**1.**

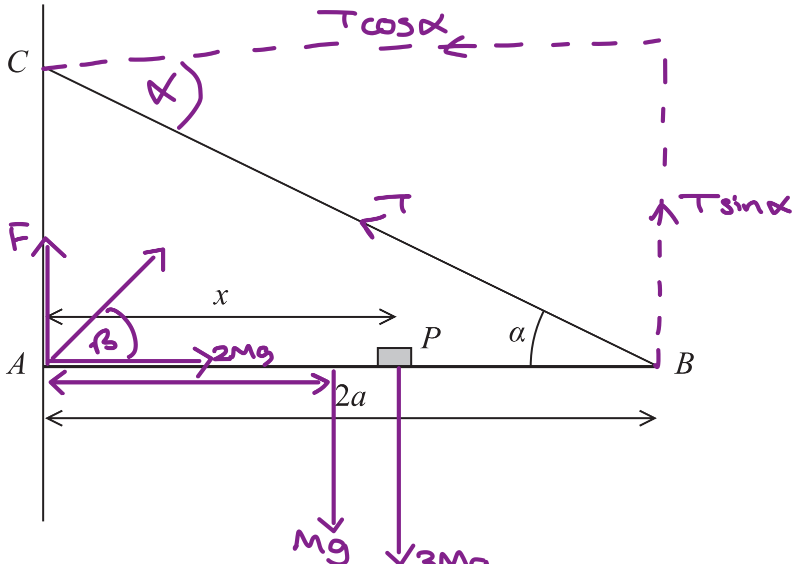



**Figure 3**

A plank, $AB$, of mass $M$ and length $2a$, rests with its end $A$ against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at $B$ and the other end is attached to the wall at the point $C$, which is vertically above $A$.

A small block of mass $3M$ is placed on the plank at the point $P$, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is $\alpha$, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

**(3)**

The magnitude of the horizontal component of the force exerted on the plank at $A$ by the wall is $2Mg$.

(b) Find $x$ in terms of $a$.

**(2)**

The force exerted on the plank at $A$ by the wall acts in a direction which makes an angle $\beta$ with the horizontal.

(c) Find the value of $\tan\beta$

**(5)**

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of $P$. You must justify your answer carefully.

**(3)**

a) Moments about A ↺ – 1

$$(a \times Mg) + (x \times 3Mg) - (2a \times T\sin\alpha) = 0$$

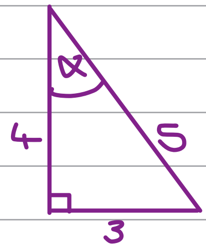
$$aMg + 3xMg - \frac{6Ta}{5} = 0$$

$$\frac{6Ta}{5} = aMg + 3xMg \quad - 1$$

$$\frac{6Ta}{5} = Mg(a + 3x)$$

$$Ta = \frac{5Mg(3x + a)}{6}$$

$$T = \frac{5Mg(3x + a)}{6a} \quad - 1$$




$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

b) R(↑):

$$T\cos\alpha - 2Mg = 0$$

$$T\cos\alpha = 2Mg$$

$$\frac{5Mg(3x + a)}{6a} \times \frac{4}{5} = 2Mg \quad - 1$$

$$\frac{20Mg(3x + a)}{30a} = 2Mg$$

$$20Mg(3x + a) = 2Mg \times 30a$$

$$20(3x + a) = 60a$$

$$3x + a = 3a$$

$$3x = 2a$$

$$x = \frac{2}{3}a \quad - 1$$

c) Moments about B $\curvearrowright$ - 1

$$(2a \times F) - (a \times Mg) - (2a - x)(3Mg) = 0$$

$$2aF = aMg + 6aMg - 3xMg$$

$$2aF = 7aMg - 3xMg$$

$$2aF = 7aMg - 2aMg$$

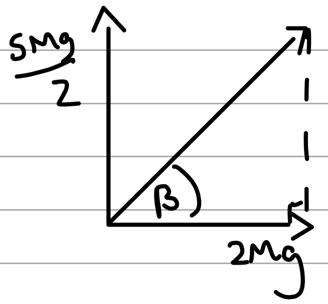
$$2F = 7Mg - 2Mg \quad - 1$$

$$F = \frac{7Mg - 2Mg}{2}$$

$$F = \frac{5Mg}{2} \quad - 1$$

(from b))

