Model Solutions

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

AS
FURTHER MATHEMATICS
Paper 1

Monday 11 May 2020 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Express the complex number $1 - i\sqrt{3}$ in modulus-argument form.

Tick (✓) **one** box.

let $z = 1 - i\sqrt{3}$

[1 mark]

$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ☐

$2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ ☐

$2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ ☑

$2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$ ☐

$|z| = \sqrt{1^2 + (-\sqrt{3})^2}$

$= 2$

$\arg z = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$

$= -\frac{\pi}{3}$

$\therefore z = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

2 Given that $1 - i$ is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.

Tick (✓) **one** box.

[1 mark]

$-1 + i$ and -1 ☐

$1 + i$ and 1 ☑

$-1 + i$ and 1 ☐

$1 + i$ and -1 ☐

$1+i$ must be a root as it is the conjugate pair of $1-i$.

checking if -1 is a root:

$(-1)^3 - 3(-1)^2 + 4(-1) - 2 = -10$

$\therefore -1$ is not a root

checking if 1 is a root:

$(1)^3 - 3(1)^2 + 4(1) - 2 = 0$

$\therefore 1$ is a root.

Do not write outside the box

3 Given $(x-1)(x-2)(x-a) < 0$ and $a > 2$

Find the set of possible values of x.

Tick (✓) **one** box.

[1 mark]

$\{x : x < 1\} \cup \{x : 2 < x < a\}$ ☑

$\{x : 1 < x < 2\} \cup \{x : x > a\}$ ☐

$\{x : x < -a\} \cup \{x : -2 < x < -1\}$ ☐

$\{x : -a < x < -2\} \cup \{x : x > -1\}$ ☐

As $(x-1)(x-2)(x-a) \neq 0$

$x \neq 1,\ x \neq 2,\ x \neq a$

As $(x-1)(x-2)(x-a) < 0$,

$x < 1$ or $2 < x < a$

$\therefore \{x : x < 1\} \cup \{x : 2 < x < a\}$

Turn over for the next question

Do not write outside the box

4 The matrices **A** and **B** are such that

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

4 (a) Find the product **AB** in terms of a.

[2 marks]

$$\underset{(2\times3)}{\begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix}} \underset{(3\times2)}{\begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}} = \begin{bmatrix} 2-2a+0 & -6+4a^2+15 \\ 0+4+0 & 0-8a+5 \end{bmatrix}$$

$\therefore$ must be a 2x2 matrix.

$$= \begin{bmatrix} 2-2a & 4a^2+9 \\ 4 & -8a+5 \end{bmatrix}$$

4 (b) Find the determinant of **AB** in terms of a.

[1 mark]

$$\det \begin{bmatrix} 2-2a & 4a^2+9 \\ 4 & -8a+5 \end{bmatrix}$$

$$= (2-2a)(-8a+5) - 4(4a^2+9)$$

$$= -16a + 10 + \cancel{16a^2} - 10a - \cancel{16a^2} - 36$$

$$= -26a - 26$$

Do not write outside the box

4 (c) Show that **AB** is singular when $a = -1$

[2 marks]

If AB is singular, det AB = 0

$-26 - 26a = 0$

$-26a = 26$

$a = -1$

Turn over for the next question

Do not write outside the box

5 (a) Show that

$$r^2(r+1)^2 - (r-1)^2r^2 = pr^3$$

where p is an integer to be found.

[1 mark]

$r^2(r+1)^2 - (r-1)^2r^2 =$ *Expand*

$(r^2(r^2+2r+1)) - (r^2(r^2-2r+1)) =$

$(\cancel{r^4} + 2r^3 + \cancel{r^2}) - (\cancel{r^4} - 2r^3 + \cancel{r^2}) =$

$4r^3$

$\therefore p = 4$

Do not write outside the box

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[3 marks]

$$\sum_{r=1}^{n} 4r^3 = \sum_{r=1}^{n} \left[r^2(r+1)^2 - (r-1)^2 r^2 \right]$$

Plugging in numbers:

$f(1)$ $\cancel{1^2(2)^2} - \cancel{0^2(1)^2}$

$f(2)$ $\cancel{2^2(3)^2} - \cancel{1^2(2)^2}$

$f(3)$ $\cancel{3^2(4)^2} - \cancel{2^2(3)^2}$ ...

