AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

AS
FURTHER MATHEMATICS

MODEL ANSWERS

Paper 2 – Discrete

Thursday 17 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

J U N 1 8 7 3 6 6 2 D 0 1

PB/Jun18/E6

7366/2D

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Answer **all** questions in the spaces provided.

1 The table shows some of the outcomes of performing a modular arithmetic operation.

	2	3
2		1
3	1	

Which pair are operations that could each be represented by the table?

Tick (✓) **one** box.

- ☐ Addition mod 6 and multiplication mod 5
- ☐ Addition mod 6 and multiplication mod 6
- ☑ Addition mod 4 and multiplication mod 5
- ☐ Addition mod 4 and multiplication mod 6

[1 mark]

2 The binary operation $\otimes$ is given by

$a \otimes b = 3a(5 + b) \pmod 8$

where $a, b \in \mathbb{Z}$

Given that $2 \otimes x = 6$, which of the integers below is a possible value of x?

Circle your answer.

[1 mark]

(0) 1 2 3

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3 Alex and Sam are playing a zero-sum game.

The game is represented by the pay-off matrix for Alex.

		Sam		
	Strategy	S_1	S_2	S_3
Alex	A_1	2	2	3
	A_2	0	3	5
	A_3	−1	2	−2

3 (a) Explain why the value of the game is 2

[3 marks]

col. maxima = (2,3,5)
min (col. maxima) = 2
row minima = (2, 0, -2)
max (row minima) = 2
min(col. max) = max(row min) = 2 so there is a stable solution & the game has value 2

3 (b) Identify the play-safe strategy for each player.

[1 mark]

Alex plays A1
Sam plays S1

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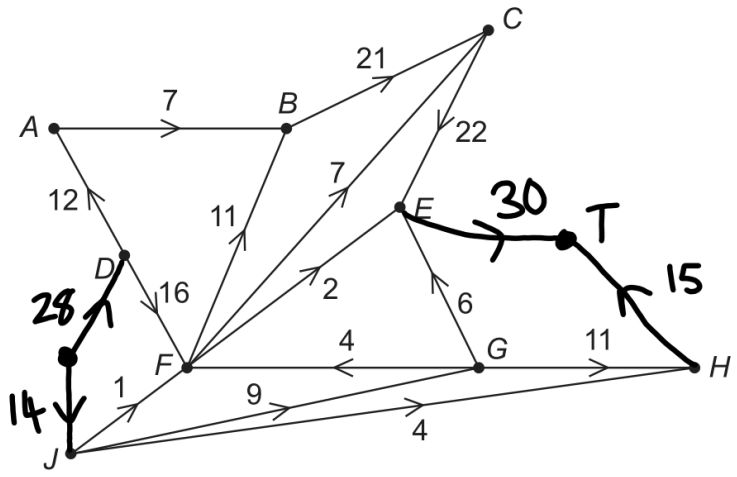
4 The diagram shows a network of pipes.

Each pipe is labelled with its upper capacity in $cm^3\,s^{-1}$



4 (a) (i) Find the value of the cut given by $\{A, B, C, D, F, J\}$ $\{E, G, H\}$.

[1 mark]

37

4 (a) (ii) State what can be deduced about the maximum flow through the network.

[1 mark]

Max flow ≤ 37

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4 (b) (i) List the nodes which are sources of the network.

[1 mark]

D & J, these 2 nodes only have arrows pointing away from them

4 (b) (ii) Add a supersource *S* to the network.

[1 mark]

4 (c) (i) List the nodes which are sinks of the network.

[1 mark]

E & H, both only have arrows pointing towards them

4 (c) (ii) Add a supersink *T* to the network.

[1 mark]

Turn over for the next question

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5 A group of friends want to prepare a meal. They start preparing the meal at 6:30 pm

Activities to prepare the meal are shown in **Figure 1** below.

