SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

AS MODEL SOLUTIONS

FURTHER MATHEMATICS

Paper 2 – Mechanics

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A child, of mass 40 kg, moves at constant speed of 5 m s^{-1} on a fairground ride.

The path of the child is a circle of radius 4 metres.

Find the magnitude of the resultant force acting on the child.

Circle your answer.

[1 mark]

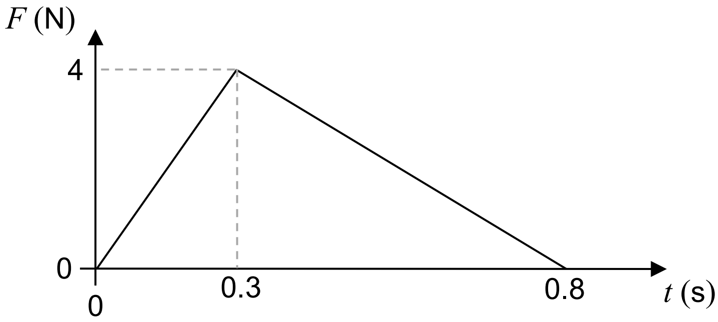
6.3 N   50 N   130 N   250 N

Circular motion:

$F = ma$ and $a = \frac{v^2}{r}$

$\Rightarrow F = \frac{mv^2}{r} = \frac{40(5)^2}{4} = 250N$

2 The graph shows how a force, F, varies with time during a period of 0.8 seconds.



Find the magnitude of the impulse of F during the 0.8 seconds.

Circle your answer.

[1 mark]

1.0 Ns   1.6 Ns   2.2 Ns   3.2 Ns

I = Area under a force-time graph

$= \frac{1}{2}(0.8)(4) = 1.6$ Ns

Turn over for the next question

3 A tank full of liquid has a hole made in its base.

Two students, Sarah and David, propose two different models for the speed, v, at which liquid exits the tank.

David thinks that v will depend on the height of the liquid in the tank, h, the acceleration due to gravity, g, and the density of the liquid, ρ, such that $v \propto g^a h^b \rho^c$
where a, b and c are constants.

Sarah thinks that v will not depend on the density of the liquid and suggests the model $v \propto g^a h^b$

3 (a) By considering dimensions, explain which student's model should be rejected.

[2 marks]

The units of velocity are LT^{-1}

$LT^{-1} = (LT^{-2})^a (L)^b (MT^{-3})^c$

There is no M term on the LHS so $c=0$.

So David's model should be rejected

3 (b) Find the values of the constants in order for the model that you did **not** reject in part **(a)** to be dimensionally consistent.

[2 marks]

$LT^{-1} = (LT^{-2})^a (L)^b$

$= L^{a+b} T^{-2a}$

$\Rightarrow a+b=1$ and $-2a=-1$

$a=\frac{1}{2}$ and $b=\frac{1}{2}$

4 A cricket ball of mass 156 grams is thrown from a point which is 1.5 metres above the ground, with a speed of 12 m s^{-1}

A tennis ball of mass 58 grams is thrown from the same point, with the same speed.

Prove that both balls hit the ground with the same speed.

Clearly state any assumptions you have made and how you have used them.

[5 marks]

Assuming that no external forces act, I can use conservation of energy. Also, I will model both balls as particles, so they move the same vertical distance

	Before	After
GPE	mgh	0
KE	$\frac{1}{2}mu^2$	$\frac{1}{2}mv^2$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$$

$$v^2 = u^2 + 2gh$$

$$v = \sqrt{u^2 + 2gh} = \sqrt{(12)^2 + 2g(1.5)}$$

$$v = \sqrt{144 + 3g}$$

Therefore, the speeds of both balls are independent of their masses (m) so both balls hit the ground with the same speed

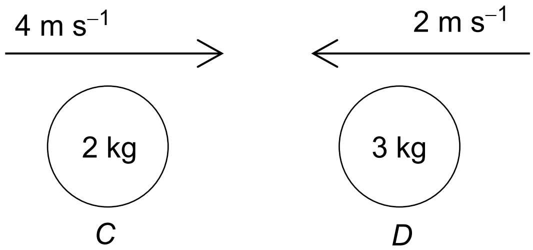
5 Two small smooth discs, *C* and *D*, have equal radii and masses of 2 kg and 3 kg respectively.

The discs are sliding on a smooth horizontal surface towards each other and collide directly.

Disc *C* has speed 4 m s^{-1} and disc *D* has speed 2 m s^{-1} as they collide.

The coefficient of restitution between *C* and *D* is 0.6

The diagram shows the discs, viewed from above, before the collision.



5 (a) Show that the speed of *D* immediately after the collision is 1.8 m s^{-1}, correct to 2 significant figures.

[4 marks]

Conservation of momentum: $2(4) + 3(-2) = 2V_C + 3V_D$

$\therefore 2V_C + 3V_D = 2$ ①

Newton's Law of Restitution:

$e = 0.6 = \frac{V_D - V_C}{4 - -2} \Rightarrow 0.6(6) = V_D - V_C \Rightarrow V_D - V_C = 3.6$ ②

① + 2 × ②: $2V_C + 3V_D = 2$

$+\ 2V_D - 2V_C = 7.2$

$5V_D = 9.2$

$V_D = 1.84$

$V_D = 1.8\ \text{ms}^{-1}$ (2sf)

5 (b) Find the speed of *C* immediately after the collision.