$// \; x = \frac{2}{3}a$




$$\tan\beta = \frac{\frac{5Mg}{2}}{2Mg} \quad - 1$$

$$\tan\beta = \frac{5}{4} \quad - 1$$

d) $T \leq 5Mg$

$$\frac{5Mg(3x + a)}{6a} \leq 5Mg \quad - 1$$

$$5Mg(3x + a) \leq 5Mg \times 6a$$

$$3x + a \leq 6a$$

$$3x \leq 5a$$

$$x \leq \frac{5}{3}a \quad - 1$$

P must be no further away from A than $\frac{5a}{3}$. - 1

**2.**

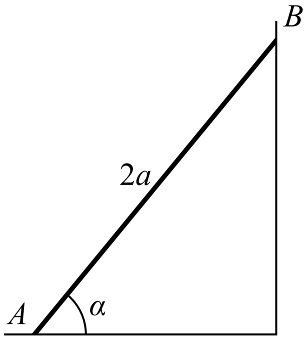



**Figure 1**

A uniform ladder $AB$, of length $2a$ and weight $W$, has its end $A$ on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end $B$ of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude $P$ to the ladder at $A$, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle $\alpha$ with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at $B$ has magnitude $3W$.

**(5)**

(b) Find, in terms of $W$, the range of possible values of $P$ for which the ladder remains in equilibrium.

**(5)**

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

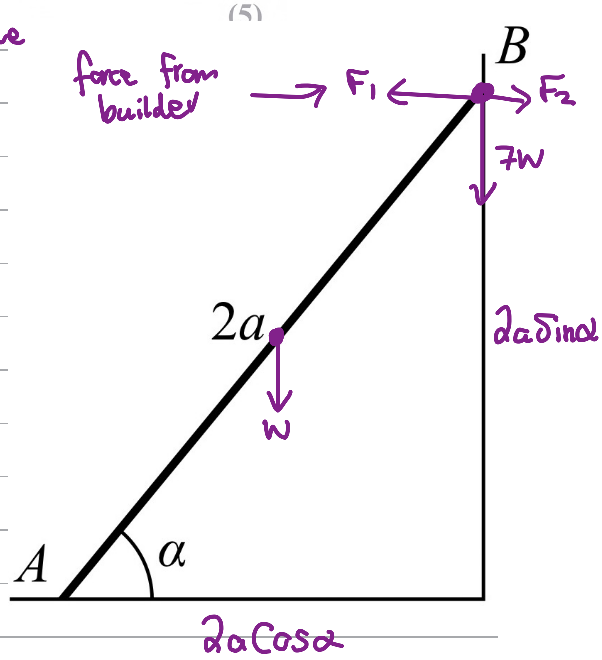
(c) Explain briefly how this helps to stop the ladder from slipping.

**(3)**

(a) Show that the reaction of the wall on the ladder at $B$ has magnitude $3W$.

(5)

Moments: at point A there is a pivot point and we have multiple forces acting on A, and they are all in equilibrium since there is no movement.




=> Take moments about A, and since all the forces are in equilibrium, their sum will add to 0. ①

• the builder • weight of ladder • Push back force

$\Rightarrow 7W \cdot 2a\cos\alpha + Wa\cos\alpha - S \cdot 2a\sin\alpha = 0$ ②

$\Rightarrow 14Wa\cos\alpha + Wa\cos\alpha - 2Sa\sin\alpha = 0$

$\Rightarrow 15Wa\cos\alpha = 2a \cdot S \cdot \sin\alpha$

$\Rightarrow 15W \cdot \cos\alpha = 2S \cdot \sin\alpha$

$\Rightarrow 15W = 2 \cdot S \cdot \frac{\sin\alpha}{\cos\alpha}$  $\left(\tan\alpha = 5/2 = \frac{\sin\alpha}{\cos\alpha}\right)$

$\Rightarrow 15W = 2 \cdot S \cdot 5/2$ ①

$\Rightarrow 15W = 5 \cdot S$  $\Rightarrow \underline{S = 3W}$ as required. ①

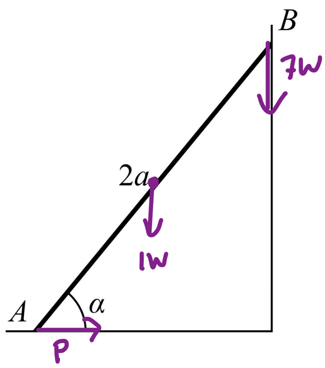
b) Perpendicular Force $= 1W + 7W = 8W$ $(F_1)$ ①

Friction $= \mu F_1 = 1/4 \times 8 = 2W$ ①

$\Rightarrow$ Max $P = 3W + 2W = 5W$ ① 3W comes from part a of the q.