Figure 1

Label	Activity	Duration (mins)	Immediate predecessors
A	Weigh rice	1	–
B	Cook rice	18	*A*
C	Drain rice	1	*B*
D	Chop vegetables	10	–
E	Fry vegetables	12	D
F	Combine fried vegetables and drained rice	1	C, E
G	Prepare sauce ingredients	4	–
H	Boil sauce	12	G
I	Serve meal on plates	2	F, H

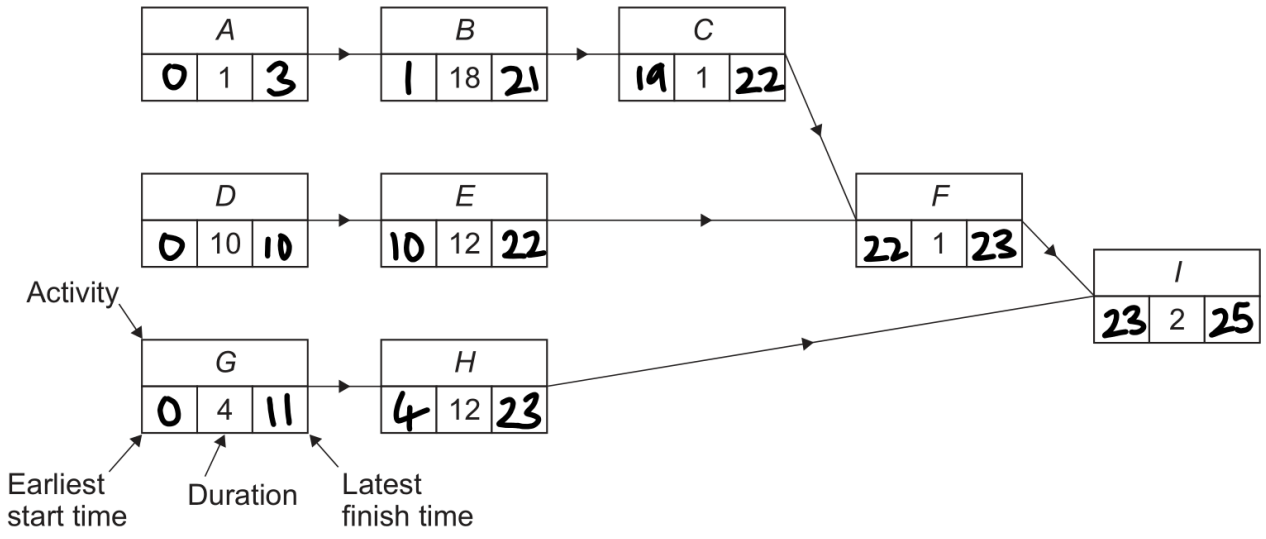
5 (a) (i) Use **Figure 2** shown below to complete **Figure 1** above.

[1 mark]

5 (a) (ii) Complete **Figure 2** showing the earliest start time and latest finishing time for each activity.

[2 marks]

Figure 2



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5 (b) (i) State the activity which must be started first so that the meal is served in the shortest possible time.

Fully justify your answer.

[2 marks]

D must be started first because it is the first critical path & has float zero

5 (b) (ii) Determine the earliest possible time at which the preparation of the meal can be completed.

[1 mark]

the finish time for I is 25 minutes
6:30 pm + 25 minutes = 6:55pm

Question 5 continues on the next page

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5 (c) The group of friends want to cook spring rolls so that they are served at the same time as the rest of the meal. This requires the additional activities shown in **Figure 3**.

Figure 3

Label	Activity	Duration	Immediate predecessors
J	Switch on and heat oven	10·25	–
K	Put spring rolls in oven and cook	8·25	J
L	Transfer spring rolls to serving dish	0·5	K

It takes 15 seconds to switch on the oven.

The oven must be allowed to heat up for 10 minutes before the spring rolls are put in the oven.

It takes 15 seconds to put the spring rolls in the oven.

The spring rolls must cook in the hot oven for 8 minutes.

