AQA

AS

FURTHER MATHEMATICS

Paper 2 Discrete

Thursday 16 May 2019   **Afternoon**   **Time allowed: 1 hour 30 minutes**

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The network represents a system of pipes.

The number on each arc represents the upper capacity for each pipe in $cm^3 s^{-1}$
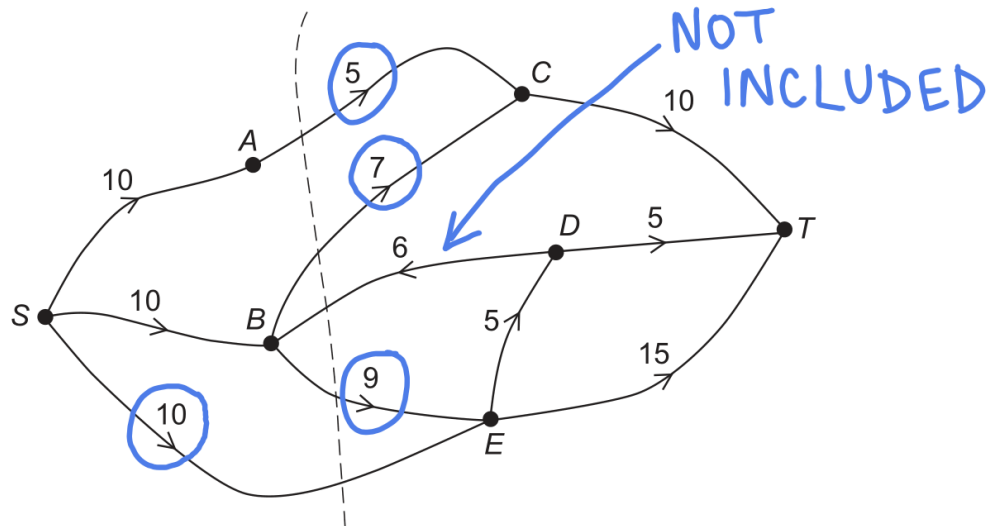
The value of the cut $\{S, A, B\} \{C, D, E, T\}$ is $V cm^3 s^{-1}$

Find V.

Circle your answer.

[1 mark]

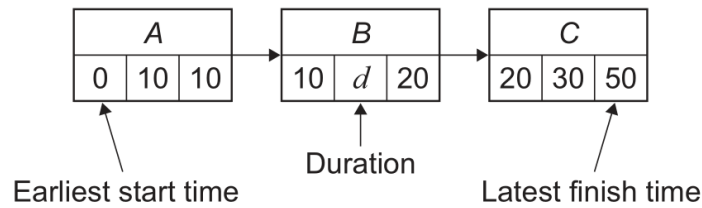25   30   (31)   37

$5 + 7 + 9 + 10 = 31$

Turn over for the next question

Turn over ▶

2 Part of an activity network is shown in the diagram below.

ABC is part of the critical path of the activity network.

A				B				C		
0	10	10		10	d	20		20	30	50

Earliest start time   Duration   Latest finish time

The duration of activity *B* is d.

Which of the following statements about d is correct?

Circle your answer.

[1 mark]

$0 < d < 10$ $d = 10$ $10 < d < 20$ $d = 20$

$20 - 10 - 0 = 10$

$\therefore d = 10$

critical path so float = 0

i.e. late time – duration – early time = 0

3 Manon makes apple cakes and banana cakes.

Each apple cake is made with 3 eggs and 100 grams of flour.
Each banana cake is made with 2 eggs and 150 grams of flour.

Manon has 36 eggs and 1500 grams of flour.

Manon wants to make as many cakes as possible.

Formulate Manon's situation as a linear programming problem, clearly defining any variables you introduce.

[4 marks]

Let x = number of apple cakes.
Let y = number of banana cakes.
Maximise

$P_{max} = x + y$ ← both cakes contribute equally to total

Subject to:

$3x + 2y \leq 36$

$100x + 150y \leq 1500$

$x \geq 0, y \geq 0$

x, y are integer

Turn over for the next question

Turn over ▶

4 (a)   State the definition of a bipartite graph.