$\Rightarrow$ Min $P = 3W - 2W = 1W = W$ ①

$\Rightarrow$ The range of P will be : $\underline{W \leq P \leq 5W}$ ①




c) By standing on the bottom of the ladder this helps to stop the ladder from slipping since :

• by taking moments at A, the reaction on the ladder at B is unchanged. ①

• More weight on the ladder, so the perpendicular force $(F_1)$ will increase ①

• Since $F_1$ increases, friction will increase since friction $= F_1 \cdot \mu$. ①

**3.**

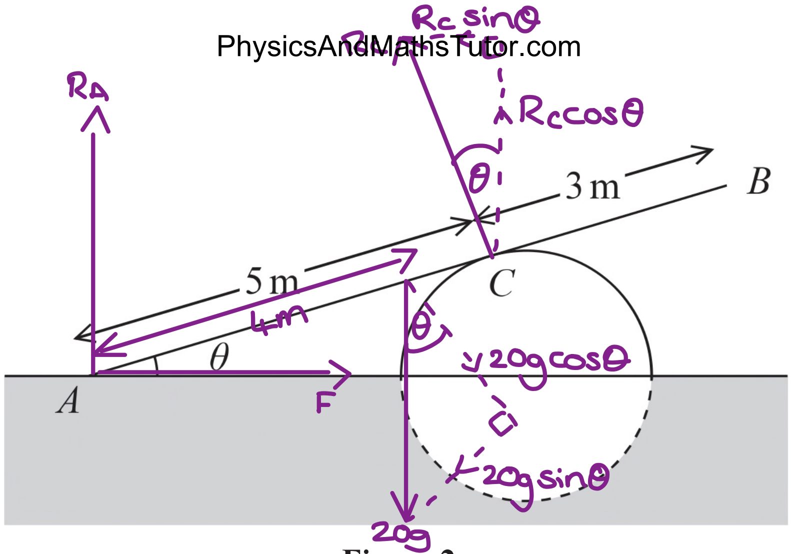



**Figure 2**

A ramp, *AB*, of length 8 m and mass 20 kg, rests in equilibrium with the end *A* on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as *A*.

The point of contact between the ramp and the drum is *C*, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle $\theta$ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point *C* acts in a direction which is perpendicular to the ramp.

**(1)**

(b) Find the magnitude of the resultant force acting on the ramp at *A*.

**(9)**

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to *A* than to *B*,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at *C*.

**(1)**

a) As the drum is smooth (no friction), the reaction is therefore perpendicular to the ramp. - 1

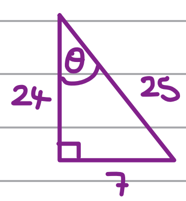
b) <u>moments about A</u>

$(4 \times 20g\cos\theta) - (5 \times R_c) = 0$

$5R_c = 80g\cos\theta$ 2

$R_c = 16g\cos\theta$

$R_c = \frac{384g}{25}$




$\tan\theta = \frac{7}{24}$

$\cos\theta = \frac{24}{25}$

$\sin\theta = \frac{7}{25}$

b) R(↑) $\Rightarrow F = ma$

$$R_A + R_C\cos\theta - 20g = 0 \quad - 2$$

$$R_A + \frac{9216g}{625} - 20g = 0$$

$$R_A = \frac{3284g}{625} \quad - 1$$

R(→)

$$F - R_C\sin\theta = 0 \quad - 2$$

$$F - \frac{2688g}{625} = 0$$

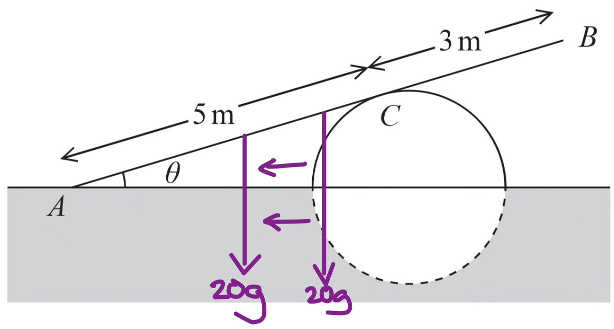
$$F = \frac{2688g}{625} \quad - 1$$


$$\text{Resultant} = \sqrt{(R_A)^2 + (F)^2} \quad - 1$$

$$= \sqrt{\left(\frac{3284g}{625}\right)^2 + \left(\frac{2688g}{625}\right)^2}$$

$$= 66.5\text{ N (3s.f.)} \quad - 1$$

c)




centre of mass of AB has moved towards A, and so the clockwise moment about A decreases (as distance from A to the force decreases) - which means the anticlockwise moment does too.

