

Please write clearly in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 1

Friday 22 May 2020

Morning

Time allowed: 2 hours

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
TOTAL	



Answer **all** questions in the spaces provided.

1 Which of the integrals below is **not** an improper integral?

Circle your answer.

[1 mark]

$$\int_0^{\infty} e^{-x} dx$$

$$\int_0^2 \frac{1}{1-x^2} dx$$

$$\int_0^1 \sqrt{x} dx$$

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

2 Which one of the matrices below represents a rotation of 90° about the x -axis?

Circle your answer.

[1 mark]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$



3 The quadratic equation $ax^2 + bx + c = 0$ ($a, b, c \in \mathbb{R}$) has real roots α and β .

One of the four statements below is incorrect.

Which statement is **incorrect**?

Tick (✓) **one** box.

[1 mark]

$$c = 0 \Rightarrow \alpha = 0 \text{ or } \beta = 0$$

$$c = a \Rightarrow \alpha \text{ is the reciprocal of } \beta$$

$$b < 0 \text{ and } c < 0 \Rightarrow \alpha > 0 \text{ and } \beta > 0$$

$$b = 0 \Rightarrow \alpha = -\beta$$

Turn over for the next question

Turn over ►



Do not write
outside the
box

4 It is given that $1 - 3i$ is one root of the quartic equation

$$z^4 - 2z^3 + pz^2 + rz + 80 = 0$$

where p and r are real numbers.

4 (a) Express $z^4 - 2z^3 + pz^2 + rz + 80$ as the product of two quadratic factors with real coefficients.

[4 marks]



4 (b) Find the value of p and the value of r .

[2 marks]

Turn over for the next question

Turn over ►



5 H_1 is the locus of points such that the distance from the point $(5, 0)$ is twice the distance from the line $x = 2$

5 (a) Show that the equation of H_1 can be written in the form

$$(x - 1)^2 - \frac{y^2}{q} = r$$

where q and r are integers.

[5 marks]



6 Let w be the root of the equation $z^7 = 1$ that has the smallest argument α in the interval $0 < \alpha < \pi$

6 (a) Prove that w^n is also a root of the equation $z^7 = 1$ for any integer n .

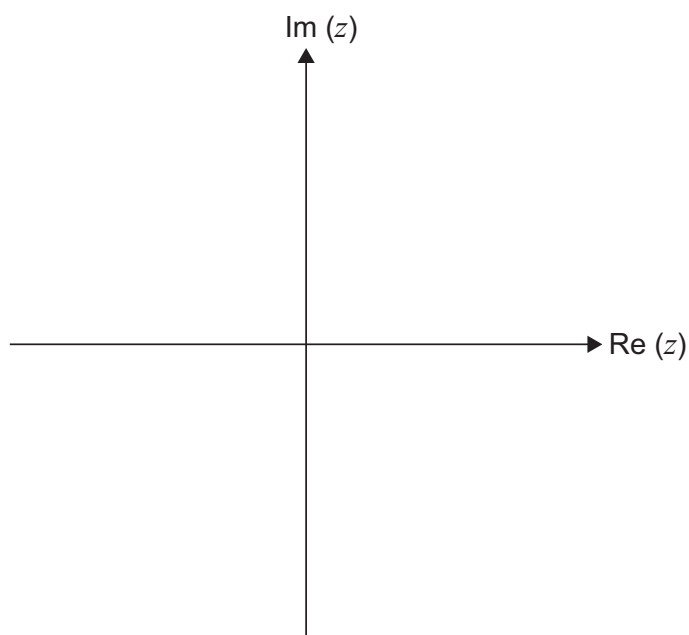
[1 mark]

6 (b) Prove that $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$

[2 marks]

6 (c) Show the positions of $w, w^2, w^3, w^4, w^5,$ and w^6 on the Argand diagram below.

[2 marks]



Do not write
outside the
box

Turn over for the next question

Turn over ►



9 (b) Find the coordinates of the two stationary points of the graph of $y = f(x)$

[2 marks]

9 (c) Show that the graph of $y = f(x)$ has an oblique asymptote and find its equation.