[1 mark]

A bipartite graph is one in which the vertices can be split into two sets where no edge connects vertices in the same set.

4 (b)   A jazz quintet has five musical instruments: bassoon, clarinet, flute, oboe and violin.

Jay, Kay, Lee, Mel and Nish are musicians and each plays a musical instrument in the jazz quintet.

Jay knows how to play the bassoon and the clarinet.

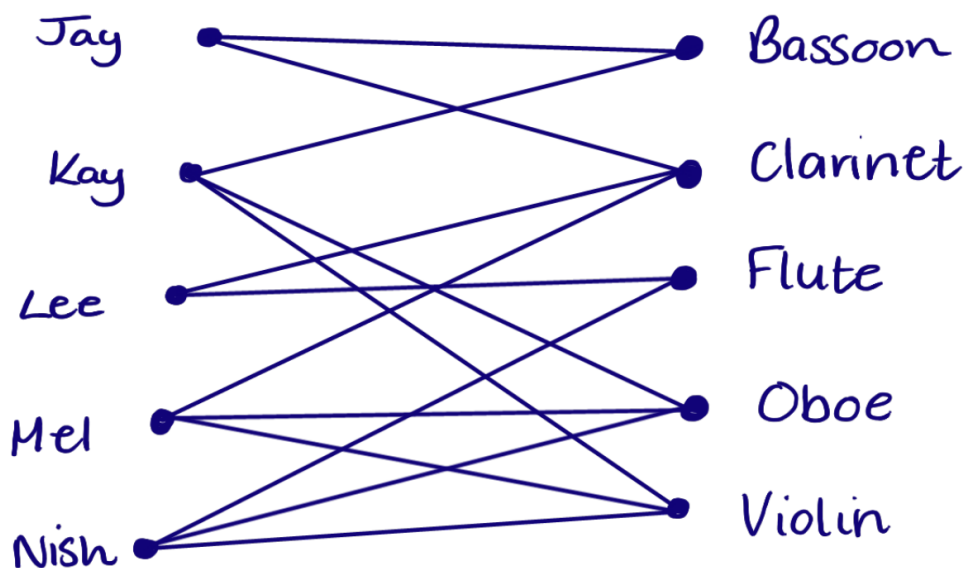Kay knows how to play the bassoon, the oboe and the violin.

Lee knows how to play the clarinet and the flute.

Mel knows how to play the clarinet, the oboe and the violin.

Nish knows how to play the flute, the oboe and the violin.

4 (b) (i)   Draw a graph to show which musicians know how to play which instruments.

[2 marks]

4 (b) (ii) Nish arrives late to a jazz quintet rehearsal.

Each of the other four musicians is already playing an instrument:

Jay is playing the clarinet
Kay is playing the oboe
Lee is playing the flute
Mel is playing the violin.

Explain how the graph in part **(b)(i)** shows that there is no instrument available that Nish knows how to play.

[1 mark]

The vertex for Nish is not connected to the vertex for bassoon.

4 (b) (iii) When Nish arrives the rehearsal stops. When they restart the rehearsal, Nish is playing the flute.

Draw all possible subgraphs of the graph in part **(b)(i)** that show how Jay, Kay, Lee and Mel can each be assigned a unique musical instrument they know how to play.

