AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
FURTHER MATHEMATICS
Paper 1

Exam Date | Morning | Time allowed: 2 hours

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A vector is given by $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

Which vector is **not** perpendicular to **a**?

Circle your answer.

[1 mark]

$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Vectors are perpendicular if their dot product = 0

$$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} = 10 + 1 - 9 = 2$$

$2 \neq 0 \therefore \underline{a}$ is not perpendicular to $\begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix}$

2 Use the definitions of cosh x and sinh x in terms of e^x and e^{-x} to show that $\cosh^2 x - \sinh^2 x \equiv 1$

[2 marks]

LHS : $\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - (\cancel{e^{2x}} - 2 + \cancel{e^{-2x}})}{4}$

$= \frac{4}{4}$

$= 1 = \text{RHS}$

3 (a) Given that

$$\frac{2}{(r+1)(r+2)(r+3)} \equiv \frac{A}{(r+1)(r+2)} + \frac{B}{(r+2)(r+3)}$$

find the values of the integers A and B

[2 marks]

$\Rightarrow \quad 2 = A(r+3) + B(r+1)$

$2 = r(A+B) + 3A + B$

$A + B = 0 \Rightarrow A = -B$

$\Rightarrow 3A + B = 2$

$3A - A = 2$

$\underline{A = 1} \quad \Rightarrow \underline{B = -1}$

3 (b) Use the method of differences to show clearly that

$$\sum_{r=9}^{97} \frac{1}{(r+1)(r+2)(r+3)} = \frac{89}{19800}$$

[4 marks]

$\sum_{r=9}^{97}\left(\frac{1}{(r+1)(r+2)(r+3)}\right) = \frac{1}{2}\sum_{r=9}^{97}\left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}\right)$

$= \frac{1}{2}\left[\left(\frac{1}{10\times11} - \cancel{\frac{1}{11\times12}}\right) + \left(\cancel{\frac{1}{11\times12}} - \cancel{\frac{1}{12\times13}}\right)\right.$

$\left. \ldots + \left(\cancel{\frac{1}{97\times98}} - \cancel{\frac{1}{98\times99}}\right) + \left(\cancel{\frac{1}{98\times99}} - \frac{1}{99\times100}\right)\right]$

$= \frac{1}{2}\left(\frac{1}{10\times11} - \frac{1}{99\times100}\right) = \frac{1}{2} \times \frac{89}{9900}$

$= \frac{89}{19800}$

4 A student states that $\int_0^{\frac{\pi}{2}} \frac{\cos x+\sin x}{\cos x-\sin x}\,\mathrm{d}x$ is not an improper integral because

$\frac{\cos x+\sin x}{\cos x-\sin x}$ is defined at both $x=0$ and $x=\frac{\pi}{2}$

Assess the validity of the student's argument.

[2 marks]

$0 < \pi/4 < \pi/2$

and at $x = \frac{\pi}{4}$, $\cos x - \sin x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

$\therefore$ at $x = \frac{\pi}{4}$, $\frac{\cos x + \sin x}{\cos x - \sin x}$ is undefined.

$\therefore$ The integral is improper, but not for the reason the student states

5 $\quad p(z) = z^4 + 3z^2 + az + b$, $a \in \mathbb{R}, b \in \mathbb{R}$

$2 - 3i$ is a root of the equation $p(z) = 0$

5 (a) Express $p(z)$ as a product of quadratic factors with real coefficients.

[5 marks]

If $2-3i$ is a root, so is $2+3i$.
Therefore $z-(2-3i)$ and $z-(2+3i)$ are both factors of $p(z)$:

$(z-(2-3i))(z-(2+3i)) = (z-2+3i)(z-2-3i)$
$= z^2 + 2z - 3zi - 2z + 4 + 6i + 3iz - 6i - 9i^2$
$= z^2 - 4z + 13$

$\Rightarrow$ Quadratic factors are: $(z^2-4z+13)$ and an unknown, (z^2+cz+d)

$\therefore (z^2-4z+13)(z^2+cz+d) = z^4 + 3z^2 + az + b$

(expand + compare coefficients)

$z^4 + z^3(c-4) + z^2(d-4c+13) + z(13c-4d) + 13d$

$c - 4 = 0 \qquad d - 4c + 13 = 3$
$\Rightarrow c = 4 \qquad \Rightarrow d = 6$

$\Rightarrow p(z) = (z^2-4z+13)(z^2+4z+6)$

5 (b) Solve the equation $p(z) = 0$.

