AQA

Please write clearly in block capitals.

Centre number | | | | | |        Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

# AS
# FURTHER MATHEMATICS
## Paper 2 Mechanics

Thursday 14 May 2020 Afternoon Time allowed: 1 hour 30 minutes

### Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

### Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

### Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| TOTAL | |

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Answer **all** questions in the spaces provided.

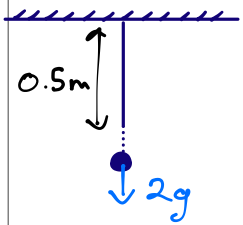
**1** **In this question use $g = 10\,\text{m s}^{-2}$**

A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.5 metres and modulus of elasticity 100 N. The other end of the string is attached to the point *O*.

Find the extension of the elastic string when the particle hangs in equilibrium vertically below *O*.

Circle your answer.

**[1 mark]**




0.01 m    0.1 m    0.2 m    0.4 m

$F = \frac{\lambda x}{l} = mg$

$\frac{100x}{0.5} = 2 \times 10 \Rightarrow x = 0.1\text{ m}$

**2** An object moves under the action of a single force $F$ newtons.

It is given that $F = 6x^2$, where $x$ represents the displacement in metres from the initial position of the object.

Find the work done by $F$ in moving the object from $x = 1$ to $x = 2$

Circle your answer.

**[1 mark]**

12 J    14 J    18 J    42 J


$W = \int F\,dx$

$= \int_1^2 6x^2\,dx$

$= \left[2x^3\right]_1^2$

$= 16 - 2$

$= 14\text{ J}$

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**3** The time taken for the moon to make one complete orbit around Earth is approximately 27.3 days.

Model this orbit as circular, with a radius of $3.84 \times 10^8$ metres.

Find the approximate speed of the moon relative to Earth, in metres per second.

**[3 marks]**

Period = T = 27.3 × 24 × 3600 s

= 2358720 s

$V = \omega r = \frac{2\pi}{T} r$

$= \frac{2\pi \times 3.84 \times 10^8}{2358720}$

= 1022.90359...

≈ 1020 $ms^{-1}$

**Turn over for the next question**

**Turn over ▶**

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**4** A particle $P$, of mass $m$ kg, collides with a particle $Q$, of mass 2 kg

Immediately before the collision the velocity of $P$ is $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ m s$^{-1}$ and the velocity of $Q$ is $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ m s$^{-1}$

As a result of the collision the particles coalesce into a single particle which moves with velocity $\begin{bmatrix} k \\ 0 \end{bmatrix}$ m s$^{-1}$, where $k$ is a constant.

Find the value of $k$.

**[4 marks]**

By conservation of linear momentum:

Initial Momentum = Final Momentum

$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{(1+2)f}$

$m\begin{pmatrix} 4 \\ -2 \end{pmatrix} + 2\begin{pmatrix} -3 \\ 5 \end{pmatrix} = (m+2)\begin{pmatrix} k \\ 0 \end{pmatrix}$

$\begin{pmatrix} 4m-6 \\ -2m+10 \end{pmatrix} = \begin{pmatrix} km+2k \\ 0 \end{pmatrix}$

From y-components,

$-2m+10=0$

$m = \underline{\underline{5}}$

From x-components,

$4(5)-6 = 5k+2k$

$14 = 7k$

$\underline{\underline{k = 2}}$

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**5** A train consisting of an engine and eight carriages moves on a straight horizontal track.

A constant resistive force of 2400 N acts on the engine.

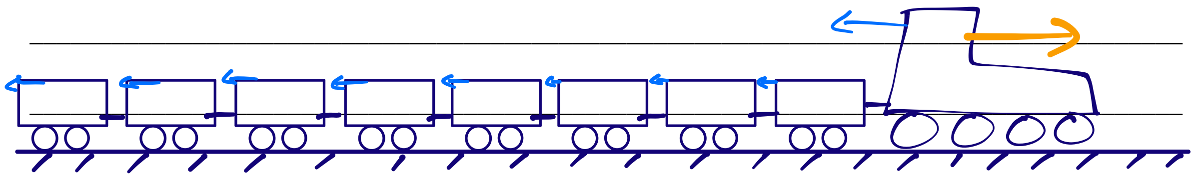
A constant resistive force of 300 N acts on **each** of the eight carriages.