[2 marks]

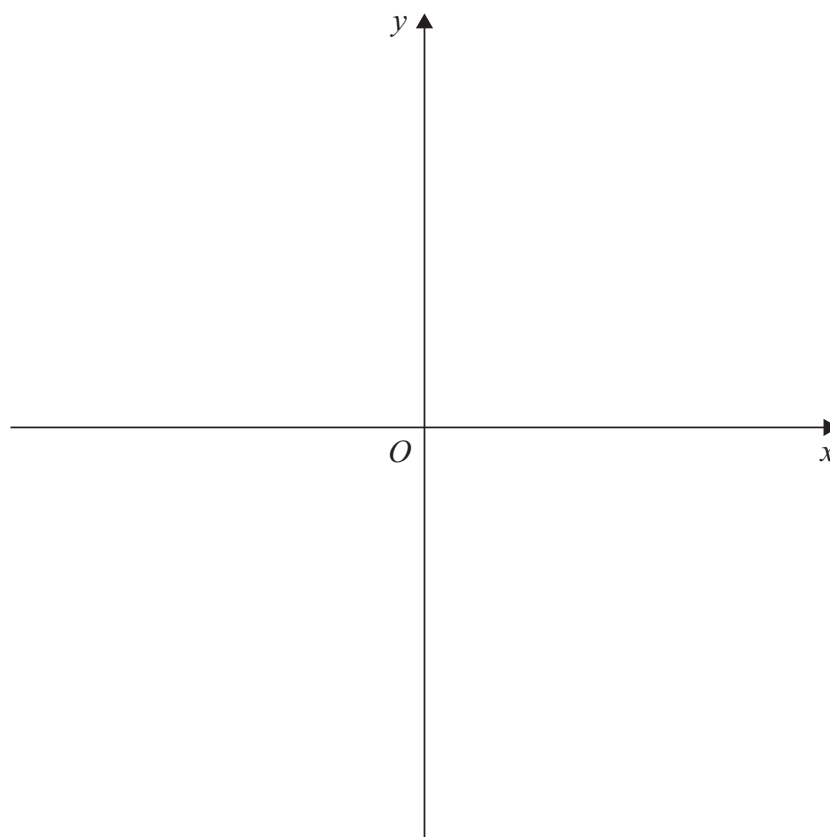
Question 9 continues on the next page

Turn over ►



9 (d) Sketch the graph of $y = f(x)$ on the axes below.

[4 marks]



Turn over for the next question

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**

Turn over ►



Do not write
outside the
box

10 (b) Find the particular solution for which $y = 0$ when $x = 3$

Give your answer in the form $y = f(x)$

[2 marks]

Turn over ►



11 The lines l_1 , l_2 and l_3 are defined as follows.

$$l_1 : \left(\mathbf{r} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix} \right) \times \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \mathbf{0}$$

$$l_2 : \left(\mathbf{r} - \begin{bmatrix} -3 \\ 2 \\ 7 \end{bmatrix} \right) \times \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \mathbf{0}$$

$$l_3 : \left(\mathbf{r} - \begin{bmatrix} -5 \\ 12 \\ -4 \end{bmatrix} \right) \times \begin{bmatrix} 4 \\ 0 \\ 9 \end{bmatrix} = \mathbf{0}$$

11 (a) (i) Explain how you know that two of the lines are parallel.

[1 mark]



11 (b) Show that the lines l_1 and l_3 meet, and find the coordinates of their point of intersection.

[5 marks]



Turn over for the next question

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**

Turn over ►



12 (b) The formulae booklet gives the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ as

$$\cosh^{-1}\left(\frac{x}{a}\right) \quad \text{or} \quad \ln(x + \sqrt{x^2 - a^2}) + c$$

Ronald says that this contradicts the result given in part **(a)**.

Explain why Ronald is wrong.

[2 marks]

Turn over ►



- 13 (b)** Find the speed of the particle when it is at a point P , a distance $\frac{1}{4}$ metre from the equilibrium position. Give your answer to two significant figures.

[4 marks]

Turn over ►



14 (a) Given that

$$\sinh(A + B) = \sinh A \cosh B + \cosh A \sinh B$$

express $\sinh(m + 1)x$ and $\sinh(m - 1)x$ in terms of $\sinh mx$, $\cosh mx$, $\sinh x$ and $\cosh x$

[1 mark]

14 (b) Hence find the sum of the series

$$C_n = \cosh x + \cosh 2x + \dots + \cosh nx$$

in terms of $\sinh x$, $\sinh nx$ and $\sinh(n + 1)x$

[5 marks]



There are no questions printed on this page

*Do not write
outside the
box*

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