[1 mark]

$z = 2 \pm 3i$ from part (a)

$z = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 6}}{2} = -2 \pm \sqrt{-2}$
$= -2 \pm \sqrt{2}\, i$

6 (a) Obtain the general solution of the differential equation

$$\tan x \frac{dy}{dx} + y = \sin x \tan x$$

where $0 < x < \frac{\pi}{2}$

[5 marks]

$\frac{dy}{dx} + y\cot x = \sin x$ $(\div \tan x)$

$\text{I.f.} = e^{\int(\cot x)dx} = e^{\ln \sin x} = \sin x$

$\Rightarrow \sin x \frac{dy}{dx} + y\cot x \sin x = \sin^2 x$

$\therefore y \sin x = \int(\sin^2 x)\, dx$

$y \sin x = \int\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$y \sin x = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

6 (b) Hence find the particular solution of this differential equation, given that $y = \frac{1}{2\sqrt{2}}$ when $x = \frac{\pi}{4}$

[2 marks]

at $x = \frac{\pi}{4}$, $y = \frac{1}{2\sqrt{2}}$: $\frac{\sin \pi/4}{2\sqrt{2}} = \frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2} + C$

$\Rightarrow C = \frac{\sqrt{2}/2}{2\sqrt{2}} - \frac{\pi}{8} + \frac{1}{4} = \frac{1}{2} - \frac{\pi}{8}$

$\therefore$ particular solution : $y \sin x = \frac{1}{2}x - \frac{1}{2}\sin 2x + \frac{1}{2} - \frac{\pi}{8}$

7 Three planes have equations,

$$x - y + kz = 3$$

$$kx - 3y + 5z = -1$$

$$x - 2y + 3z = -4$$

Where k is a real constant. The planes do not meet at a unique point.

7 (a) Find the possible values of k

[3 marks]

$$\underline{M} = \begin{bmatrix} 1 & -1 & k \\ k & -3 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

do not meet at unique point $\Rightarrow \det \underline{M} = 0$

$$\therefore 1(-9+10) - -1(3k-5) + k(-2k+3) = 0$$

$$1 + 3k - 5 - 2k^2 + 3k = 0$$

$$2k^2 - 6k + 4 = 0$$

$$(k-2)(k-1) = 0$$

$k = 2$ or $k = 1$

7 (b) There are two possible geometric configurations of the given planes.

Identify each possible configurations, stating the corresponding value of k

Fully justify your answer.

[5 marks]

$k=1$:
$x - y + z = 3$ ①
$x - 3y + 5z = -1$ ②
$x - 2y + 3z = -4$ ③
① − ③ : $y - 2z = 7$
③ − ② : $y - 2z = -3$ $\Big\}$ $7 \neq -3$

$\Rightarrow$ inconsistent, three planes form a prism

$k=2$:
$x - y + 2z = 3$ ① $(\times 2)$: $2x - 2y + 4z = 6$ ④
$2x - 3y + 5z = -1$ ②
$x - 2y + 3z = -4$ ③ $(\times 2)$: $2x - 4y + 6z = -8$ ⑤
① − ③ : $y - z = 7$, ② − ⑤ : $y - z = 7$
④ − ② : $y - z = 7$

$\Rightarrow$ consistent, planes meet in a line + form a sheaf

7 (c) Given further that the equations of the planes form a consistent system, find the solution of the system of equations.

[3 marks]

