Questions

Q1.

Unless otherwise stated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

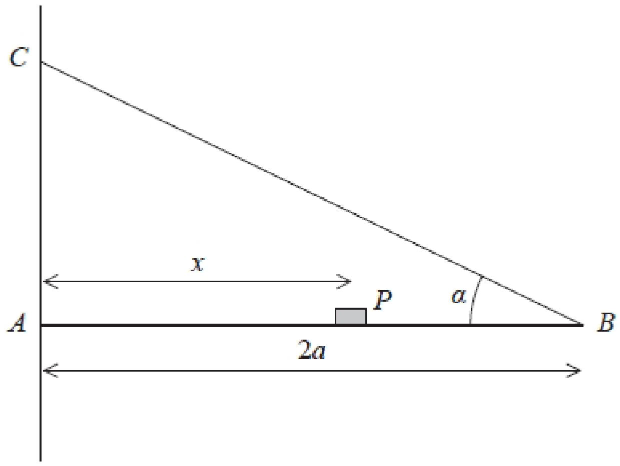


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall.
The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$.
The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is 2 Mg.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of tan β

(5)

The rope will break if the tension in it exceeds 5 Mg.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

(Total for question = 13 marks)

Q2.

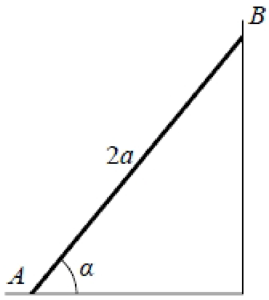


Figure 1

A uniform ladder *AB*, of length 2*a* and weight *W*, has its end *A* on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end *B* of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight 7*W* stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude *P* to the ladder at *A*, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at *B* has magnitude 3*W*.

(5)

(b) Find, in terms of *W*, the range of possible values of *P* for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for question = 13 marks)

Q3.

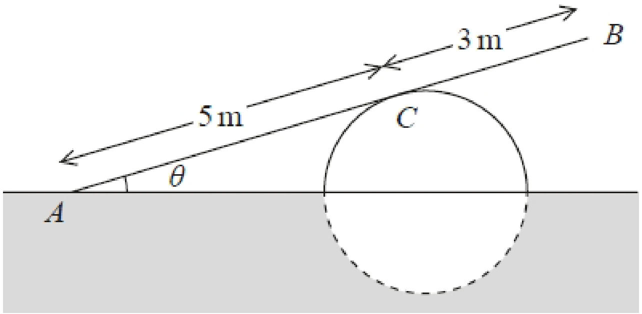


Figure 2

A ramp, *AB*, of length 8 m and mass 20 kg, rests in equilibrium with the end *A* on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as *A*.

The point of contact between the ramp and the drum is *C*, where *AC* = 5 m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum,

at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point *C* acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at *A*.

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to *A* than to *B*,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at *C*.

(1)

(Total for question = 11 marks)

Q4.

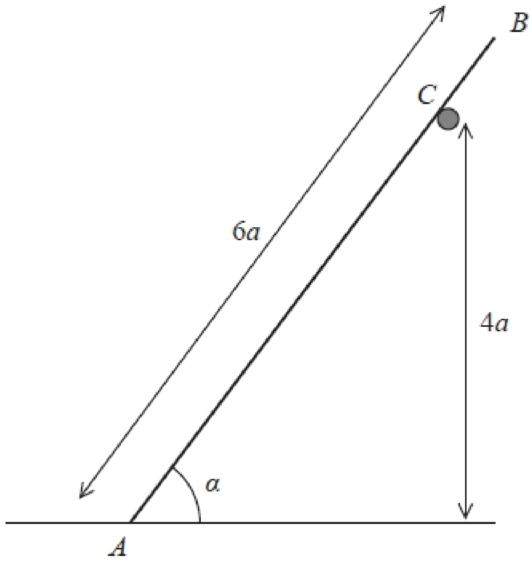


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

(Total for question = 10 marks)

Q5.