$f(n-1)$ $\cancel{(n-1)^2 n^2} - \cancel{(n-2)^2(n-1)^2}$

$f(n)$ $n^2(n+1)^2 - \cancel{(n-1)^2 n^2}$

$$\therefore \sum_{r=1}^{n} 4r^3 = n^2(n+1)^2$$

$$\Rightarrow \frac{1}{4}\sum_{r=1}^{n} 4r^3 = \sum_{r=1}^{n} r^3$$

$$\therefore \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$ (as required)

Do not write outside the box

6 Anna has been asked to describe the transformation given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

She writes her answer as follows:

The transformation is a rotation about the x-axis through an angle of θ, where

$$\sin\theta = \frac{1}{2} \quad \text{and} \quad -\sin\theta = -\frac{1}{2}$$

$$\theta = 30°$$

Identify and correct the error in Anna's work.

[2 marks]

Anna gave the wrong angle (the angle is not 30°).

$\sin\theta = \frac{1}{2}$ $\quad \theta = 30°$ or $\theta = 150°$

$\cos\theta = \frac{-\sqrt{3}}{2}$ $\quad \theta = 150°$ or $\theta = -150°$

$\therefore \theta = 150°$

Do not write outside the box

7 Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4

[4 marks]

when $n=1$

$7^1 - 3^1 = 4 = 4(1)$ As 4 is divible by 4, it is true for $n=1$.

Assume $n=k$ is divisible by 4.

$7^k - 3^k$ is divisible by 4.

when $n=k+1$

$f(k+1) = 7^{k+1} - 3^{k+1}$

$= 7 \times 7^k - 3 \times 3^k$

$f(k+1) - f(k) = 7 \times 7^k - 3 \times 3^k - (7^k - 3^k)$

$= 6 \times 7^k - 2 \times 3^k$

$= 4 \times 7^k + 2(7^k - 3^k)$

$= 4 \times 7^k + 2f(k)$

$\therefore f(k+1) = 4 \times 7^k + 3f(k)$

Conclusion

$f(k+1)$ is divisible by 4, as 4×7^k is divisible by 4 and $f(k)$ is assumed to be divisible by 4.

As $f(1)$ is divisible by 4, by induction.

$7^n - 3^n$ is divisible by 4 for $n \geq 1$.

Do not write outside the box

8 (a) Prove that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[5 marks]

Let $y = \tanh^{-1} x$

$\tanh y = x$

Write tanhy in exponential form:

$$x = \frac{\frac{e^y - e^{-y}}{2}}{\frac{e^y + e^{-y}}{2}} = \frac{e^{2y}-1}{e^{2y}+1}$$

$x(e^{2y}+1) = e^{2y} - 1$

$xe^{2y} + x = e^{2y} - 1$

$x + 1 = e^{2y} - xe^{2y}$

Isolate e^{2y}

$x + 1 = e^{2y}(1-x)$

$\frac{x+1}{1-x} = e^{2y}$

$\ln\left|\frac{x+1}{1-x}\right| = 2y$

$\frac{1}{2}\ln\left|\frac{x+1}{1-x}\right| = y$

As $y = \tanh^{-1}(x)$

$\frac{1}{2}\ln\left|\frac{x+1}{1-x}\right| = \tanh^{-1}(x)$ (as required)

Do not write outside the box

8 (b) Prove that the graphs of

$$y = \sinh x \quad \text{and} \quad y = \cosh x$$

do **not** intersect.

[3 marks]

Write both equations in exponential forms

$y = \frac{e^{x} - e^{-x}}{2}$, $y = \frac{e^{x} + e^{-x}}{2}$

Equate them

$\frac{e^{x} - e^{-x}}{2} = \frac{e^{x} + e^{-x}}{2}$

$\cancel{e^{x}} - e^{-x} = \cancel{e^{x}} + e^{-x}$

$0 = 2e^{-x}$

But $e^{-x} > 0$

$\therefore$ the graphs $y = \sinh x$ and $y = \cosh x$ do not intersect.

Do not write outside the box

9 The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β, where $\alpha > \beta$.

9 (a) (i) Write down the value of $\alpha\beta$.