The magnitude of the normal reaction at C will decrease - 1

**4.**

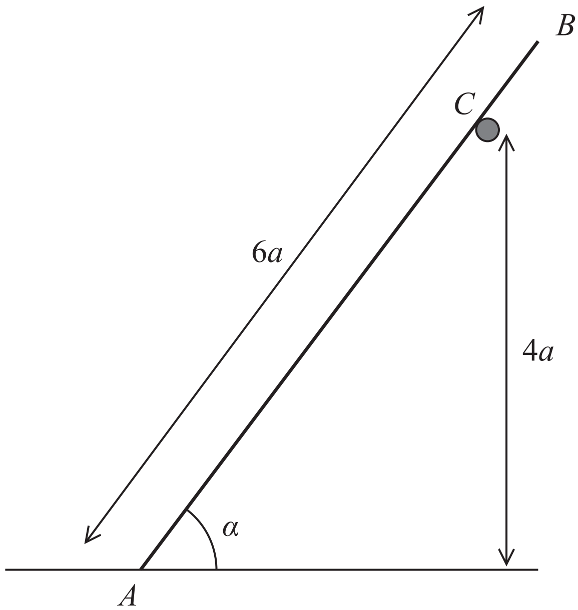



**Figure 1**

A ladder $AB$ has mass $M$ and length $6a$.

The end $A$ of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point $C$.

The point $C$ is at a vertical height $4a$ above the ground.

The vertical plane containing $AB$ is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle $\alpha$, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is $\mu$.

The ladder rests in limiting equilibrium. $\rightarrow F = \mu R$

The ladder is modelled as a uniform rod.

$\rightarrow$ Centre of mass is half-way.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at $C$ is $\frac{9Mg}{25}$

$\hookrightarrow N$

**(3)**

(b) Hence, or otherwise, find the value of $\mu$.

**(7)**

a)

Take moments about A ✓

Moment = Force × Distance to the Pivot (A)

acw moment = cw moment

acw = anticlockwise  cw = clockwise.

$$N \times \frac{4\cancel{a}}{\sin\alpha} = Mg\cos\alpha \times 3\cancel{a} \checkmark$$

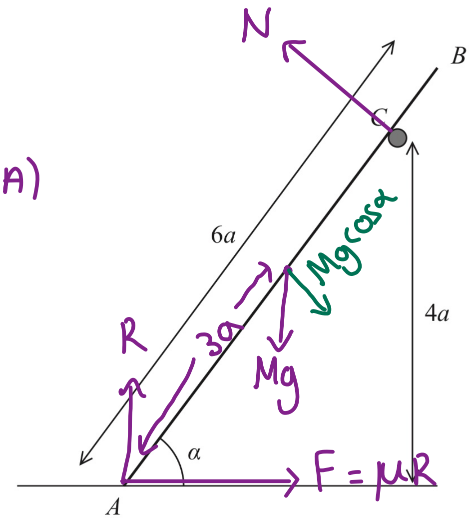
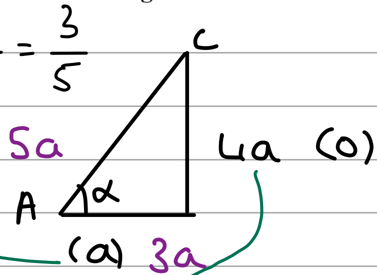



Figure 1

$$N = Mg\cos\alpha \times \frac{3}{4}\sin\alpha$$

$$\cos\alpha = \frac{a}{h} = \frac{3\cancel{a}}{5\cancel{a}} = \frac{3}{5}$$



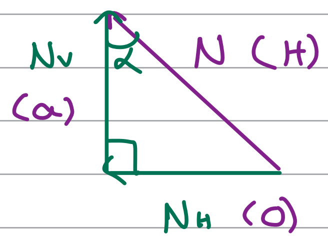

$$= Mg \times \frac{3}{5} \times \frac{3}{\cancel{4}} \times \frac{\cancel{4}}{5}$$

$$\sin\alpha = \frac{4}{5} = \frac{o}{h}$$

$$N = \frac{9}{25}Mg \text{ (as required)} \checkmark$$

$$\sin\alpha = \frac{o}{h} \rightarrow h = \frac{o}{\sin\alpha}$$

$N = \frac{9Mg}{25}$




Resolving Horizontally :