The maximum speed of the train on the track is 120 km h$^{-1}$

Find the maximum power output of the engine.

Fully justify your answer.

**[5 marks]**



When the train travels at max. speed,

Driving Force from Engine = $\sum$ Resistive Forces

$F = 2400 + 8(300) = 4800\,N$

At max power output,

$P = Fv$

$= 4800\,N \times \dfrac{120\times10^3\,m}{3600\,s}$

$= 160\,000\,W$

$= \underline{\underline{160\,kW}}$

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**6** The magnitude of the gravitational force $F$ between two planets of masses $m_1$ and $m_2$ with centres at a distance $d$ apart is given by

$$F = \frac{Gm_1m_2}{d^2}$$

where $G$ is a constant.

**6 (a)** Show that $G$ must have dimensions $L^3M^{-1}T^{-2}$, where $L$ represents length, $M$ represents mass and $T$ represents time.

**[2 marks]**

$F = \frac{G m_1 m_2}{d^2} \Rightarrow G = \frac{F d^2}{m_1 m_2}$

$\therefore [G] = \frac{[F][d^2]}{[m_1][m_2]}$, where $[x]$ represents the units $x$ is measured in

By definition, $[m_1] = [m_2] = M$

$[d^2] = [d][d] = L \cdot L = L^2$

By Newton's 2nd Law, $[F] = [m][a]$

$= M\ L\ T^{-2}$

$\Rightarrow [G] = \frac{MLT^{-2} \cdot L^2}{M \cdot M}$

$= L^3 M^{-1} T^{-2}$

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**6 (b)** The lifetime $t$ of a planet is thought to depend on its mass $m$, its radius $r$, the constant $G$ and a dimensionless constant $k$ such that

$$t = km^a r^b G^c$$

where $a$, $b$ and $c$ are constants.

Determine the values of $a$, $b$ and $c$.

**[3 marks]**

Using dimensional analysis,

$[t] = T$ $\quad [r^b] = L^b$

$[k] = \text{None}$ $\quad [G^c] = L^{3c} M^{-c} T^{-2c}$

$[m^a] = M^a$

Comparing units on either side,

$T^1 = M^a L^b L^{3c} M^{-c} T^{-2c}$

$M^0 L^0 T^1 = M^{a-c} L^{b+3c} T^{-2c}$

Equating exponents of time,

$1 = -2c$

$\Rightarrow c = -1/2$

Equating exponents of mass,

$0 = a - c$

$a = c = -1/2$

Equating exponents of length,

$0 = b + 3c$

$b = -3c = 3/2$

$\therefore a = -0.5,\ b = 1.5,\ c = -0.5$

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**7** **In this question use $g = 9.8\,\text{m s}^{-2}$**

As part of a competition, Jo-Jo makes a small pop-up rocket.

It is operated by pressing the rocket vertically downwards to compress a light spring, which is positioned underneath the rocket.

The rocket is released from rest and moves vertically upwards.

The mass of the rocket is 18 grams and the stiffness constant of the spring is $60\,\text{N m}^{-1}$

Initially the spring is compressed by 3 cm

**7 (a)** Find the speed of the rocket when the spring first reaches its natural length.

**[4 marks]**

By conservation of mechanical energy:

Elastic PE lost by spring = Grav. PE gained by rocket + KE gained by rocket

$\frac{1}{2}kx^2 = mgx + \frac{1}{2}mv^2$

$\frac{1}{2} \times 60 \times 0.03^2 = 0.018 \times 9.8 \times 0.03 + \frac{1}{2} \times 0.018 \times v^2$

$0.027 = 0.005292 + 0.009v^2$

$v^2 = 2.412$

$v = 1.5530614...$

$\approx 1.6\ \text{ms}^{-1}$ (to 2 sf)

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**7 (b)** By considering energy find the distance that the rocket rises.

**[2 marks]**

Initial EPE = GPE at peak of trajectory

$\frac{1}{2}kx^2 = mgh$

$\frac{1}{2} \times 60 \times 0.03^2 = 0.018 \times 9.8 \times h$

$h = \frac{0.027}{0.1764} = 0.15306122...$

$\approx 0.15$ m (To 2sf)

**7 (c)** In order to win a prize in the competition, the rocket must reach a point which is 15 cm vertically above its starting position.