It takes 30 seconds to transfer the spring rolls to a serving dish.

5 (c) (i) Complete **Figure 3** above.

[1 mark]

5 (c) (ii) Determine the latest time at which the oven can be switched on in order for the spring rolls to be served at the same time as the rest of the meal.

[2 marks]

the spring rolls take 10·25 + 8·25 + 0·5 = 19 minutes if we want to have them ready at 6:55 then they need to start at 6:55 - 19 minutes = 6:36pm

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Turn over for the next question

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6 An animal sanctuary has a rainwater collection site.

The manager of the sanctuary is installing a pipe system to connect the rainwater collection site to five other sites in the sanctuary.

Each site does **not** need to be connected directly to the rainwater collection site.

There are nine possible routes between the sites that are suitable for water pipes.

The distances, in metres, of the nine possible routes are given in the table below.

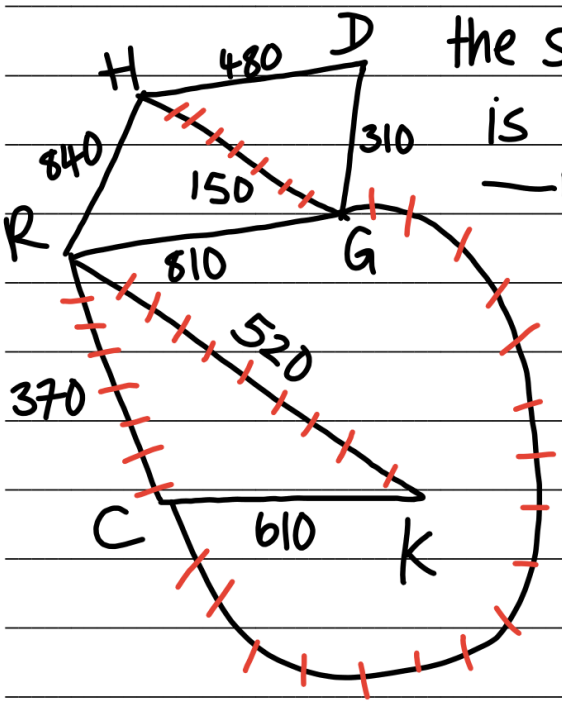
From/To	**Henhouse (*H*)**	**Goatshed (*G*)**	**Kennels (*K*)**	**Cattery (*C*)**
Rainwater collection site (*R*)	840	810	520	370
Cattery (*C*)	–	680	610	
Duckpond (*D*)	480	310		
Goatshed (*G*)	150			

Water pipe costs 60 pence per metre.

Find the minimum cost of connecting all the sites to the rainwater collection site.

Fully justify your answer.

[5 marks]



the shortest route from RC is RC
— " — R to K is RK since the pipes are going to go to C, the shortest route to G is via C, so we also need CG.
now we have got to G, the shortest routes to H & D are GH & GD.

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So we need RC, RK, CG, GH, GD