$F = N_H \longrightarrow F = N\sin\alpha$

$\therefore F = \frac{9Mg}{25}\sin\alpha$

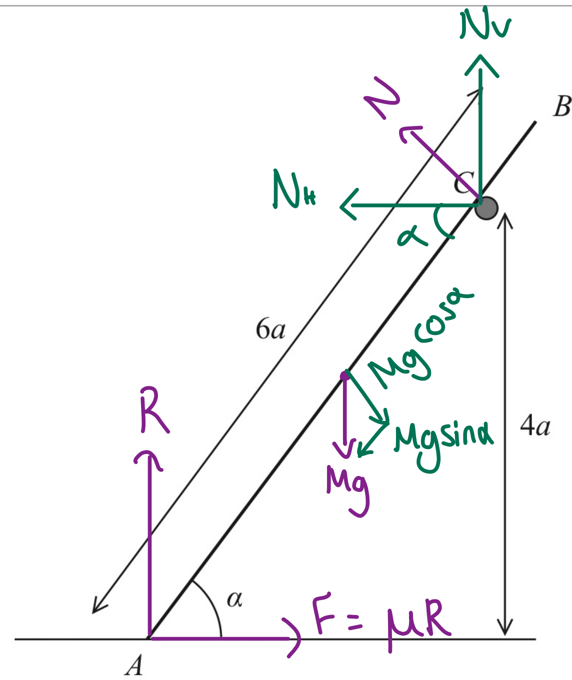



**Figure 1**

Resolving Vertically :

$R + N_V = Mg \longrightarrow R + N\cos\alpha = Mg$

$R + \frac{9Mg}{25}\cos\alpha = Mg$

Using $F = \mu R$

$$\frac{9\cancel{Mg}}{25}\sin\alpha = \mu\left(\cancel{Mg} - \frac{9\cancel{Mg}}{25}\cos\alpha\right)$$

$$\frac{9\sin\alpha}{25} = \mu\left(1 - \frac{9\cos\alpha}{25}\right)$$

$\sin\alpha = 4/5$
$\cos\alpha = 3/5$

$$\frac{9}{25}\times\frac{4}{5} = \mu\left(1 - \frac{9}{25}\times\frac{3}{5}\right)$$

$$\mu = \frac{(9/25 \times 4/5)}{(1 - 9/25 \times 3/5)} = \frac{18}{49}$$

**5.**

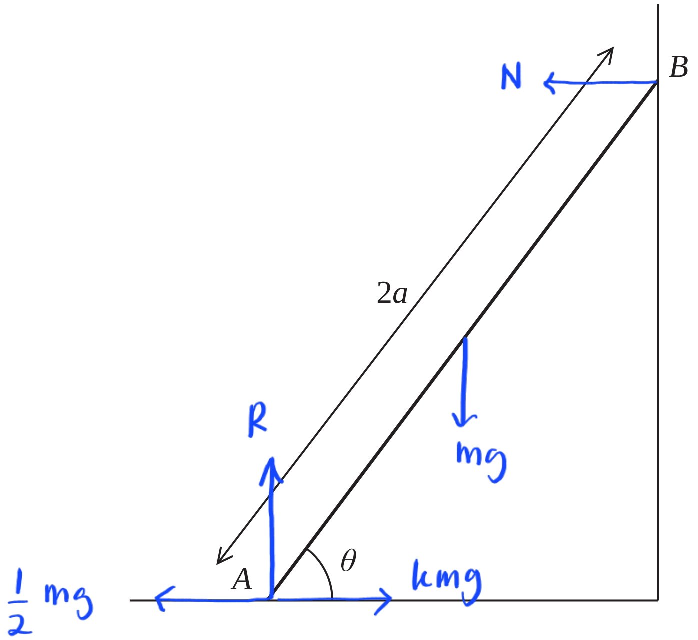



**Figure 2**

A beam $AB$ has mass $m$ and length $2a$.

The beam rests in equilibrium with $A$ on rough horizontal ground and with $B$ against a smooth vertical wall.