With reference to the assumptions you have made, determine if Jo-Jo wins a prize or not.

Fully justify your answer.

**[3 marks]**

→ According to our calculations while Jo-Jo's rocket theoretically passes 15cm in height, air resistance was not accounted for, and this would lower the max. height reached

→ The rocket was modelled as a particle. The actual max. height would depend on its dimensions.

It is unlikely for Jo-Jo to win as the theoretical max height is so close to the prize cutoff. Even with the added dimensions of the rocket, air resistance, orientation and experimental error would likely prevent the rocket from hitting 15cm.

Turn over ▶

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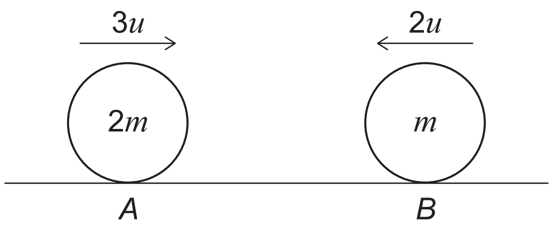
**8** Two smooth spheres *A* and *B* have the same radius and are free to move on a smooth horizontal surface.

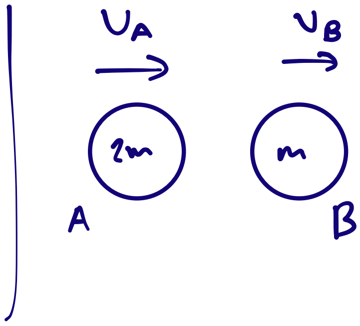
The masses of *A* and *B* are $2m$ and $m$ respectively.

Both *A* and *B* are initially at rest.

The sphere *A* is set in motion directly towards *B* with speed $3u$ and at the same time *B* is set in motion directly towards *A* with speed $2u$.

Subsequently *A* and *B* collide directly.




The coefficient of restitution between the spheres is $e$.

**8 (a)** Show that the speed of *B* after the collision is given by

$$\frac{2u(2+5e)}{3}$$

**[4 marks]**

By conservation of linear momentum:

Initial Momentum = Final Momentum

$2m(3u) + m(-2u) = 2mV_A + mV_B$

$2V_A + V_B = 4u \longrightarrow (1)$

By Newton's Law of Restitution:

$e = \dfrac{\text{separation speed}}{\text{approach speed}} = \dfrac{V_B - V_A}{3u + 2u}$

$\Rightarrow V_B - V_A = 5eu \longrightarrow (2)$

$(1) + 2\times(2)$: $3V_B = 4u + 10eu$

$V_B = \dfrac{2u(2+5e)}{3}$

**Question 8 continues on the next page**

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**8 (b)** Given that the direction of the velocity of *A* is reversed during the collision, find the range of possible values of $e$.

Fully justify your answer.

**[4 marks]**

By definition, we know: $0 \le e \le 1$

$V_B = \frac{2u(2+5e)}{3}$ , $V_A = V_B - 5eu$

$\Rightarrow V_A = \frac{4u - 5eu}{3}$

Since $V_A < 0$ (reversed direction)

$\frac{4u - 5eu}{3} < 0$

$4 - 5e < 0$

$e > \frac{4}{5}$ ↪ i.e, the collision must be at least this bouncy for A to reverse direction

$\therefore \frac{4}{5} < e \le 1$

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**8 (c)** Given that the magnitude of the impulse that *A* exerts on *B* is $\frac{19mu}{3}$, find the value of $e$.

**[4 marks]**

Change in momentum of B $= mV_B - m(-2u)$

$= m(V_B + 2u)$

$= \frac{19mu}{3}$ (impulse on B)

$\Rightarrow V_B + 2u = \frac{19u}{3}$

$\frac{2u(2+5e)}{3} + 2u = \frac{19u}{3}$

$\frac{4+10e}{3} + 2 = \frac{19}{3}$

$4 + 10e + 6 = 19$

$10e = 9$

$e = \frac{9}{10}$

**END OF QUESTIONS**

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