[1 mark]

$\alpha\beta = \frac{c}{a} = \frac{3}{2}$

let $2x^2 + px + 3 = 0$

be $ax^2 + bx + c = 0$

9 (a) (ii) Express $\alpha + \beta$ in terms of p.

[1 mark]

$\alpha + \beta = -\frac{b}{a} = \frac{-p}{2}$

9 (b) Hence find $(\alpha - \beta)^2$ in terms of p.

[2 marks]

Expand $(\alpha - \beta)^2$

$(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= [\alpha^2 + \beta^2] - 2\alpha\beta$

$= [(\alpha+\beta)^2 - 2\alpha\beta] - 2\alpha\beta$

$= (\alpha+\beta)^2 - 4\alpha\beta$

As $\alpha + \beta = -\frac{p}{2}$ and $\alpha\beta = \frac{3}{2}$

$\Rightarrow \left(-\frac{p}{2}\right)^2 - 4\left(\frac{3}{2}\right)$

$= \frac{p^2}{4} - 6$

Do not write outside the box

9 (c) **Hence** find, in terms of p, a quadratic equation with roots $\alpha - 1$ and $\beta + 1$

[4 marks]

$(\alpha - 1) + (\beta + 1) = \alpha + \beta = \frac{-b}{a} = -\frac{p}{2}$

$(\alpha - 1)(\beta + 1) = \alpha\beta + \alpha - \beta - 1$

From part b, we know $\alpha - \beta = \sqrt{\frac{p^2}{4} - 6}$

$\Rightarrow \frac{3}{2} + \sqrt{\frac{p^2}{4} - 6} - 1$

$= \frac{1}{2} + \sqrt{\frac{p^2}{4} - 6}$

Insert $(\alpha-1)+(\beta+1)$ and $(\alpha-1)(\beta+1)$ values into a quadratic equation:

$x^2 - x\left(-\frac{p}{2}\right) + \left(\frac{1}{2} + \sqrt{\frac{p^2}{4} - 6}\right) = 0$

$x^2 + \frac{p}{2}x + \left(\frac{1}{2} + \sqrt{\frac{p^2}{4} - 6}\right) = 0$

Do not write outside the box

10 (a) Show that the equation

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a, b and c are constants to be found.

[3 marks]

$y(2x+4) = 3x-5$

$\div 2$ $2xy + 4y = 3x - 5$ $\div 2$

$xy + 2y = \frac{3}{2}x - \frac{5}{2}$

$xy + 2y - \frac{3}{2}x + \frac{5}{2} = 0$

$xy + 2y - \frac{3}{2}x - 3 = -\frac{11}{2}$

$(x-2)(y-\frac{3}{2}) = -\frac{11}{2}$

$a = -2$, $b = -\frac{3}{2}$, $c = -\frac{11}{2}$

10 (b) Write down the equations of the asymptotes of the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[2 marks]

Using part a answer

- $(x-2) \neq 0$, $x \neq 2$

$\therefore$ graph asymptotes at $x = -2$.

- $(y - \frac{3}{2}) \neq 0$, $y \neq \frac{3}{2}$

$\therefore$ graph asymptotes at $y = \frac{3}{2}$.

Do not write outside the box

10 (c) Sketch, on the axes provided, the graph of

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]