[2 marks]

Kay & Mel
have 2 instruments
in common

Bassoon → can swap
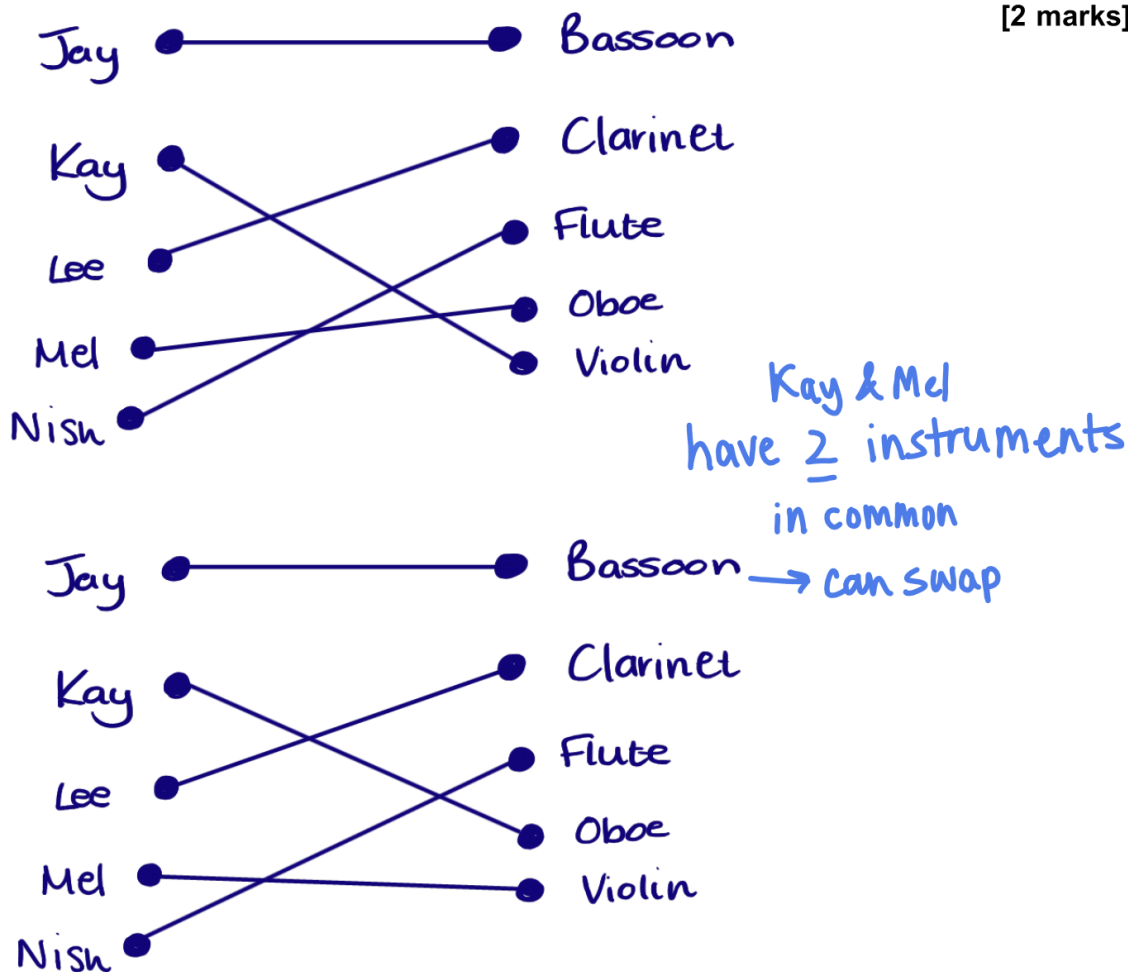
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5 (a)   Complete the Cayley table in **Figure 1** for multiplication modulo 4

[2 marks]

Figure 1

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

5 (b)   The set S is defined as

$$S = \{a, b, c, d\}$$

Figure 2 shows an incomplete Cayley table for S under the commutative binary operation •

Figure 2

•	a	b	c	d
a	b	a	a	c
b	a	c	d	c
c	a	d	d	d
d	c	c	d	d

5 (b) (i)   Complete the Cayley table in **Figure 2**.

[1 mark]

5 (b) (ii) Determine whether the binary operation • is associative when acting on the elements of *S*.

Fully justify your answer.