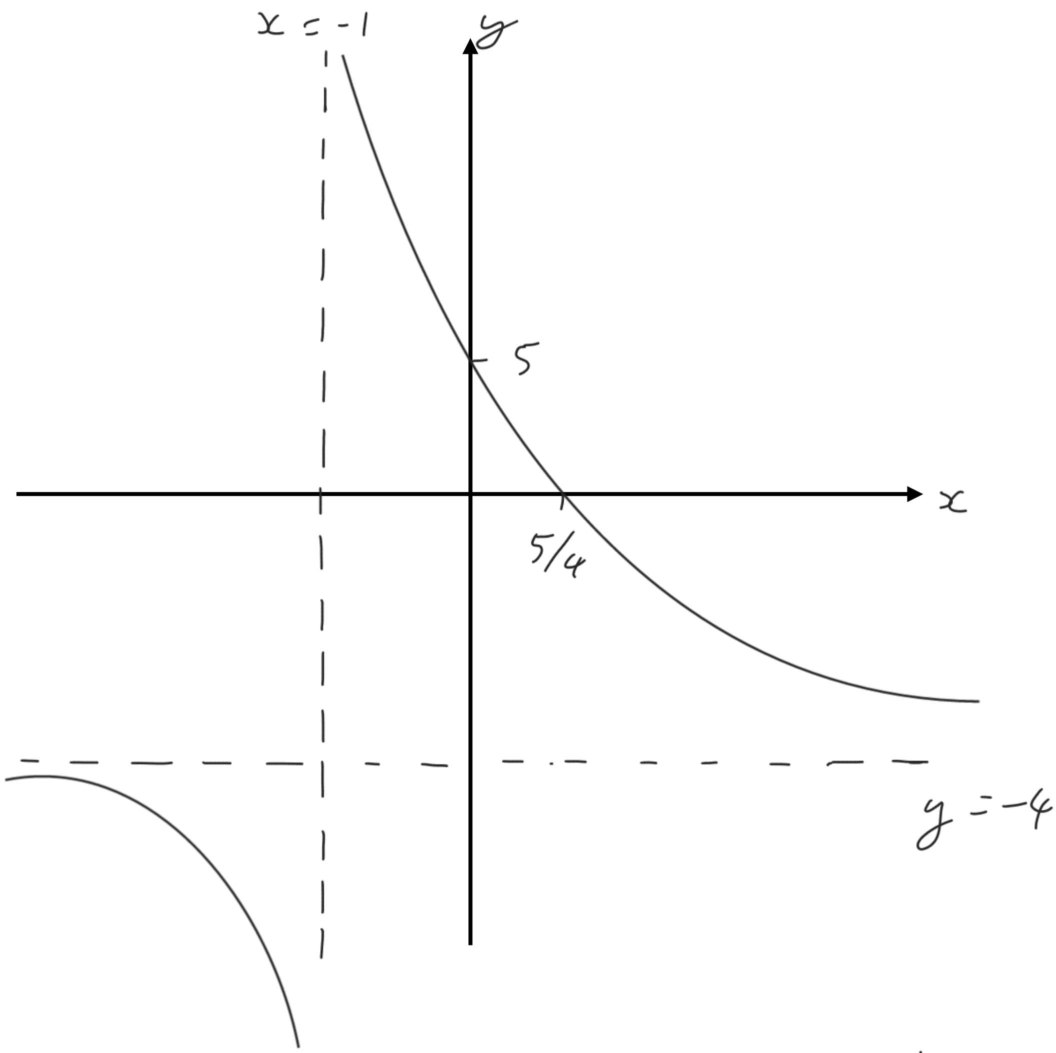
Only consistent at $k = 2$
$\Rightarrow$ $x - y + 2z = 3$ ①
$2x - 3y + 5z = -1$ ②
$x - 2y + 3z = -4$ ③
(from (b): $y - z = 7$ ④)
let $z = k$, $y = 7 + k$ $\therefore$ $x = 10 - k$

8 A curve has equation

$$y = \frac{5 - 4x}{1 + x}$$

8 (a) Sketch the curve.

[4 marks]