The beam is inclined to the horizontal at an angle $\theta$, as shown in Figure 2.

The coefficient of friction between the beam and the ground is $\mu$

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

**(5)**

A horizontal force of magnitude $kmg$, where $k$ is a constant, is now applied to the beam at $A$.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of $k$.

**(5)**

(a)

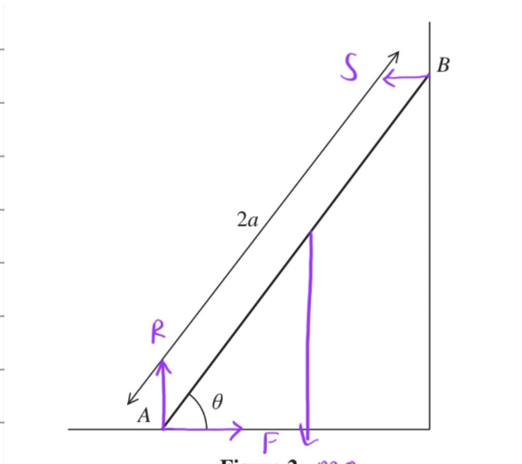



Figure 2

moment equation at A $= mg \times a\cos\theta = S \times 2a\sin\theta$ — ①

Vertically : $R = mg$ — ②

Horizontally : $F = S$ — ③ substitute ② and ③ into ①

$\therefore \quad Ra\cos\theta = 2Fa\sin\theta$

$$F = \frac{Ra\cos\theta}{2a\sin\theta}$$

$$F = \frac{1}{2}R\cot\theta$$

However, $F \leq \mu R$ which means : $\frac{1}{2}R\cot\theta \leq \mu R$

$$\mu \geq \frac{1}{2}\cot\theta$$

the $\mu R$ should be more than F because the beam is at rest.

(b)

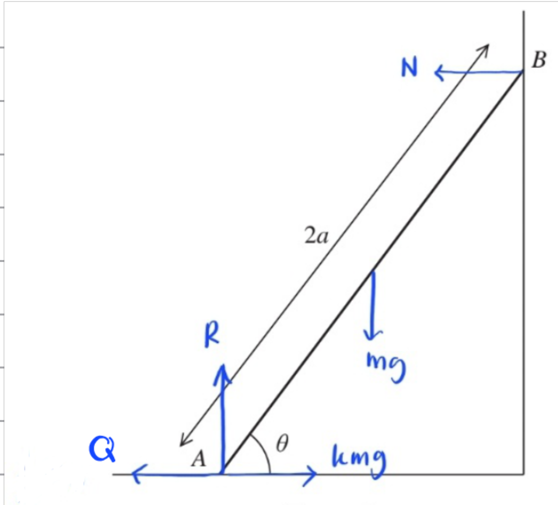



**Figure 2**

1 1

Moments equation at A : $mg \times a\cos\theta = N \times 2a\sin\theta$ (1)

1 Vertical forces : $R = mg$ (2)

$\mu = 1/2$ (given in question)

Horizontal forces : $N + Q = kmg$

However, $Q = \mu R$ (limiting equilibrium) : $Q = \frac{1}{2}mg$

$R = mg$ derived from (2)

$\therefore N + Q = kmg$

$N + \frac{1}{2}mg = kmg$

$N = kmg - \frac{1}{2}mg$

$\therefore N = kR - \frac{1}{2}R$ — (3)

Substitute (2) and (3) into (1)

$R\cancel{a}\cos\theta = (kR - \frac{1}{2}R) \times 2\cancel{a}\sin\theta$ 1

$\cancel{R}\cos\theta = 2\cancel{R}(k - \frac{1}{2})\sin\theta$

$$\cos\theta = 2\sin\theta\left(k - \frac{1}{2}\right)$$

$$\frac{\cos\theta}{2\sin\theta} = k - \frac{1}{2}$$

$$\frac{1}{2}(\cot\theta) = k - \frac{1}{2}$$

Given, $\tan\theta = \frac{5}{4}$

$$\frac{1}{2}\left(\frac{1}{\tan\theta}\right) = k - \frac{1}{2}$$

$$\frac{1}{2}\left(\frac{4}{5}\right) = k - \frac{1}{2}$$

$$\frac{4}{10} = k - \frac{1}{2}$$

$$k = \frac{4}{10} + \frac{1}{2}$$

$$k = \frac{9}{10}$$

$$k = 0.9 \quad \# \quad 1$$