AQA MODEL ANSWERS

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

AS
FURTHER MATHEMATICS
Paper 2 Statistics

Thursday 16 May 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has the following probability distribution function

$$P(X = x) = \begin{cases} \dfrac{5-x}{10} & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X \geq 3)$ $\rightarrow$ non-zero values: $X=3,4$

Circle your answer. $P(X \geq 3) = \frac{5-3}{10} + \frac{5-4}{10} = \frac{3}{10}$

[1 mark]

0.1 0.15 0.2 0.3

2 A binomial hypothesis test was carried out at the 5% level of significance with the hypotheses

$H_0 : p = 0.6$

$H_1 : p > 0.6$

A sample of size 30 was used to carry out the test.

Find the probability that a Type I error was made.

Circle your answer. $\rightarrow$ false positive $\rightarrow$ $p=0.6$ but $P<0.05$

[1 mark]

4.4% 4.8% 5.0% 9.4%

$X \sim B(30, 0.6)$

Use calculator to find first probability < 0.05

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3 Fiona is studying the heights of corn plants on a farm.

She measures the height, x cm, of a random sample of 200 corn plants on the farm.

The summarised results are as follows:

$$\sum x = 60\,255 \quad \text{and} \quad \sum (x - \bar{x})^2 = 995$$

Calculate a 96% confidence interval for the population mean of heights of corn plants on the farm, giving your values to one decimal place.

[5 marks]

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{60255}{200} = 301.275$

$s^2 = \dfrac{\sum(x-\bar{x})^2}{n-1} = \dfrac{995}{199} = 5$ $(s = \sqrt{5})$

standard normal

sample $\sim N(301.275, 5)$, $Z \sim N(0,1)$

confidence interval 96%, critical value $= \dfrac{0.96+1}{2} = 0.98$

Inverse $Z \sim N(0,1)$ for $0.98 = 2.054 = z$

$\Rightarrow \bar{x} \pm z\sqrt{\dfrac{s^2}{n}} = 301.275 \pm 2.05\sqrt{\dfrac{5}{200}}$

$= (301.0, 301.6)$

Turn over for the next question

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4 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{4}{99}(12x - x^2 - x^3) & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

only non-zero values

4 (a) Find $P(X > 1)$

[3 marks]

$$P(X>1) = \int_1^{\infty} f(x) = \frac{4}{99}\int_1^3 (12x - x^2 - x^3)\,dx$$

$$= \frac{4}{99}\left[\frac{12x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right]_1^3$$

$$= \frac{4}{99}\left(6(9) - \frac{(27)}{3} - \frac{(81)}{4} - 6(1) + \frac{(1)}{3} + \frac{(1)^4}{4}\right)$$

$$= \frac{4}{99} \times \frac{58}{3}$$

$$= \frac{232}{297}$$

4 (b) Show that $E(X^{-1}) = \frac{10}{11}$

[3 marks]

$$E(X^{-1}) = \int x^{-1} f(x)\,dx = \frac{4}{99}\int_0^3 x^{-1}(12x - x^2 - x^3)\,dx$$

$$= \frac{4}{99}\int_0^3 (12 - x - x^2)\,dx$$

$$= \frac{4}{99}\left[12x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{4}{99}\left(12(3) - \frac{(9)}{2} - \frac{(27)}{3}\right)$$

$$= \frac{10}{11}$$

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4 (c) Find $\mathrm{E}(2X^{-1} - 3)$

[2 marks]

$E(aY+b) = aE(Y)+b$

so $E(2X^{-1}-3) = 2E(X^{-1})-3$

$= 2 \times \frac{10}{11} - 3$

part(b)

$= -\frac{13}{11}$

Turn over for the next question

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5 The discrete random variable X has the following probability distribution function

$$P(X = x) = \begin{cases} \frac{1}{n} & x = 1, 2, ..., n \\ 0 & \text{otherwise} \end{cases}$$

5 (a) (i) Prove that $E(X) = \frac{n+1}{2}$

[3 marks]

$E(X) = \sum_{x=1}^{n} \frac{x}{n} = \frac{1}{n}\sum_{x=1}^{n} x$

can take outside sum ∵ n is constant

standard result

$\sum_{x=1}^{n} x = \frac{n}{2}(1+n) \Rightarrow E(X) = \frac{\frac{n}{2}(1+n)}{n} \boxed{= \frac{n+1}{2}}$

5 (a) (ii) Prove that $\text{Var}(X) = \frac{n^2 - 1}{12}$

[4 marks]