Undefined at $x = -1 \Rightarrow$ asymptote at $x = -1$

as $x \to \infty$, $y \to -4 \Rightarrow$ asymptote at $y = -4$

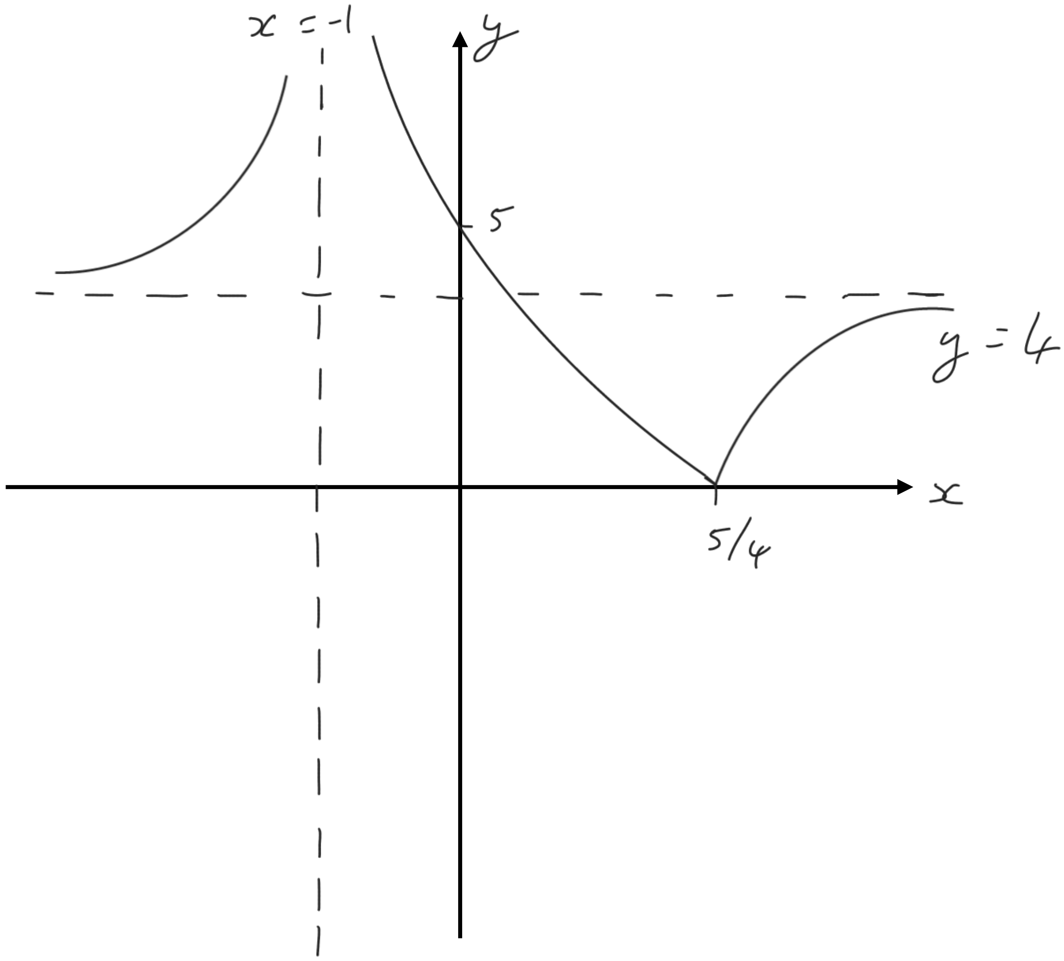
at $x = 0$, $y = 5$

at $y = 0$, $x = 5/4$

8 (b) Hence sketch the graph of $y = \left|\dfrac{5-4x}{1+x}\right|$.

$\Rightarrow x$ is not negative

[1 mark]



9 A line has Cartesian equations $x - p = \dfrac{y+2}{q} = 3 - z$ and a plane has equation $\mathbf{r}.\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = -3$

9 (a) In the case where the plane fully contains the line, find the values of p and q.

[3 marks]

Let $\lambda = x - p = \dfrac{y+2}{q} = 3 - z$

$\Rightarrow x = \lambda + p \quad , \quad y = \lambda q - 2 \quad , \quad z = 3 - \lambda$

$\begin{pmatrix} \lambda + p \\ \lambda q - 2 \\ 3 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = -3$, as line is on plane

$\Rightarrow \lambda + p - \lambda q + 2 - 6 + 2\lambda = -3$

$\lambda(3 - q) + (p - 1) = 0$ (true for all λ values)

$\Rightarrow p = 1 \quad , \quad q = 3$

9 (b) In the case where the line intersects the plane at a single point, find the range of values of p and q.

[3 marks]

Single point of intersection $\Rightarrow$ lines not parallel

$\Rightarrow \begin{pmatrix} 1 \\ q \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \neq 0$

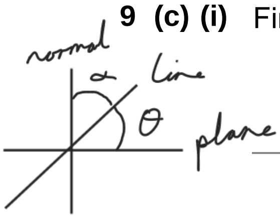
$\Rightarrow 1 - q + 2 \neq 0 \quad \rightarrow \quad q \neq 3$

p can take on any value

9 (c) In the case where the angle θ between the line and the plane satisfies $\sin\theta = \dfrac{1}{\sqrt{6}}$ and the line intersects the plane at $z = 0$

9 (c) (i) Find the value of q.

[4 marks]



$\Rightarrow$ If $\sin\theta = \dfrac{1}{\sqrt{6}}$, $\cos\alpha = \dfrac{1}{\sqrt{6}}$

$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$, where $\underline{a}$ = line and $\underline{b}$ = normal to plane

$\Rightarrow \begin{pmatrix} 1 \\ q \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \sqrt{1^2+q^2+1^2}\sqrt{1^2+1^2+2^2}\cos\alpha$

$1 - q + 2 = \sqrt{2+q^2} \times \sqrt{6}/\sqrt{6}$

$(3-q)^2 = 2+q^2$, $9 - 6q + q^2 = 2 + q^2$

$7 = 6q$

$\underline{q = 7/6}$

9 (c) (ii) Find the value of p.