total distance = 370 + 520 + 680 + 150 + 310

= 2030 m

cost = 2030 x 0·6 = £1218

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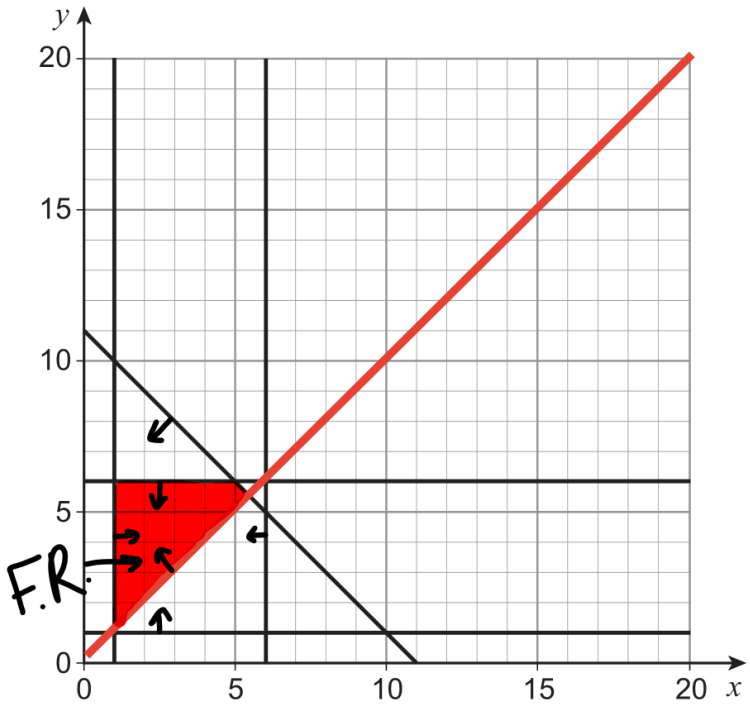
7 A linear programming problem has the constraints

$$1 \leq x \leq 6$$
$$1 \leq y \leq 6$$
$$y \geq x$$
$$x + y \leq 11$$

7 (a) (i) Complete **Figure 4** to identify the feasible region for the problem.

[2 marks]

Figure 4



7 (a) (ii) Determine the maximum value of $5x + 4y$ subject to the constraints.

[2 marks]

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7 (b) The simple-connected graph G has seven vertices.

The vertices of G have degree 1, 2, 3, v, w, x and y

7 (b) (i) Explain why $x \geq 1$ and $y \geq 1$

[1 mark]

7 (b) (ii) Explain why $x \leq 6$ and $y \leq 6$

[1 mark]

7 (b) (iii) Explain why $x + y \leq 11$

[2 marks]

7 (b) (iv) State an additional constraint that applies to the values of x and y in this context.

[1 mark]

Turn over ▶

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7 (c) The graph G also has eight edges. The inequalities used in part **(a)(i)** apply to the graph G.

7 (c) (i) Given that $v + w = 4$, find all the feasible values of x and y.

[3 marks]

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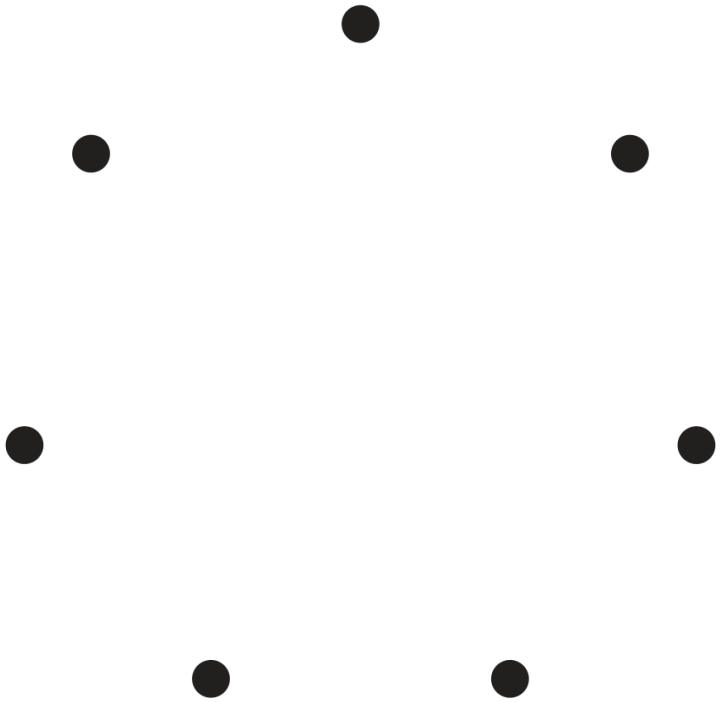
7 (c) (ii) It is also given that the graph G is semi-Eulerian.

On **Figure 5**, draw G.

[2 marks]

Figure 5



END OF QUESTIONS

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