AQA

Please write clearly in block capitals.

Centre number | | | | | |    Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

# AS
# FURTHER MATHEMATICS
## Paper 1

Time allowed: 1 hour 30 minutes

**Materials**
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
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| **TOTAL** | |

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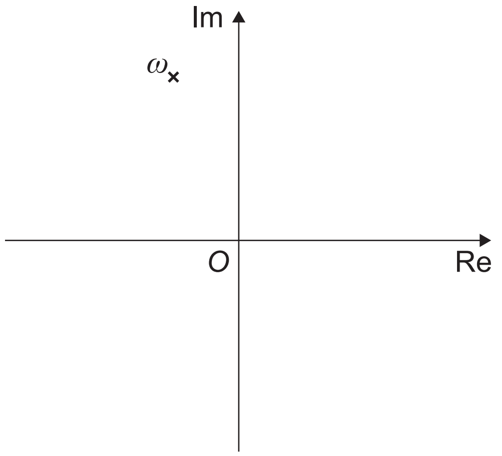
Answer **all** questions in the spaces provided.

**1** The complex number $\omega$ is shown below on the Argand diagram.




Which of the following complex numbers could be $\omega$?

Tick (✓) **one** box.

**[1 mark]**

$\cos(-2) + \mathrm{i}\sin(-2)$ ☐

$\cos(-1) + \mathrm{i}\sin(-1)$ ☐

$\cos(1) + \mathrm{i}\sin(1)$ ☐

$\cos(2) + \mathrm{i}\sin(2)$ ☐

**2** Given that $\mathrm{f}(x) = 3x - 1$ find the mean value of $\mathrm{f}(x)$ over the interval $4 \le x \le 8$

Circle your answer.

**[1 mark]**

6 11 17 23

Do not write outside the box

**3** The matrix **M** represents a rotation about the $x$-axis.

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & \frac{\sqrt{3}}{2} \\ 0 & b & -\frac{1}{2} \end{bmatrix}$$

Which of the following pairs of values is correct?

Tick (✓) **one** box.

**[1 mark]**

$a = \frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}$ ☐

$a = \frac{1}{2}$ and $b = -\frac{\sqrt{3}}{2}$ ☐

$a = -\frac{1}{2}$ and $b = \frac{\sqrt{3}}{2}$ ☐

$a = -\frac{1}{2}$ and $b = -\frac{\sqrt{3}}{2}$ ☐

**4** The point (2, −1) is invariant under the transformation represented by the matrix **N**

Which of the following matrices could be **N**?

Circle your answer.

**[1 mark]**

$\begin{bmatrix} 4 & 6 \\ 2 & 5 \end{bmatrix}$ $\begin{bmatrix} 6 & 5 \\ 4 & 2 \end{bmatrix}$ $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}$ $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

**Turn over for the next question**

**5** Show that the vectors $\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$ are perpendicular.

**[2 marks]**

**6** Prove the identity

$$\cosh^2 x - \sinh^2 x = 1$$

**[2 marks]**

Do not write outside the box

**7** Show that the Maclaurin series for $\ln(\mathrm{e} + 2\mathrm{e}x)$ is

$$1 + 2x - 2x^2 + ax^3 - \ldots$$

where $a$ is to be determined.

**[3 marks]**

**Turn over for the next question**

**8** Stephen is correctly told that $(1 + \mathrm{i})$ and $-1$ are two roots of the polynomial equation

$$z^3 - 2\mathrm{i}z^2 + pz + q = 0$$

where $p$ and $q$ are complex numbers.

**8 (a)** Stephen states that $(1 - \mathrm{i})$ **must** also be a root of the equation because roots of polynomial equations occur in conjugate pairs.

Explain why Stephen's reasoning is wrong.

**[1 mark]**

**8 (b)** Find $p$ and $q$

**[5 marks]**

Do not write outside the box

**Turn over for the next question**

Do not write outside the box

**9 (a)** Use the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+3) = an(n+1)(n+b)$$

where $a$ and $b$ are constants to be determined.