[3 marks]

from equation of line

at $z = 0$, $q = \dfrac{7}{6}$: $x - p = 3 \Rightarrow x = 3 + p$

$\dfrac{6(y+2)}{7} = 3 \Rightarrow y = 1.5$

$\Rightarrow \begin{pmatrix} p+3 \\ 1.5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = -3$

$p + 3 - 1.5 = -3 \Rightarrow \underline{p = -4.5}$

10 The curve, C, has equation $y = \dfrac{x}{\cosh x}$

10 (a) Show that the x-coordinates of any stationary points of C satisfy the equation $\tanh x = \dfrac{1}{x}$

[3 marks]

Quotient rule : $\dfrac{dy}{dx} = \dfrac{\cosh x - x\sinh x}{\cosh^2 x}$

Stationary points : $\dfrac{dy}{dx} = 0 = \cosh x - x\sinh x$

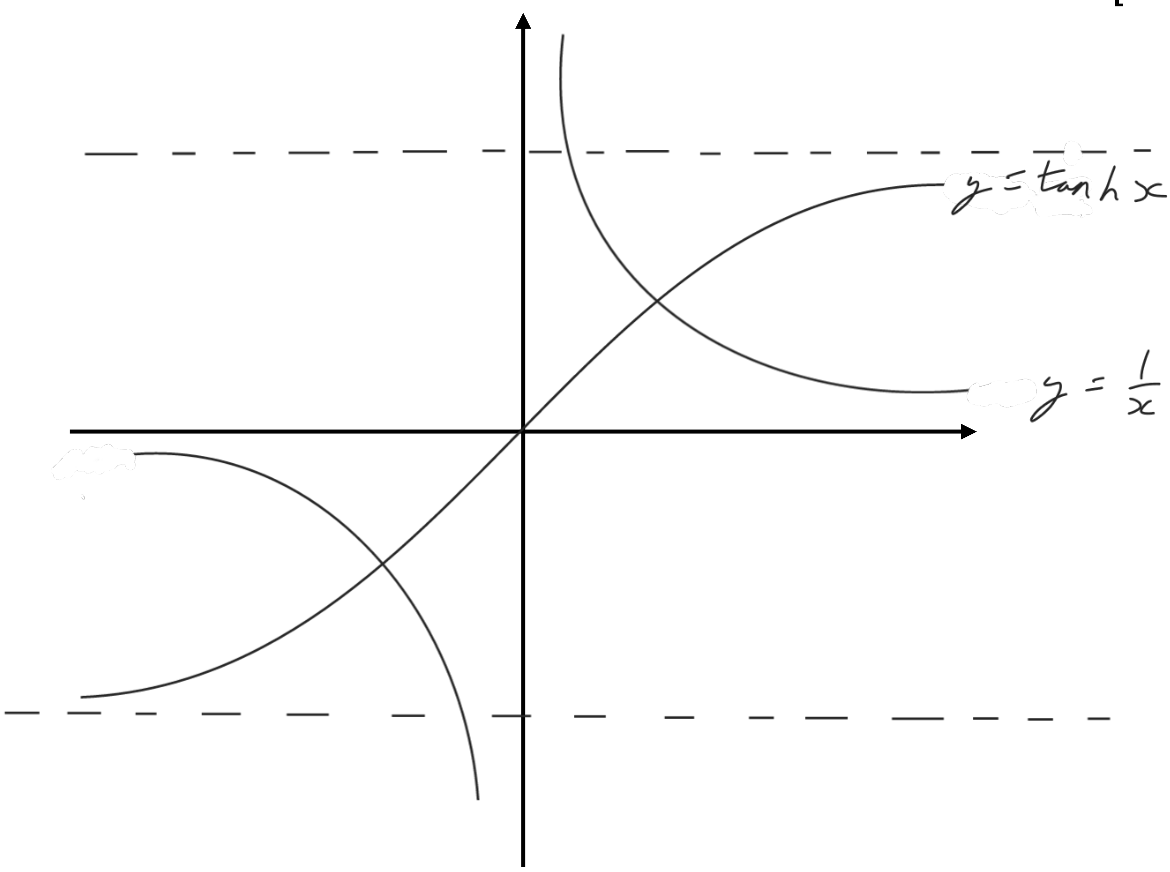
$\cosh x = x\sinh x$

$\dfrac{\cosh x}{\sinh x} = x$

$\tanh x = \dfrac{1}{x}$

10 (b) (i) Sketch the graphs of $y = \tanh x$ and $y = \dfrac{1}{x}$ on the axes below.