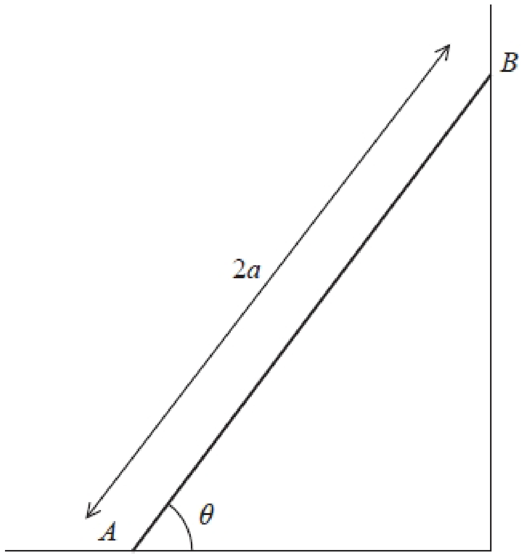


Figure 2

A beam *AB* has mass *m* and length 2*a*.

The beam rests in equilibrium with *A* on rough horizontal ground and with *B* against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geq \frac{1}{2} \cot \theta$

(5)

A horizontal force of magnitude *kmg*, where *k* is a constant, is now applied to the beam at *A*.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan \theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of *k*.

(5)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a+3x)}{2a \times \frac{3}{5}} = \frac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \frac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \frac{Y}{X}$ or $\frac{R\sin\beta}{R\cos\beta}$ $= \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \le \frac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	
			(13 marks)

Notes:
(a) **M1**: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only. **A1**: Correct equation **A1***: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.
(b) **M1**: Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the <u>given</u> expression for T. **A1**: Accept $0.67a$ or better
(c) **M1**: Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules **A1 ft**: Correct equation <u>with their x substituted in T expression</u> or using $T = \frac{2Mg}{\cos\alpha}$ **A1**: Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$ **M1**: For finding an equation **in tan** β **only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$ This is independent but must have found a Y. **A1**: Accept $\frac{-5}{4}$ if it follows from their working.
(d) **M1**: Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$) **A1**: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better. **B1**: **Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Q2.

Question	Scheme	Marks	AOs
(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Notes:

(a)

1st M1: for producing an equation in S, W and α only

1st A1: for an equation that is correct, or which has one error or omission

2nd A1: for a fully correct equation

2nd M1: for use of $\tan\alpha = \frac{5}{2}$ to obtain S in terms of W only

3rd A1*: for given answer $S = 3W$ correctly obtained

(b)

B1: for $R = 8W$

1st M1: for use of $F = \frac{1}{4}R$

2nd M1: for either $P = (3W +$ their $F)$ or $P = (3W -$ their $F)$

1st A1: for a correct max or min value for a correct range for P

2nd A1: for a correct range for P

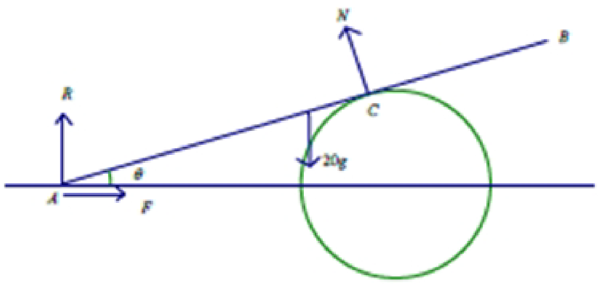
(c)