Asymptotes at $x = -2$ and $y = \frac{3}{2}$

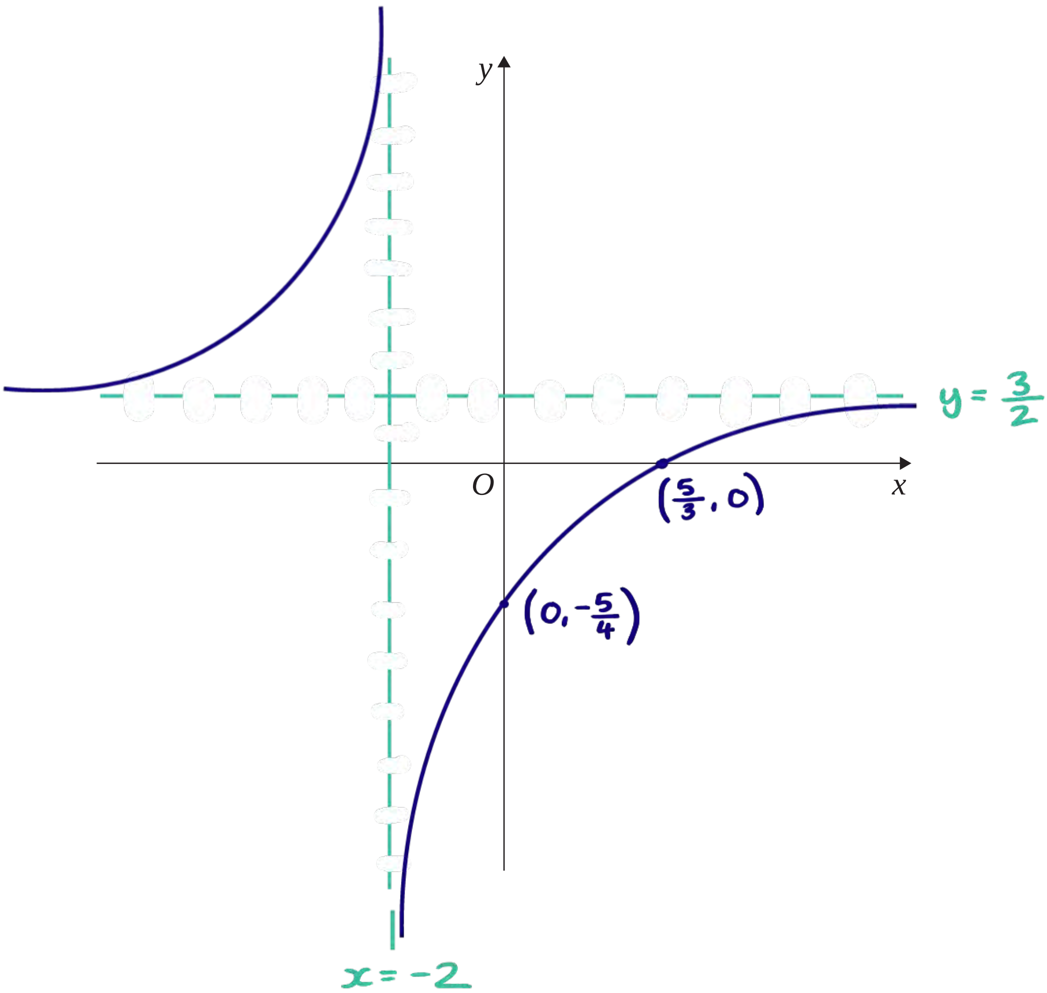
At $y = 0$,

$0 = 3x - 5$

$x = \frac{5}{3}$

At $x = 0$,

$y = -\frac{5}{4}$



Do not write outside the box

11 Sketch the polar graph of

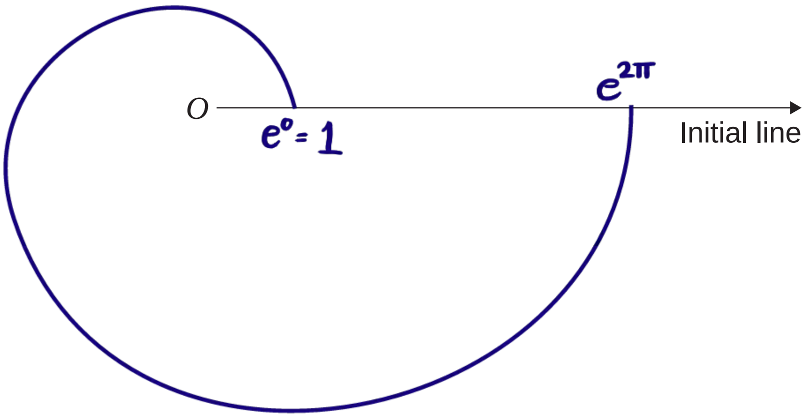
$$r = \sinh\theta + \cosh\theta$$

for $0 \le \theta \le 2\pi$

[3 marks]

$$\sinh\theta + \cosh\theta = \frac{e^{\theta} - e^{-\theta}}{2} + \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\Rightarrow \sinh\theta + \cosh\theta = e^{\theta}$$



Do not write outside the box

12 The mean value of the function f over the interval $1 \le x \le 5$ is m.

The graph of $y = \mathrm{g}(x)$ is a reflection in the x-axis of $y = \mathrm{f}(x)$.