[2 marks]



10 (b) (ii) Hence determine the number of stationary points of the curve C.

[1 mark]

Stationary points represented by intersections.

$\therefore$ 2 stationary points

10 (c) Show that $\frac{d^2y}{dx^2} + y = 0$ at each of the stationary points of the curve C.

[4 marks]

$$\frac{dy}{dx} = \frac{\cosh x - x \sinh x}{\cosh^2 x}$$

$$\frac{d^2y}{dx^2} = \frac{\cosh^2 x(\sinh x - x\cosh x - \sinh x) - 2\cosh x \sinh x(\cosh x - \sinh x)}{\cosh^4 x}$$

$$\frac{d^2y}{dx^2} = \frac{-x\cosh^2 x - 2\cosh x \sinh x - 2x\sinh^2 x}{\cosh^3 x}$$

$$\frac{d^2y}{dx^2} + y = \frac{-x\cosh^2 x - 2\cosh x \sinh x - 2x\sinh^2 x}{\cosh^3 x} + \frac{x}{\cosh x}$$

$$\frac{d^2y}{dx^2} + y = \frac{-x\cosh^2 x - 2\cosh x \sinh x - 2x\sinh^2 x + x\cosh^2 x}{\cosh^3 x}$$

$$\frac{d^2y}{dx^2} + y = \frac{-2\sinh x(\cosh x - x\sinh x)}{\cosh^3 x}$$

We know $\cosh x - x\sinh x = 0$, as x satisfies $\frac{dy}{dx} = 0$

stationary point

$$\therefore \frac{d^2y}{dx^2} + y = 0$$

$\times \frac{\sinh\theta}{\sinh\theta}$

11 (a) Prove that $\dfrac{\sinh\theta}{1+\cosh\theta}+\dfrac{1+\cosh\theta}{\sinh\theta}\equiv 2\coth\theta$ } 1st step

$\times \frac{1+\cosh\theta}{1+\cosh\theta}$

Explicitly state any hyperbolic identities that you use within your proof.

[4 marks]

$$LHS: \frac{\sinh^2\theta + (1+\cosh\theta)^2}{\sinh\theta(1+\cosh\theta)}$$

$$= \frac{\sinh^2\theta + \cosh^2\theta + 2\cosh\theta + 1}{\sinh\theta(1+\cosh\theta)}$$

$$(\sinh^2\theta + 1 = \cosh^2\theta)$$

$$\Rightarrow \frac{2\cosh\theta(\cosh^2\theta + 1)}{\sinh\theta(1+\cosh^2\theta)}$$

$$= \frac{2\cosh\theta}{\sinh\theta} = 2\coth\theta = RHS$$

11 (b) Solve $\dfrac{\sinh\theta}{1+\cosh\theta}+\dfrac{1+\cosh\theta}{\sinh\theta}=4$ giving your answer in an exact form.

[2 marks]

$$2\coth\theta = 4$$

$$\coth\theta = 2$$

$$\tanh\theta = 1/2$$

$$\theta = \tan^{-1}(1/2)$$

$$= \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-1/2}\right)$$

$$= \frac{1}{2}\ln\left(\frac{3/2}{1/2}\right)$$

$$\theta = \frac{1}{2}\ln 3$$

12 The function $\mathrm{f}(x) = \cosh(\mathrm{i}x)$ is defined over the domain $\{x \in \mathbb{R} : -a\pi \le x \le a\pi\}$, where a is a positive integer.

By considering the graph of $y = [\mathrm{f}(x)]^n$, find the mean value of $[\mathrm{f}(x)]^n$, when n is an odd positive integer.

