AQA

AS
FURTHER MATHEMATICS
7366/2S

Paper 2 Statistics

Mark scheme

June 2019

Version: 1.0 Final

196A7366/2s/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	0.3
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	4.4%
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Calculates the sample mean to be $\frac{60255}{200}$ (OE)	AO1.1b	B1	$\bar{x} = \frac{60255}{200}$ = 301.275
	Calculates the sample variance to be $\frac{995}{199}$ (OE) or standard deviation to be $\sqrt{\frac{995}{199}}$ (OE) or AWRT 2.24	AO1.1b	B1	$s^2 = \frac{995}{199}$ = 5 ($s = \sqrt{5}$)
	Finds z value to at least 3 significant figures Can be implied by a correct confidence interval.	AO1.1a	B1	z = 2.05(374891)
	Uses formula for confidence interval (PI) but values substituted must be clear	AO1.1a	M1	$\bar{x} \pm z\sqrt{\frac{s^2}{n}}$ = $301.275 \pm 2.05\sqrt{\frac{5}{200}}$ = (301.0, 301.6)
	Obtains correct confidence interval correct way round AWRT values to 1 d.p. Allow 301 for 301.0 Allow if population variance $\left(\frac{995}{200}\right)$ used	AO1.1b	A1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Forms an integral with the correct integrand (PI) (limits not needed)	AO1.1a	M1	P(X > 1) = $\frac{4}{99}\int_1^3 12x - x^2 - x^3\, dx$
	Integrates correctly and applies the correct limits the correct way round (PI)	AO1.1b	A1	= $\frac{4}{99}\left[\frac{12x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right]_1^3$
	Obtains the correct answer of $\frac{232}{297}$ or 0.781 (AWRT)	AO1.1b	A1	= $\frac{4}{99} \times \frac{58}{3}$ = $\frac{232}{297}$
4(b)	Selects correct integral, can be unsimplified (limits not needed)	AO1.1a	M1	E(X^{-1}) = $\frac{4}{99}\int_0^3 x^{-1}(12x - x^2 - x^3)\, dx$
	Obtains $\frac{4}{99}\int_0^3 (12 - x - x^2)\, dx$ or $\int_0^3 \left(\frac{16}{33} - \frac{4}{99}x - \frac{4}{99}x^2\right) dx$ (OE)	AO1.1b	A1	= $\frac{4}{99}\int_0^3 (12 - x - x^2)\, dx$ = $\frac{4}{99}\left[12x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^3$
	Shows that E(X^{-1}) = $\frac{10}{11}$ Mark awarded if they have a completely correct solution with correct notation, which is clear and easy to follow	AO2.1	R1	= $\frac{10}{11}$
4(c)	Applies expectation formula E($aY + b$) = aE(Y) + b or selects correct integral	AO1.1a	M1	E($2X^{-1}$ – 3) = 2E(X^{-1}) – 3 = 2 × $\frac{10}{11}$ – 3
	Obtains $-\frac{13}{11}$ or –1.18 (AWRT)	AO1.1b	A1	= $-\frac{13}{11}$
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)(i)	Applies formula for E(X)	AO1.1a	M1	E(X) = $\sum_{x=1}^{n}\frac{x}{n}=\frac{1}{n}\sum_{x=1}^{n}x$
	Applies formula for $\sum x$	AO1.1b	A1	= $\frac{\frac{n}{2}(1+n)}{n}$
	Shows that E(X) = $\frac{n+1}{2}$ Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips Condone missing E(X)	AO2.1	R1	= $\frac{n+1}{2}$
5(a)(ii)	Applies formula for E(X^2)	AO1.1a	M1	E(X^2) = $\sum_{i=1}^{n}\frac{x^2}{n}=\frac{1}{n}\sum_{i=1}^{n}x^2$
	Applies formula for $\sum x^2$	AO1.1b	A1	= $\frac{\frac{1}{6}n(n+1)(2n+1)}{n}$ = $\frac{(n+1)(2n+1)}{6}$
	Applies Var(X) = E(X^2) – (E(X))^2 using their E(X^2) and E(X) = $\frac{n+1}{2}$	AO1.1a	M1	Var(X) = $\frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2$
	Shows that Var(X) = $\frac{n^2-1}{12}$ Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips Need an intermediate line of working after substituting into the variance formula	AO2.1	R1	= $\frac{2n^2+3n+1}{6}-\frac{n^2+2n+1}{4}$ = $\frac{4n^2+6n+2-3n^2-6n-3}{12}$ = $\frac{n^2-1}{12}$
5(b)	States that the dice is unbiased or fair or each score has equal probability (condone chance) of occurring	AO3.5b	E1	Dice is unbiased The dice is numbered 1 to 6