$\text{Var}(X) = E(X^2) - (E(X))^2$ so we need to find $E(X^2)$

$E(X^2) = \sum_{x=1}^{n} \frac{x^2}{n} = \frac{1}{n}\sum_{x=1}^{n} x^2$

$\sum_{x=1}^{n} x^2 = \frac{1}{6}n(n+1)(2n+1)$ by standard result

$\Rightarrow E(X^2) = \frac{\frac{1}{6}n(n+1)(2n+1)}{n} = \frac{(n+1)(2n+1)}{6}$

$\text{Var}(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$

$\times\frac{2}{2}$ $\times\frac{3}{3}$

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$$= \frac{4n^2+6n+2-3n^2-6n-3}{12}$$

$$= \frac{n^2-1}{12}$$

5 (b) State **two** conditions under which a discrete uniform distribution can be used to model the score when a cubic dice is rolled.

[2 marks]

→ 6 sides

dice is unbiased (so each score has equal probability)

the dice is numbered 1 to 6

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6 A company owns two machines, *A* and *B*, which make toys. Both machines run continuously and independently.

Machine *A* makes an average of 2 errors per hour.

6 (a) Using a Poisson model, find the probability that the machine makes exactly 5 errors in 4 hours, giving your answer to three significant figures.

[2 marks]

2 errors/hour ⇒ 2x4 errors in 4 hours

$X \sim Po(8)$

$P(X=5) = 0.0916$ using calculator
(3sf)

6 (b) Machine *B* makes an average of 5 errors per hour. Both machines are switched on and run for 1 hour.

The company finds the probability that the total number of errors made by machines *A* and *B* in 1 hour is greater than 8.

If the probability is greater than 0.4, a new machine will be purchased.

Using a Poisson model, determine whether or not the toy company will purchase a new machine.

[3 marks]

A: 2, B: 5 ⇒ 7 errors/hour average

$X+Y \sim Po(7)$

$P((X+Y)>8) = 0.27$ by calculator

this probability is < 40% so they won't purchase a new machine

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6 (c) After investigation, the standard deviation of errors made by machine *A* is found to be 0.5 errors per hour and the standard deviation of errors made by machine *B* is also found to be 0.5 errors per hour.

Explain whether or not the use of Poisson models in parts **(a)** and **(b)** is appropriate.

[2 marks]

variance $= \sqrt{0.5} = 0.25$

for a Poisson distribution, mean = variance, but $\lambda_A = 2$ & $\lambda_B = 5$, which $\neq 0.25$. therefore, a Poisson distribution is not appropriate.

Turn over for the next question

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7 Mohammed is conducting a medical trial to study the effect of two drugs, *A* and *B*, on the amount of time it takes to recover from a particular illness.

Drug *A* is used by one group of 60 patients and drug *B* is used by a second group of 60 patients.

The results are summarised in the table:

		Recovery time			
		1 week	**2 weeks**	**3 weeks**	**Total**
Drug	***A***	36	19	5	60
	B	21	24	15	60
	Total	57	43	20	120

Mohammed claims that there is an association between recovery time and drug used.

7 (a) Investigate Mohammed's claim using the 1% level of significance.

[7 marks]

1. State your hypotheses

H_0: there is no association between recovery time & drug used

H_1: there is an association between recovery time & drug used

2. Find expected values using fractions of totals

E	1	2	3
A	$\frac{60}{120} \times \frac{57}{120} \times 120 = 28.5$	$\frac{43 \times 60}{120} = 21.5$	$\frac{60 \times 20}{120} = 10$
B	$\frac{60 \times 57}{120} = 28.5$	$\frac{43 \times 60}{120} = 21.5$	$\frac{60 \times 20}{120} = 10$

3. Calculate χ^2-test statistic

$$\sum \frac{(O-E)^2}{E} = \frac{(36-28.5)^2}{28.5} + \frac{(19-21.5)^2}{21.5} + \frac{(5-10)^2}{10} + \frac{(21-28.5)^2}{28.5}$$

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$$+\frac{(24-21.5)^2}{21.5}+\frac{(15-10)^2}{10}$$

$$=\underline{9.53}$$

4. find critical value for degrees of freedom of q

d.o.f. $=(3-1)\times(2-1)=2$

$\chi^2_{2,0.01}=9.210$

$9.53>9.210$ $\therefore$ reject H_0, there is sufficient evidence to suggest that recovery time & drug used are not independent.

7 (b) Interpret any association between recovery time and the drug used.

[2 marks]

largest sources of association (biggest $\frac{(O-E)^2}{E}$) are A/3 weeks & B/3 weeks. for each, $\frac{(O-E)^2}{E}=2.5$. fewer people than expected have a recovery time of 3 weeks when taking drug A.

link to context

END OF QUESTIONS

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