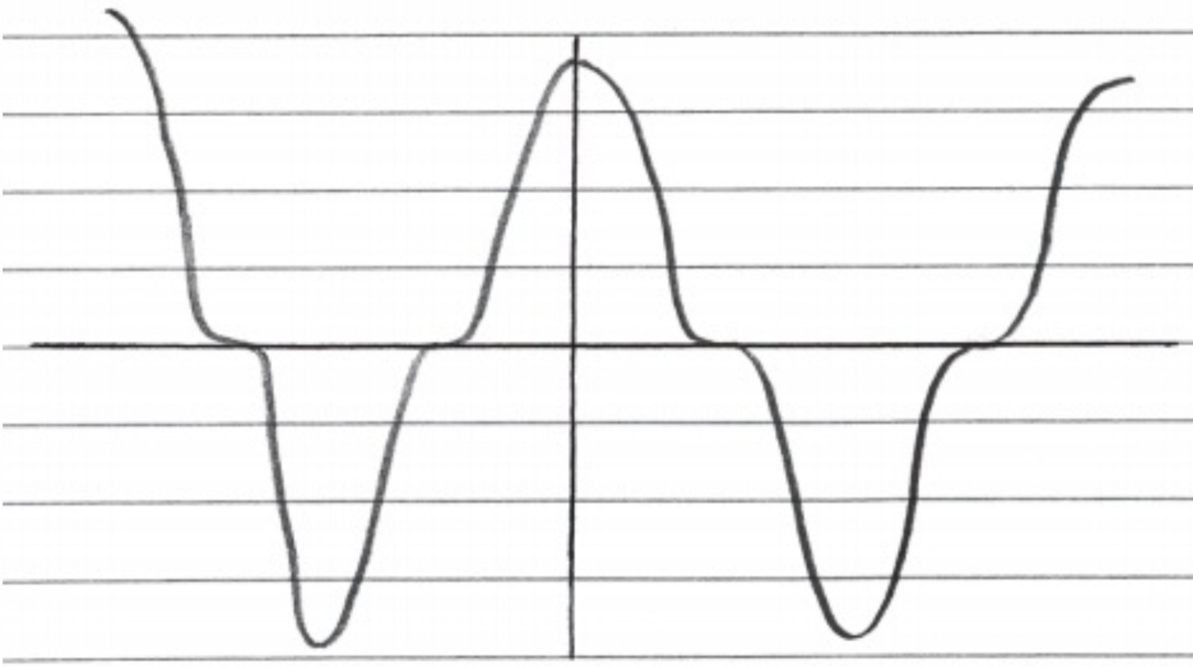
Fully justify your answer.

[3 marks]

$\cosh(ix) = \frac{e^{ix} + e^{-ix}}{2} = \frac{\cos x + i\sin x + \cos x - i\sin x}{2}$

$= \frac{2\cos x}{2} = \cos x$

$\Rightarrow [f(x)]^n = [\cosh(ix)]^n = [\cos^n x]$



Same shape above and below x-axis

$\Rightarrow$ mean value : $y = 0$

13 Given that $\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $\mathbf{M}^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for all $n \in \mathbb{N}$

[5 marks]

$\underline{n=1:}$ LHS : $\underline{M}^1 = \underline{M} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

RHS : $\begin{pmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$\therefore$ true for $n = 1$.

Assume true for $n = k$:

$\underline{M}^k = \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$

$\underline{n = k+1}$

$\underline{M}^{k+1} = \underline{M} \times \underline{M}^k = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$

and $3^{(k-1)} + 3^{(k-1)} + 3^{(k-1)} = 3 \times 3^{(k-1)} = 3^k = 3^{(k-1)+1}$

$\underline{M}^{k+1} = \begin{pmatrix} 3^{(k-1)+1} & 3^{(k-1)+1} & 3^{(k-1)+1} \\ 3^{(k-1)+1} & 3^{(k-1)+1} & 3^{(k-1)+1} \\ 3^{(k-1)+1} & 3^{(k-1)+1} & 3^{(k-1)+1} \end{pmatrix}$

$\therefore$ True for $n = k+1$ when true for $n = k$, and true for $n = 1$. $\therefore$ True for all positive integers n.

14 A particle, P, of mass M is released from rest and moves along a horizontal straight line through a point O. When P is at a displacement of x metres from O, moving with a speed v ms^{-1}, a force of magnitude $|8Mx|$ acts on the particle directed towards O. A resistive force, of magnitude $4Mv$, also acts on P.

14 (a) Initially P is held at rest at a displacement of 1 metre from O. Describe completely the motion of P after it is released.

Fully justify your answer.

[8 marks]

$F = ma$: $\quad -4Mv - 8Mx = Ma$

$$-4M\frac{dx}{dt} - 8Mx = M\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + 4M\frac{dx}{dt} + 8Mx = 0$$

Aux: $\lambda^2 + 4\lambda + 8 = 0$

$\lambda = -2 \pm 2i$

General sol^n: $x = Ae^{-2t}\cos(2t + B)$

$$\frac{dx}{dt} = -2Ae^{-2t}\cos(2t+B) - 2Ae^{-2t}\sin(2t+B)$$

Initially: $t = 0$, $x = 0$, $v = 0$

$\Rightarrow 0 = -2Ae^0\cos B - 2Ae^0\sin B$

$0 = \cos B + \sin B$

$\therefore \tan B = -1 \rightarrow B = -\pi/4$

$x = 1 = Ae^0\cos(-\pi/4)$

$1 = A/\sqrt{2} \rightarrow A = \sqrt{2}$

$\therefore x = \sqrt{2}e^{-2t}\cos(2t - \pi/4)$

$\therefore$ Damped SHM, particle oscillates about O with period π sec and exponentially decreasing amplitude.