The graph of $y = \mathrm{h}(x)$ is a translation of $y = \mathrm{g}(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Determine, in terms of m, the mean value of the function h over the interval $4 \le x \le 8$

[2 marks]

- Reflection in the x-axis changes the mean from m to $-m$.
- The translation of $y = g(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ changes the mean, $-m$, to $-m+7$

Mean $= -m+7$

Turn over for the next question

Do not write outside the box

13 Line l_1 has equation

$$\frac{x-2}{3} = \frac{1-2y}{4} = -z$$

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$$

13 (a) In the case when l_1 and l_2 are parallel, show that $a = -11$ and find the value of b.

[4 marks]

Rearranging l_1 equation:

$$\frac{x-2}{3} = \frac{y-\frac{1}{2}}{-2} = \frac{z-0}{-1}$$

$$l_1 : r = \begin{pmatrix} 2 \\ \frac{1}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

As l_1 and l_2 are parallel, they must be a multiple of eachother

$$\begin{pmatrix} 12 \\ a+3 \\ 2b \end{pmatrix} = p \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

$12 = 3p$ $\quad$ $a+3 = -2p$

$p = 4$ $\quad$ $a+3 = -2(4)$

$a = -11$ (as required)

$2b = -p$

$2b = -4$

$b = -2$

Do not write outside the box

13 (b) In a **different** case, the lines l_1 and l_2 intersect at exactly one point, and the value of b is 3

Find the value of a.

[5 marks]

When l_1 and l_2 intersect, the line equations are both equal.

$$l_1 = \begin{pmatrix} 2+3\lambda \\ \frac{1}{2} - 2\lambda \\ -\lambda \end{pmatrix} \qquad l_2 = \begin{pmatrix} -7+12\mu \\ 4+\mu(a+3) \\ -2+6\mu \end{pmatrix}$$

$$\begin{pmatrix} 2+3\lambda \\ \frac{1}{2} - 2\lambda \\ -\lambda \end{pmatrix} = \begin{pmatrix} -7+12\mu \\ 4+\mu(a+3) \\ -2+6\mu \end{pmatrix}$$

$2+3\lambda = -7+12\mu$

$9 = 12\mu - 3\lambda$ — 1

$\frac{1}{2} - 2\lambda = 4+\mu(a+3)$

$-\frac{7}{2} = \mu(a+3) + 2\lambda$ — 2

$-\lambda = -2+6\mu$

$2 = 6\mu + \lambda$ — 3

$3 \times$ 3 $+$ 1

$(18\mu + 3\lambda) + (12\mu - 3\lambda) = 6+9$

$30\mu = 15$

$\mu = \frac{1}{2} \quad \Rightarrow \lambda = -1$

Subbing $\mu = \frac{1}{2}$ and $\lambda = -1$ into 2

$-\frac{7}{2} = \frac{1}{2}(a+3) + 2(-1)$

$-\frac{3}{2} = \frac{1}{2}(a+3)$

$-3 = a+3 \quad \Rightarrow a = -6$

Do not write outside the box

14 (a) Given

$$\frac{x+7}{x+1} \le x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \ge 0$$

where a, b, and c are integers to be found.

[4 marks]

$$0 \le x+1 - \frac{x+7}{x+1}$$

$$0 \le \frac{(x+1)^2}{x+1} - \frac{x+7}{x+1}$$

$$0 \le \frac{x^2+2x+1-x-7}{x+1}$$

$$0 \le \frac{x^2+x-6}{x+1}$$

$$0 \le \frac{(x+3)(x-2)}{x+1}$$ (as required)

$a=3$, $b=-2$, $c=1$

14 (b) Briefly explain why this statement is incorrect.

$$\frac{(x+p)(x+q)}{x+r} \ge 0 \Leftrightarrow (x+p)(x+q)(x+r) \ge 0$$

[1 mark]

$x=-r$ is a solution of the inequality on the right hand side, but not the one on the left hand side.