**[4 marks]**

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**9 (b)** Hence, or otherwise, find a fully factorised expression for

$$\sum_{r=n+1}^{5n} r(r+3)$$

**[3 marks]**

**10** Matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 3 & \mathrm{i} - 1 \\ \mathrm{i} & 2 \end{bmatrix}$$

**10 (a)** Show that $\det \mathbf{A} = a + \mathrm{i}$ where $a$ is an integer to be determined.

**[2 marks]**

**10 (b)** Matrix **B** is given by

$$\mathbf{B} = \begin{bmatrix} 14 - 2\mathrm{i} & b \\ c & d \end{bmatrix} \quad \text{and} \quad \mathbf{AB} = p\mathbf{I}$$

where $b$, $c$, $d \in \mathbb{C}$ and $p \in \mathbb{N}$

Find $b$, $c$, $d$ and $p$

**[6 marks]**

**Turn over for the next question**

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**11 (a)** Show that, for all positive integers $r$,

$$\frac{1}{(r-1)!} - \frac{1}{r!} = \frac{r-1}{r!}$$

**[1 mark]**

**11 (b)** Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{r-1}{r!} = a + \frac{b}{n!}$$

where $a$ and $b$ are integers to be determined.

**[3 marks]**

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**Turn over for the next question**

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**12** The equation $x^3 - 2x^2 - x + 2 = 0$ has three roots. One of the roots is 2

**12 (a)** Find the other two roots of the equation.

**[1 mark]**

**12 (b)** Hence, or otherwise, solve

$$\cosh^3\theta - 2\cosh^2\theta - \cosh\theta + 2 = 0$$

giving your answers in an exact form.

**[4 marks]**

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**13** Prove by induction that, for all integers $n \geq 1$

$$\sum_{r=1}^{n} 2^{-r} = 1 - 2^{-n}$$

**[4 marks]**

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**14** Curve $C_1$ has equation

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$

**14 (a)** Curve $C_2$ is a reflection of $C_1$ in the line $y = x$

Write down an equation of $C_2$

**[1 mark]**

**14 (b)** Curve $C_3$ is a circle of radius 4, centred at the origin.

Describe a single transformation which maps $C_1$ onto $C_3$

**[2 marks]**

**14 (c)** Curve $C_4$ is a translation of $C_1$
The positive $x$-axis and the positive $y$-axis are tangents to $C_4$

**14 (c) (i)** Sketch the graphs of $C_1$ and $C_4$ on the axes opposite. Indicate the coordinates of the $x$ and $y$ intercepts on your graphs.

**[2 marks]**

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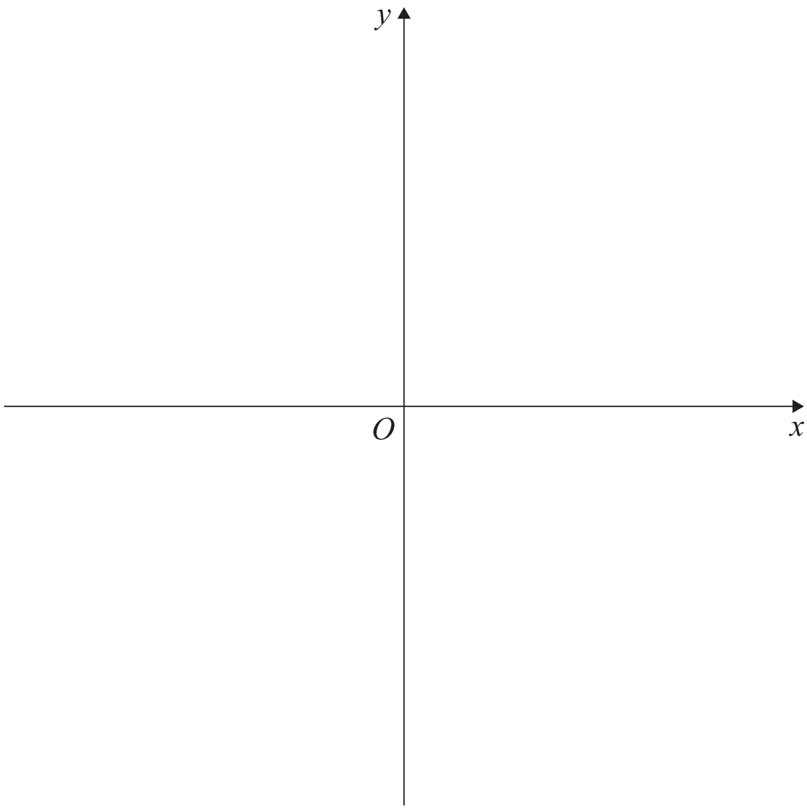


**14 (c) (ii)** Determine the translation vector.

**[2 marks]**

**14 (c) (iii)** The line $y = mx + c$ is a tangent to both $C_1$ and $C_4$
Find the value of $m$

**[2 marks]**

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**15** Two submarines are travelling on different straight lines.
The two lines are described by the equations

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4 - z$$

**15 (a) (i)** Show that the two lines intersect.

**[3 marks]**

**15 (a) (ii)** Find the position vector of the point of intersection.

**[1 mark]**

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**15 (b)** Tracey says that the submarines will collide because there is a common point on the two lines.