14 (b) It is decided to alter the resistive force so that the motion of *P* is critically damped.

Determine the magnitude of the resistive force that will produce critically damped motion.

[4 marks]

Let magnitude of resistance $= p$

$\therefore \dfrac{d^2x}{dt^2} + p\dfrac{dx}{dt} + 8x = 0$

Critical damping $\Rightarrow$ there is one solution

$\therefore$ discriminant $= 0$

$b^2 - 4ac = 0$

$p^2 - 4 \times 8 = 0$

$p = \sqrt{32} = 4\sqrt{2}$

$\therefore$ For critical damping, resistance has magnitude $= 4\sqrt{2}$ N ✓

15 An isolated island is populated by rabbits and foxes. At time t the number of rabbits is x and the number of foxes is y.

It is assumed that:

1 • The number of foxes increases at a rate proportional to the number of rabbits. When there are 200 rabbits the number of foxes is increasing at a rate of 20 foxes per unit period of time.

2 • If there were no foxes present, the number of rabbits would increase by 120% in a unit period of time.

3 • When both foxes and rabbits are present the foxes kill rabbits at a rate that is equal to 110% of the current number of foxes.

4 • At time $t = 0$, the number of foxes is 20 and the number of rabbits is 80.

15 (a) (i) Construct a mathematical model for the number of rabbits.

[9 marks]

1 $\Rightarrow \frac{dy}{dt} \propto x \quad \Rightarrow \frac{dy}{dt} = kx$

at $x = 200, \frac{dy}{dt} = 20 \quad k = \frac{20}{200} = 0.1$

$\therefore \frac{dy}{dt} = 0.1x$

2 and 3 $\Rightarrow \frac{dx}{dt} = 1.2x - 1.1y$

$\frac{1}{1.1} \times \frac{dx}{dt} = \frac{1.2}{1.1}x - y$

$y = \frac{-10}{11}\frac{dx}{dt} + \frac{12}{11}x$ ⓐ

$\frac{dy}{dt} = \frac{-10}{11}\frac{d^2x}{dt^2} + \frac{12}{11}\frac{dx}{dt}$

and $\frac{dy}{dt} = 0.1x$ from before,

$\therefore 0.1x = -\frac{10}{11}\frac{d^2x}{dt^2} + \frac{12}{11}\frac{dx}{dt}$

$10\frac{d^2x}{dt^2} + 12\frac{dx}{dt} - 1.1x = 0$

Aux : $10\lambda^2 + 12\lambda - 1.1 = 0$

$\lambda = 1.1$ or $\lambda = 0.1$

General sol^n : $x = Ae^{0.1t} + Be^{1.1t}$

(4) $\Rightarrow$ at $t = 0$, $x = 80$, $y = 20$:

$\Rightarrow 80 = A + B$ (b)

(a) : $20 = -\frac{10}{11}\frac{dx}{dt} + \frac{12}{11}(80)$

$220 = -10\frac{dx}{dt} + 960$

$10\frac{dx}{dt} = 740 \rightarrow \frac{dx}{dt} = 74$

$x = Ae^{0.1t} + Be^{1.1t}$

$\frac{dx}{dt} = 0.1Ae^{0.1t} + 1.1Be^{1.1t}$

at $t = 0$, $\frac{dx}{dt} = 74$:

$74 = 0.1A + 1.1B$

Sub in (b): $74 = 0.1(80-B) + 1.1B$

$74 = 8 + B$

$B = 66$

$A = 80 - 66 \Rightarrow A = 14$

$\Rightarrow x = 14e^{0.1t} + 66e^{1.1t}$

15 (a) (ii) Use this model to show that the number of rabbits has doubled after approximately 0.7 units of time.

[1 mark]

at $t = 0.7 \Rightarrow x = 1\ e^{0.07} + 66e^{0.77}$

$x = 157.6$

$157.6 \approx 2 \times 80$

15 (b) Suggest one way in which the model that you have used for the number of rabbits could be refined.

[1 mark]

As rabbit population grows, food may run out. Taking this into account would refine the model.

END OF QUESTIONS