Do not write outside the box

14 (c) Solve

$$\frac{x+7}{x+1} \leq x+1$$

[2 marks]

$0 \leq \dfrac{(x+3)(x-2)}{x+1}$ $\Rightarrow$ critical values: $-3, -1, 2$

when $x=-4$

$-\frac{8}{3} = \dfrac{(-1)(-8)}{-3}$ (negative, so -4 and under does not follow the inequality).

when $x=0$ (negative, so $-1, 0$ and 1 does not follow the inequality).

$-6 = \dfrac{(3)(-2)}{1}$

when $x=2$ (2 and above follows the inequality).

$0 = \dfrac{(5)(0)}{3}$

$\therefore$ $-3 \leq x < -1$ and $x \geqslant 2$

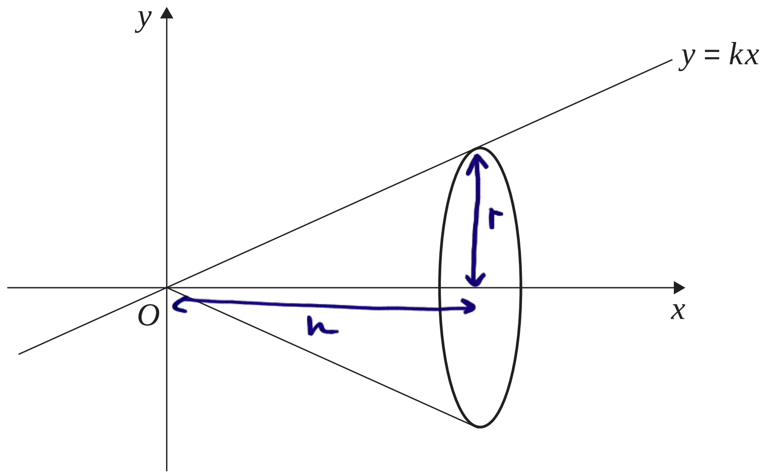
Turn over for the next question

Do not write outside the box

15 A segment of the line $y = kx$ is rotated about the x-axis to generate a cone with vertex O.

The distance of O from the centre of the base of the cone is h.

The radius of the base of the cone is r.



15 (a) Find k in terms of r and h.

[1 mark]

K is the gradient of the line.

$\boxed{K = \frac{r}{h}}$

Do not write outside the box

15 (b) Use calculus to prove that the volume of the cone is

$$\frac{1}{3}\pi r^2 h$$

[3 marks]

$$\text{Volume} = \pi \int_0^h y^2 \, dx$$

$$= \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx$$

$$= \frac{r^2\pi}{h^2} \int_0^h x^2 \, dx$$

$$= \frac{r^2\pi}{h^2} \left[\frac{x^3}{3}\right]_0^h$$

$$= \frac{r^2\pi}{\cancel{h^2}} \times \frac{h^{\cancel{3}}}{3} - 0$$

$$= \frac{1}{3}\pi r^2 h \quad \text{(as required)}$$

Do not write outside the box

16 **A** and **B** are non-singular square matrices.

16 (a) Write down the product $\mathbf{AA}^{-1}$ as a single matrix.

[1 mark]

$AA^{-1} = I$ eg. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

16 (b) **M** is a matrix such that $\mathbf{M} = \mathbf{AB}$.

Prove that $\mathbf{M}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

[3 marks]

$M = AB$

$B^{-1}A^{-1}M = B^{-1}\cancel{A^{-1}}\cancel{A}B$

$B^{-1}A^{-1}M = B^{-1}IB$

$B^{-1}A^{-1}\cancel{M}\cancel{M^{-1}} = B^{-1}IBM^{-1}$

$B^{-1}A^{-1}I = \cancel{B^{-1}}\cancel{B}M^{-1}$

$B^{-1}A^{-1}I = IM^{-1}$

$B^{-1}A^{-1} = M^{-1}$ (as required)

$AA^{-1} = I$
$BB^{-1} = I$
$MM^{-1} = I$

Do not write outside the box

17 The polar equation of the circle C is

$$r = a(\cos\theta + \sin\theta)$$

Find, in terms of a, the radius of C.

Fully justify your answer.