[2 marks]

Set up a test for associativity:

$a \cdot (b \cdot c) = a \cdot d = c$

$(a \cdot b) \cdot c = a \cdot c = a$

$a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$

∴ '•' is not as not associative
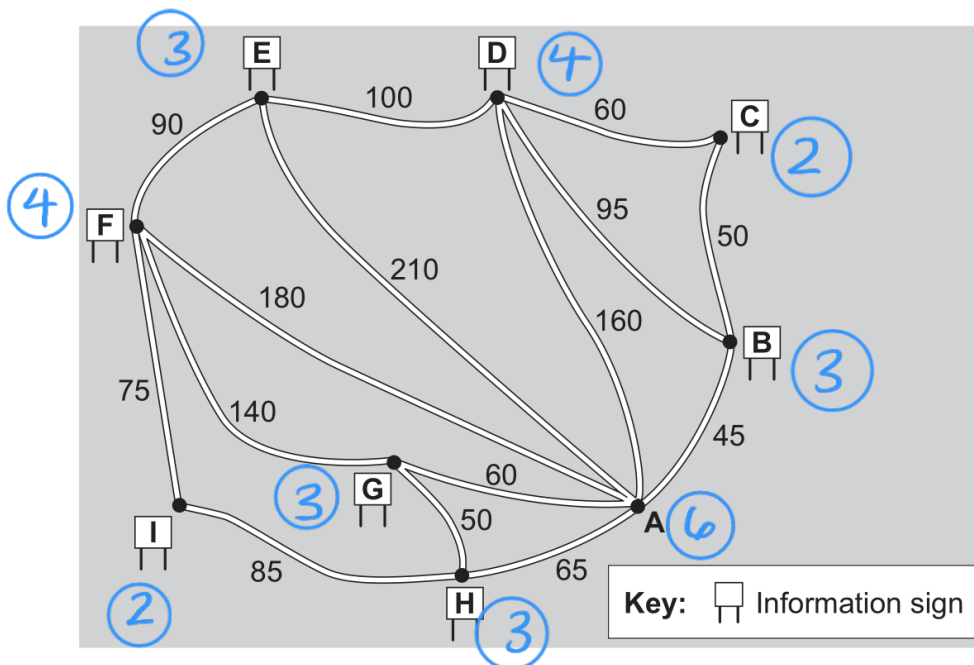
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6 The diagram shows a nature reserve which has its entrance at *A*, eight information signs at *B*, *C*, ..., *I*, and fifteen grass paths.

The length of each grass path is given in metres.

The total length of the grass paths is 1465 metres.

To cut the grass, Ashley starts at the entrance and drives a mower along every grass path in the nature reserve.

The mower moves at 7 kilometres per hour.

6 (a) Find the least possible time that it takes for Ashley to cut the grass on all fifteen paths in the nature reserve and return to the entrance.

Fully justify your answer.

[5 marks]

Odd nodes: B, E, G, H   odd nodes ➔
Shortest Distances: route inspection
B(D)E : 195   B(A)G : 105
B(A)H : 110   E(FI)H : 250
E(F)G : 230   GH : 50

Possible Pairings:

B(D)E + GH = 195 + 50 = 245   * Shortest route *
B(A)G + E(FI)H = 105 + 250 = 355
B(A)H + E(F)G = 110 + 230 = 340

Repeat BD, DE and GH as 245 is the shortest route

1465 + 245 = 1710 m

1710m ÷ 1000 = 1.710 km

1.710 ÷ 7 = 0.244 hours (3SF)

Used distance
Speed
= time

6 (b) Brook visits every information sign in the nature reserve to update them, **starting and finishing at the entrance**.

For the eight information signs, the minimum connecting distance of the grass paths is 510 metres.

6 (b) (i) Determine a lower bound for the distance Brook walks to visit every information sign.

Fully justify your answer.

[2 marks]

The two shortest arcs from the entrance are

AB : 45

AG : 60

Min. connector + shortest arcs

Lower bound = 510 + 45 + 60

= 615 m

6 (b) (ii) Using the nearest neighbour algorithm **starting from the entrance**, determine an upper bound for the distance Brook walks to visit every information sign.