	States n = 6 or the dice is numbered 1, 2, 3, 4, 5, 6	AO3.3	E1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Selects Poisson model with $\lambda = 2 \times 4 = 8$	AO3.3	M1	$2 \times 4 = 8$ $X \sim \text{Po}(8)$
	Finds $P(X = 5) = 0.0916$ AWRT	AO1.1b	A1	$P(X = 5) = 0.0916$
6(b)	Selects Poisson model with $\lambda = 2 + 5 = 7$	AO3.3	M1	$2 + 5 = 7$ $X + Y \sim \text{Po}(7)$
	Uses model to find $P(X + Y > 8) = 0.27$ (AWRT)	AO3.4	A1	$P(X + Y > 8) = 0.27$
	Concludes correctly whether or not a machine should be purchased Follow through an attempt to combine Poisson distributions	AO3.5a	E1F	The probability is less than 40% so a new machine will not be purchased
6(c)	Calculates variance = sd^2 or $\sqrt{\text{mean}}$ for machine A or machine B or the combined machines (PI) by clear argument	AO1.1a	M1	Variance = 0.25 No as for a Poisson distribution, Mean = Variance but means of 2 and 5 are not equal to a variance of 0.25
	Identifies a clear contradiction as for a Poisson distribution Mean = Variance	AO3.5b	A1	
	Total		**7**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td>7(a)</td><td>States both hypotheses using correct language
Variables must be included in at least the null hypothesis</td><td>AO2.5</td><td>B1</td><td>H_0: There is no association between recovery time and drug used
H_1: There is an association between recovery time and drug used</td></tr>
<tr><td></td><td>Calculates at least four correct expected values (PI)
Allow even if columns merged</td><td>AO1.1b</td><td>M1</td><td><table><tr><td>Expected</td><td>1</td><td>2</td><td>3</td></tr><tr><td>A</td><td>28.5</td><td>21.5</td><td>10</td></tr><tr><td>B</td><td>28.5</td><td>21.5</td><td>10</td></tr></table></td></tr>
<tr><td></td><td>Calculates χ^2-test statistic correctly
AWRT 3 significant figures
Condone 9.52</td><td>AO1.1b</td><td>A1</td><td>$\sum\frac{(O-E)^2}{E} =$
$\frac{(36-28.5)^2}{28.5}+\frac{(19-21.5)^2}{21.5}+$
$\frac{(5-10)^2}{10}+\frac{(21-28.5)^2}{28.5}+$
$+\frac{(24-21.5)^2}{21.5}+\frac{(15-10)^2}{10}$
= 9.53</td></tr>
<tr><td></td><td>States critical value (or p-value, follow through their χ^2 value)
If columns merged follow through 6.635 for 1 dof</td><td>AO1.1b</td><td>B1F</td><td>χ^2 cv for 2 df = 9.210
(p = 0.0085)</td></tr>
<tr><td></td><td>Evaluates χ^2-test statistic by comparing the cv with the ts (or p value with 0.01)</td><td>AO3.5a</td><td>M1</td><td>9.53 > 9.210
(0.0085 < 0.01)</td></tr>
<tr><td></td><td>Infers H_0 rejected, follow through with their ts and cv</td><td>AO2.2b</td><td>R1</td><td>Reject H_0</td></tr>
<tr><td></td><td>Concludes in context, based on their hypotheses
(not definite)
Should be consistent with decision to accept or reject H_0 if stated or ts (or p value) and cv (or 0.01) if not
Can be awarded if seen in part (b)</td><td>AO3.2a</td><td>E1</td><td>Some evidence to suggest/support that recovery time and drug used are not independent/Mohammed’s claim is correct</td></tr>
</table>

7(b)	Considers $\frac{(O-E)^2}{E}$ to identify largest sources of association as Drug A or B/3 weeks or Considers (O – E) to identify largest sources of association as Drug A or B/1 week [Do not allow mark if no reference as to why source selected	AO2.4	E1	Largest sources of association Drug A/3 weeks and Drug B/3 weeks $\frac{(O-E)^2}{E}$ = 2.5 Fewer people than expected using Drug A have a recovery time of 3 weeks or More people than expected using Drug B have a recovery time of 3 weeks
	Interpret main source of association in context Condone multiple comments if not contradictory	AO3.2a	E1	
	Total		**9**	