[4 marks]

Remember: $r^2 = x^2 + y^2$

$r\cos\theta = x$

$y = r\sin\theta$

$r^2 = ra(\cos\theta + \sin\theta)$

$r^2 = a(r\cos\theta + r\sin\theta)$

$x^2 + y^2 = a(x + y)$

$x^2 + y^2 = ax + ay$

$x^2 - ax - ay + y^2 = 0$

$x^2 - ax + \frac{a^2}{4} + y^2 - ay + \frac{a^2}{4} = \frac{a^2}{2}$

$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$

$\therefore$ radius $= \frac{a}{\sqrt{2}}$

Do not write outside the box

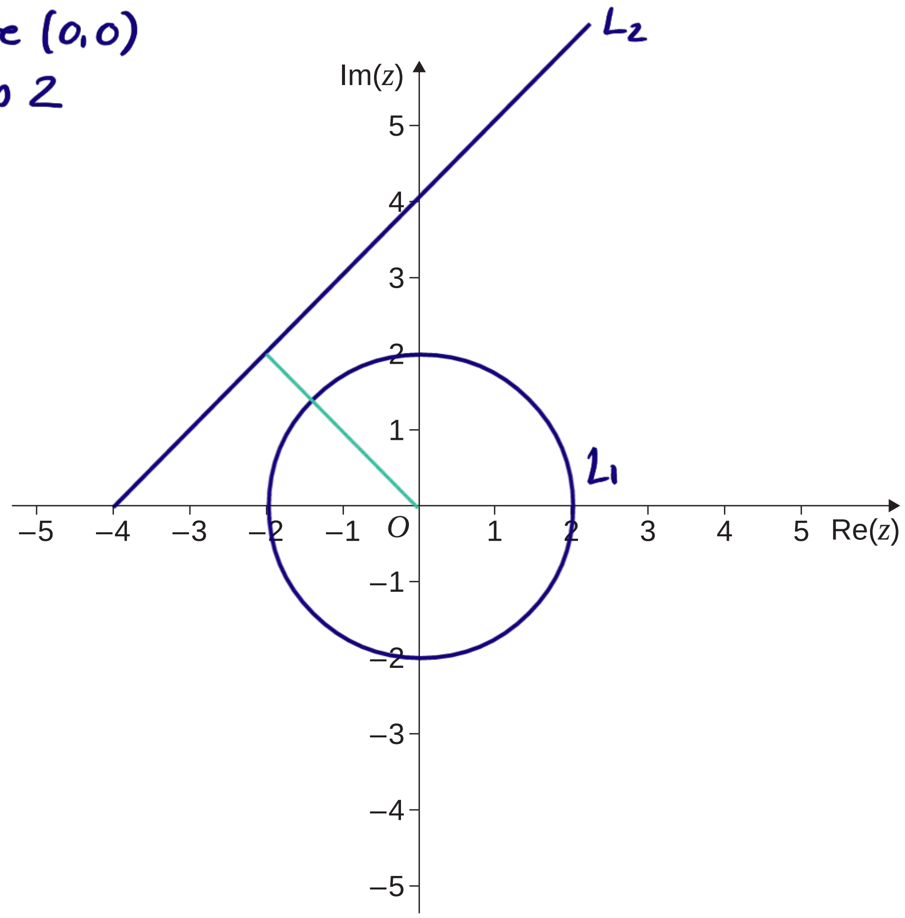
18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z+4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.

[1 mark]

L_1 : centre $(0, 0)$
Radius 2



18 (b) Sketch L_2 on the Argand diagram above.

[1 mark]

Gradient $= \tan\left(\frac{\pi}{4}\right) = 1$

Straight line from $(-4, 0)$

Do not write outside the box

18 (c) The complex number $a + \mathrm{i}b$, where a and b are real, lies on L_1

The complex number $c + \mathrm{i}d$, where c and d are real, lies on L_2

Calculate the least possible value of the expression

$$(c - a)^2 + (d - b)^2$$

[3 marks]

Find where line $y = -x$ and $y = x + 4$ intersect

$-x = x + 4$

$-2x = 4$

$x = -2, \ y = 2 \quad (-2, 2)$

Calculate distance from $(-2, 2)$ to origin.

$\sqrt{(-2-0)^2 + (2-0)^2} = 2\sqrt{2}$

Shortest distance $= \sqrt{8} - 2$

$\therefore$ least possible value $= (\sqrt{8} - 2)^2$

$= 8 - 4\sqrt{8} + 4$

$= \boxed{12 - 8\sqrt{2}}$

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**