[2 marks]

A₄₅ B₅₀ C₆₀ D₁₀₀ E₉₀ F₇₅ I₈₅ H₅₀ G₆₀ A

Upper bound :

45 + 50 + 60 + 100 + 90 + 75 + 85 + 50 + 60

= 615 m

Turn over ▶

6 (c)   Brook takes one minute to update the information at one information sign.
Brook walks on the grass paths at an average speed of 5 kilometres per hour.
Ashley and Brook start from the entrance at the same time.

6 (c) (i)   Use your answers from parts **(a)** and **(b)** to show that Ashley and Brook will return to
the entrance at approximately the same time.

Fully justify your answer.

[3 marks]

The upper bound equals the lower bound
(both equal to 615m) ∴ the optimal
distance is 615m.

time to walk → $615m \div 1000 = 0.615\,km$

$(0.615 \div 5) + (8 \times \frac{1}{60})$ time to update signs

$= 0.256$ hours   important to use consistent units!

The times for Ashley and Brook are both
about ¼ of an hour (≈15 minutes)

6 (c) (ii)   State an assumption that you have used in part **(c)(i)**.

[1 mark]

Brook and Ashley work at constant rates
with no breaks.

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7 Ali and Bex play a zero-sum game.
The game is represented by the following pay-off matrix for Ali.

		Bex		
	Strategy	B_1	B_2	B_3
Ali	A_1	2	−1	3
	A_2	−4	−2	2
	A_3	0	1	1
	A_4	−3	2	−2

7 (a) (i) Write down the pay-off matrix for Bex.

[2 marks]

		Ali			
	Strategy	A_1	A_2	A_3	A_4
Bex	B_1	−2	4	0	3
	B_2	1	2	−1	−2
	B_3	−3	−2	−1	2

7 (a) (ii) Explain why the pay-off matrix for Bex can be written as

−2	0	3
1	−1	−2

[2 marks]

$-3 < -2$, $-2 < 4$, $-1 < 0$ and $2 < 3$ hence Strategy B_1 **dominates** Strategy B_3.

$-2 < 4$, $1 < 2$ and $-3 < -2$ hence strategy A_1 **dominates** Strategy A_2.

7 (b)   The game does **not** have a stable solution.

Bex plays her optimal mixed strategy.

Find the value of the game **for Ali**.

[6 marks]

(annotations in red)
1. define probability variable
2. Calculate gains for B
3. draw graph with expected gains
4. find optimal point & value of p
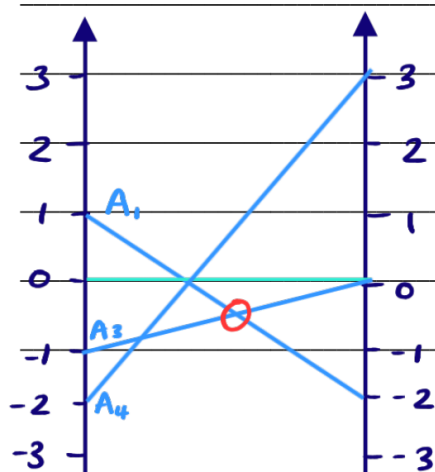5. Calculate game value for Ali.

Let Bex choose strategy B_1 with probability p and strategy B_2 with probability $1-p$.

If Ali plays:

A_1 : expected gain for Bex.
$$= -2p + 1(1-p) = -3p+1$$

A_3 : expected gain for Bex
$$= -1(1-p) = p-1$$

A_4 : expected gain for Bex
$$= 3p - 2(1-p) = 5p-2$$

$A_1 = A_3$
$$-3p+1 = p-1$$
$$4p = 2$$
$$p = \frac{1}{2}$$

Value of game for Bex :
$$p-1 = \frac{1}{2}-1 = -\frac{1}{2}$$

Hence value of game for Ali $= \frac{1}{2}$

END OF QUESTIONS

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