[2 marks]

$V_D - V_c = 3.6 \Rightarrow V_D - 3.6 = V_c$

$1.84 - 3.6 = -1.76$

$V_c = -1.76$

$|V_c| = 1.76$

$|V_c| = 1.8\,ms^{-1}$ (2sf)

So the speed of C to 2sf is $1.8 ms^{-1}$

5 (c) In fact the horizontal surface on which the discs are sliding is not smooth.

Explain how the introduction of friction will affect your answer to part **(b)**.

[2 marks]

As the collision is instantaneous, the introduction of friction will not affect my answer to part (b)

Turn over for the next question

6 A car, of mass 1200 kg, moves on a straight horizontal road where it has a maximum speed of 40 m s^{-1}

When the car travels at a speed of v m s^{-1} it experiences a resistance force which can be modelled as being of magnitude $30v$ newtons.

6 (a) Show that the power output of the car is 48 000 W, when it is travelling at its maximum speed.

[3 marks]

$F = 30v = 30 \times 40 = 1200$

$P = Fv$

$= 1200 (40)$

$= 48,000$ W

6 (b) Find the maximum acceleration of the car when it is travelling at a speed of 25 m s^{-1}

[4 marks]

$\Sigma F = ma$

$F - 30v = ma$

$F - 30(25) = 1200a$

$a = \frac{F - 30(25)}{1200} = \frac{48,000 - 750}{1200}$

$= 0.975 ms^{-2}$

$\Rightarrow \quad a = 0.98 ms^{-2}$ (2sf)

Turn over for the next question

7 A disc, of mass 0.15 kg, slides across a smooth horizontal table and collides with a vertical wall which is perpendicular to the path of the disc.

The disc is in contact with the wall for 0.02 seconds and then rebounds.

A possible model for the force, F newtons, exerted on the disc by the wall, whilst in contact, is given by

$$F = kt^2(t-b)^2 \quad \text{for} \quad 0 \le t \le 0.020$$

where k and b are constants.

The force is initially zero and becomes zero again as the disc loses contact with the wall.

7 (a) State the value of b.

[1 mark]

At $t=0$, $F=0$

$0 = k(0.02)^2(0.02-b)^2$

$\Rightarrow 0 = 0.02 - b$ so $b = 0.02$

7 (b) Find the magnitude of the impulse on the disc, giving your answer in terms of k.

[3 marks]

$I = \int_0^{0.02} kt^2(t-0.02)^2\,dt = k\int_0^{0.02} t^2(t^2 - 0.04t + 0.02^2)\,dt$

$I = k\int_0^{0.02} t^4 - 0.04t^3 + 0.0004t^2\,dt$

$= k\left[\frac{1}{5}t^5 - \frac{1}{100}t^4 + \frac{1}{7500}t^3\right]_0^{0.02}$

$= k\left[\frac{1}{5}(0.02)^5 - \frac{1}{100}(0.02)^4 + \frac{1}{7500}(0.02)^3\right]$

$= k(1.07 \times 10^{-10})$ Ns

7 (c) The disc is travelling at 4 m s^{-1} when it hits the wall.

The disc rebounds with a speed of 2 m s^{-1}

Find k .

[3 marks]

$I = \Delta p = m(\Delta v) = m(v-u)$

$k(1.07\times10^{-10}) = 0.15(4--2)$

$k = \frac{0.15(6)}{1.07\times10^{-10}} = 8.4\times10^{9}$

Turn over for the next question

8 **In this question use $g = 10$ m s^{-2}.**

A particle, of mass 2 kg, is attached to one end of a light elastic string of natural length 0.2 metres.

The other end of the string is attached to a fixed point O.

The particle is pulled down and released from rest at a point 0.6 metres directly below O.

The particle then moves vertically and next comes to rest when it is 0.1 metres below O.

Assume that no air resistance acts on the particle.

8 (a) Find the maximum speed of the particle.

[6 marks]

	Before	After
GPE	mgh	0
EPE	0	$\frac{\lambda x^2}{2l}$

$\Rightarrow mgh = \frac{\lambda x^2}{2l}$

$2(10)(0.5) = \frac{\lambda(0.4)^2}{2(0.2)}$

$10 = \frac{\lambda(0.4)(0.4)}{(0.4)}$

$\lambda = \frac{10}{0.4}$

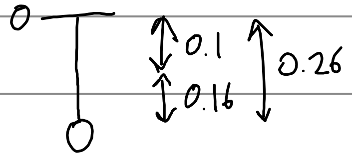
$\lambda = 25$

The maximum speed is at equilibrium:

$\Rightarrow mg = \frac{\lambda x}{l}$

$\Rightarrow 2(10) = \frac{25}{0.2}x \Rightarrow x = 0.16$ (Extension at equilibrium)

	Before	After
GPE	$2g(0.26)$	0
KE	0	$\frac{1}{2}(2)v^2$
EPE	0	$\frac{25(0.16)^2}{2(0.2)}$



$$\Rightarrow 2(10)(0.26) = \frac{1}{2}(2v^2) + \frac{25(0.16)^2}{2(0.2)}$$

$$5.2 = v^2 + 1.6$$

$$v^2 = 3.6$$

$$v = 1.897$$

$$v = 2\text{ms}^{-1} \text{ (2sf)}$$

8 (b) Describe one way in which the model you have used could be refined.

[1 mark]

I could've taken air resistance into account

END OF QUESTIONS