Explain why Tracey is not necessarily correct.

**[1 mark]**

**15 (c)** Calculate the acute angle between the lines

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad \frac{x-5}{4} = \frac{y}{2} = 4 - z$$

Give your angle to the nearest 0.1°

**[3 marks]**

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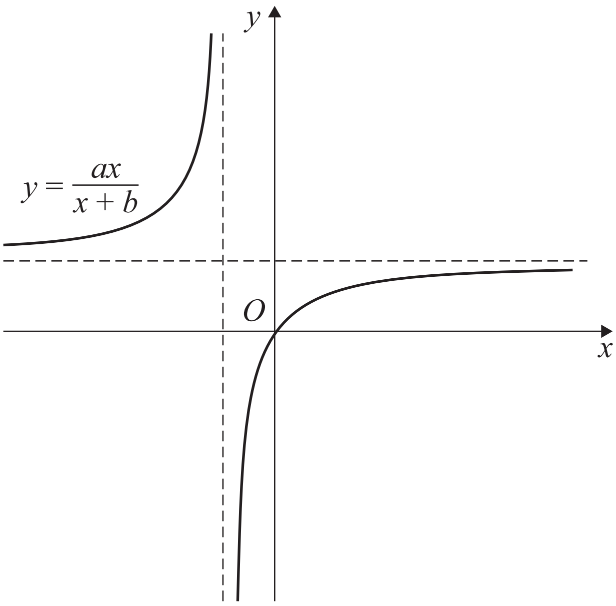
**16** Curve $C$ has equation $y = \frac{ax}{x+b}$ where $a$ and $b$ are constants.

The equations of the asymptotes to $C$ are $x = -2$ and $y = 3$




**16 (a)** Write down the value of $a$ and the value of $b$

**[2 marks]**

**16 (b)** The gradient of $C$ at the origin is $\frac{3}{2}$

With reference to the graph, explain why there is exactly one root of the equation

$$\frac{ax}{x+b} = \frac{3x}{2}$$

**[2 marks]**

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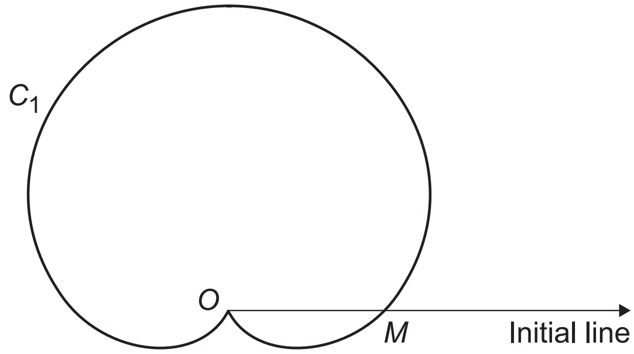
**16 (c)** Using the values found in part **(a)**, solve the inequality

$$\frac{ax}{x+b} \leq 1 - x$$

**[4 marks]**

**Turn over ▶**

**17** The curve $C_1$ has polar equation $r = 2a(1 + \sin\theta)$ for $-\pi < \theta \leq \pi$ where $a$ is a positive constant.




The point *M* lies on $C_1$ and the initial line.

**17 (a)** Write down, in terms of $a$, the polar coordinates of *M*

**[1 mark]**

**17 (b)** *N* is the point on $C_1$ that is furthest from the pole *O*

Find, in terms of $a$, the polar coordinates of *N*

**[2 marks]**

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**17 (c)** The curve $C_2$ has polar equation $r = 3a$ for $-\pi < \theta \leq \pi$
$C_2$ intersects $C_1$ at points $P$ and $Q$

Show that the area of triangle $NPQ$ can be written in the form

$$m\sqrt{3}a^2$$

where $m$ is a rational number to be determined.

**[5 marks]**

**Question 17 continues on the next page**

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**17 (d)** On the initial line below, sketch the graph of $r = 2a(1 + \cos\theta)$ for $-\pi < \theta \leq \pi$

Include the polar coordinates, in terms of $a$, of any intersection points with the initial line.

**[2 marks]**


**END OF QUESTIONS**

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**ANSWER IN THE SPACES PROVIDED**

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