta$

$= \frac{1}{2}\int_{\alpha}^{\frac{\pi}{2}} 1 + 2\cos 2\theta + \cos^2 2\theta \, d\theta$

$= \frac{1}{2}\int_{\alpha}^{\frac{\pi}{2}} 1 + 2\cos 2\theta + \frac{1}{2}(1 + \cos 4\theta) \, d\theta$

$= \frac{1}{2}\int_{\alpha}^{\frac{\pi}{2}} \frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta \, d\theta$

$= \left[\frac{3}{4}\theta + \frac{1}{2}\sin 2\theta + \frac{1}{16}\sin 4\theta\right]_{\alpha}^{\frac{\pi}{2}}$

$A_2 = \frac{3\pi}{8} - \left(\frac{3\alpha}{4} + \frac{1}{2}\sin 2\alpha + \frac{1}{16}\sin 4\alpha\right)$

Area Enclosed $= A_1 + A$

$= \alpha - \frac{1}{2}\sin 2\alpha + \frac{3\pi}{8} - \frac{3\alpha}{4} - \frac{1}{2}\sin 2\alpha - \frac{1}{16}\sin 4\alpha$

$= \frac{3\pi}{8} + \frac{\alpha}{4} - \sin 2\alpha - \frac{1}{16}\sin 4\alpha$

Turn over ▶

Do not write outside the box

15 The points $A(7, 2, 8)$, $B(7, -4, 0)$ and $C(3, 3.2, 9.6)$ all lie in the plane Π.

15 (a) Find a Cartesian equation of the plane Π.

[3 marks]

$$\vec{AB} = \begin{pmatrix} 7 \\ -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -8 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} 3 \\ 3.2 \\ 9.6 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 7.2 \\ 9.6 \end{pmatrix}$$

Using cross product:

$$\begin{vmatrix} i & j & k \\ 0 & -6 & -8 \\ -4 & 7.2 & 9.6 \end{vmatrix} = \begin{pmatrix} -57.6 + 57.6 \\ 32 \\ -24 \end{pmatrix} = \begin{pmatrix} 0 \\ 32 \\ -24 \end{pmatrix}$$

Insert point A (7, 2, 8) into $32y - 24z$

$32(2) - 24(8) = -128$

$32y - 24z = -128$ (divide both sides by -8)

$-4y + 3z = 16$

Equation of plane Π

Do not write outside the box

15 (b) The line L_1 has equation $\mathbf{r} = \begin{bmatrix} 5 \\ -0.4 \\ 4.8 \end{bmatrix} + \mu \begin{bmatrix} 15 \\ 3 \\ 4 \end{bmatrix}$

15 (b) (i) Show that L_1 lies in the plane Π.

[2 marks]

Let Q be a point along L_1, with the coordinates $\begin{pmatrix} 5+15\mu \\ -0.4+3\mu \\ 4.8+4\mu \end{pmatrix}$

Insert Q into plane equations:

$-4(-0.4+3\mu)+3(4.8+4\mu) =$

$1.6-12\mu+14.4+12\mu = 16$

So the point Q lies on the plane Π and $\therefore$ L_1 lies in the plane Π.

15 (b) (ii) Show that every point on L_1 is equidistant from B and C.

[4 marks]

Midpoint of BC is $(5, -0.4, 4.8)$, which lies on L_1.

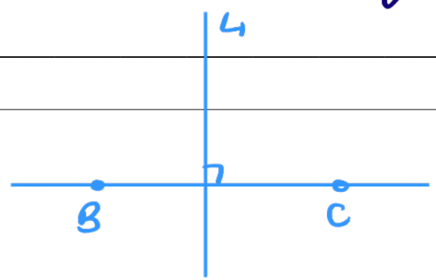
Consider the direction vectors of L_1 and BC:

$\begin{pmatrix} 15 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 9 \\ 12 \end{pmatrix} = -75+27+48=0$

$\therefore$ L_1 is perpendicular to BC.

Since the midpoint of BC also lies on L_1 then L_1 is the perpendicular bisector of BC.

$\therefore$ every point of L_1 is equidistant for B and C.



Do not write outside the box

15 (c) The line L_2 lies in the plane Π, and every point on L_2 is equidistant from A and B.

Find an equation of the line L_2

[4 marks]

L_2 is the perpendicular bisector of AB in plane Π.

Midpoint of AB $(7, -1, 4)$

As L_2 is perpendicular to $\overrightarrow{AB} = \begin{pmatrix} 0 \\ -6 \\ -8 \end{pmatrix}$ and n.

$$\begin{vmatrix} i & j & k \\ 0 & -6 & -8 \\ 0 & -4 & 3 \end{vmatrix} = \begin{pmatrix} -50 \\ 0 \\ 0 \end{pmatrix}$$

So let direction vector for L_2 be $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$L_2 : \underline{r} = \begin{pmatrix} 7 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Do not write outside the box

15 (d) The points A, B and C all lie on a circle G.
The point D is the centre of circle G.

Find the coordinates of D.

[3 marks]

D is point of intersection of L_1 and L_2 (when $L_1 = L_2$)

$$\begin{pmatrix} 5+15\mu \\ -0.4+3\mu \\ 4.8+4\mu \end{pmatrix} = \begin{pmatrix} \lambda+7 \\ -1 \\ 4 \end{pmatrix}$$

$5+15\mu = \lambda+7$

$15\mu - \lambda = 2$

$4.8+4\mu = 4$

$\mu = -0.2$

$-0.4+3\mu = -1$

$3\mu = -0.6$

$\mu = -0.2$

Sub $\mu = -0.2$ into L_1

$$\begin{pmatrix} 5+15(-0.2) \\ -0.4+3(-0.2) \\ 4.8+4(-0.2) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$D(2, -1, 4)$

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**