1st M1: for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder

2nd M1: for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder

3rd M1: for concluding that this increases the limiting friction at A

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The drum is smooth so there is no friction; thus there is no component parallel to the ramp and therefore the reaction is perpendicular to the ramp	B1	This mark is given for a correct explanation stated
(b)			
	M(A): $5N = 20g \times 4 \cos \theta$	M1	This mark is given for a method to find moments about A
	$N = 16g \cos \theta$ $N = 150$	A1	This mark is given for a correct value for N
	$\updownarrow$ $R + N \cos \theta = 20g$	M1	This mark is given for finding an equation in R by resolving vertically
	$R + N \times \frac{24}{25} = 20g$	A1	This mark is given for a correct equation in R
	$\updownarrow$ $F = N \sin \theta = 20g$	M1	This mark is given for finding an equation in F by resolving vertically
	$F = N \times \frac{7}{25}$	A1	This mark is given for a correct equation for F
	$R = 51.5$ N, $F = 42.1$ N	M1	This mark is given for using trigonometry to correctly solve for R and F
	$\lvert \text{Force} \rvert = \sqrt{51.5^2 + 42.1^2} = 66.5$ N	M1	This mark is given for a method to find the resultant force
		A1	This mark is given for correctly finding the resultant force
(c)	The magnitude of the normal reaction will decrease	B1	This mark is given for a correct reason given

Q4.

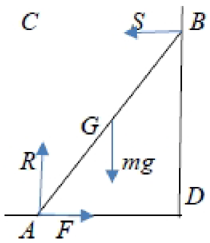
Question	Scheme	Marks	AOs
(a)	Take moments about A	M1	3.3
	$N\times\dfrac{4a}{\sin\alpha}=Mg\times 3a\cos\alpha$	A1	1.1b
	$\dfrac{9Mg}{25}$ *	A1*	1.1b
		(3)	

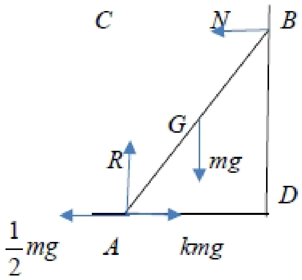
(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F=\dfrac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow)\ R+\dfrac{9Mg}{25}\cos\alpha=Mg$	A1	1.1b
	Other possible equations: $(\nwarrow),\ R\cos\alpha+\dfrac{9Mg}{25}=Mg\cos\alpha+F\sin\alpha$ $(\nearrow),\ Mg\sin\alpha=F\cos\alpha+R\sin\alpha$ $\text{M}(C),\ Mg.2a\cos\alpha+F.5a\sin\alpha=R.5a\cos\alpha$ $\text{M}(G),\ \dfrac{9Mg}{25}.2a+F.3a\sin\alpha=R.3a\cos\alpha$ $\text{M}(B),\ Mg.3a\cos\alpha+F.6a\sin\alpha=R.6a\cos\alpha+\dfrac{9Mg}{25}a$ $(F=\dfrac{36Mg}{125}\ ,\ R=\dfrac{98Mg}{125})$		
	$F=\mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b

	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow), Y + \frac{9Mg}{25} = Mg\cos\alpha + X\sin\alpha$ $(\nearrow), Mg\sin\alpha = X\cos\alpha$ $(\uparrow), \frac{9Mg}{25}\cos\alpha + Y\cos\alpha = Mg$ $(\rightarrow), Y\sin\alpha + \frac{9Mg}{25}\sin\alpha = X$		
	$\text{M}(C), Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ $\text{M}(G), \frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 $\text{M}(B), Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1		
	$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

Notes:		
a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of : $\frac{4a}{\sin\alpha}, \frac{4a}{\cos\alpha}, 5a$ but use of any of : $6a, 5a\sin\alpha, 4a\cos\alpha,\dots$ or anything involving $\tan\alpha$ is M0 Also M0 if no a's in their first equation.
	A1	Correct equation, trig does not need to be substituted
	A1*	Given answer correctly obtained.
b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)
	M1	Must have 3 equations (and all 3 previous M marks)
	A1	Accept 0.37 or better

Q5.

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations <u>(they must have enough)</u> and $F \leq \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \geq \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	

(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$ S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1 + \frac{5}{4} = \frac{5k}{2}$ **M1** $k = 0.9$ **A1**	A1	1.1b
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:		
a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F \leq \mu R$ to give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)
b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao