Moments Cheat Sheet

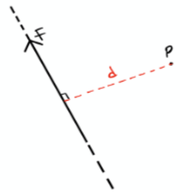
In this chapter, we will start to deal with objects that are modelled as rigid bodies. A rigid body is a solid body which does not deform when it is subject to a force.

Moments

The moment of a force is a measure of the turning effect produced by the force on a rigid body. When describing a moment, we must specify the direction of rotation as clockwise or anti-clockwise.

The moment of a force F about a point P is defined as the magnitude of the force multiplied by the perpendicular distance from the line of action of the force to the pivot P:

- Moment of F about $P = |F| \times d$

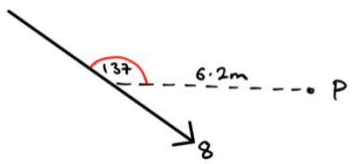


In the above case, the moment is clockwise as the force produces a turning effect in the clockwise direction. Note that since the moment is force (N) multiplied by distance (m), the units for the moment of a force is Nm.

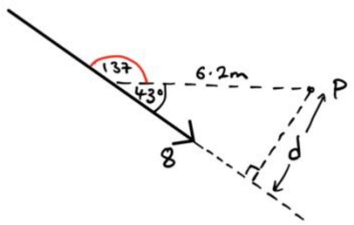
Often, you will need to use basic trigonometry to find the perpendicular distance required to calculate the moment.

Example 1: Calculate the moment about P of the force 8N acting on a lamina.

To find the moment, we need to multiply 8 by the perpendicular distance from the line of action of the force to the point P.



We draw a new diagram, labelling the required distance, d. We can use trigonometry to see that the distance d is given by $6.2\sin(43°)$.



Therefore, the moment will be $8 \times 6.2\sin(43°)$.
$= 33.83\ Nm$ anticlockwise.

Resultant moments

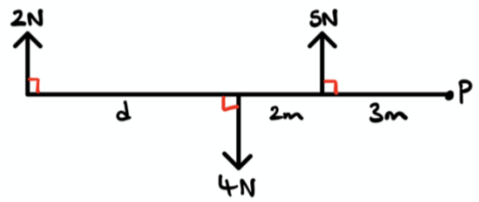
When we have multiple coplanar forces acting upon a rigid body, we can work out the resultant turning effect on the body by calculating the sum of moments produced by each force. You must pick one direction to be positive and the other to be negative in such situations. It does not matter which you pick.

Important Terminology

Uniform = All the mass of the rod acts through the middle.
Non-uniform = The mass does not act through the middle of the rod.
Light = The rod has no mass.

Often questions contain a lot of information, so it can be useful to highlight the important aspects.

Example 2: The diagram shows a set of forces acting on a light rod. The resultant moment about P is $17Nm$ clockwise. Find the length, d.



We calculate the moment of each force about P:	Moment of 2N force $= 2(5+d)$ clockwise Moment of 4N force $= 4(5)$ anti-clockwise Moment of 5N force $= 5(3)$ clockwise
Working out the resultant moment	Resultant moment $=$ $2(5+d) - 4(5) + 5(3)$ clockwise
Equate this to 17 and solve for d	$\Rightarrow 2(5+d) - 4(5) + 5(3) = 17$ $\therefore\ 5 + d = 11 \therefore d = 6m$

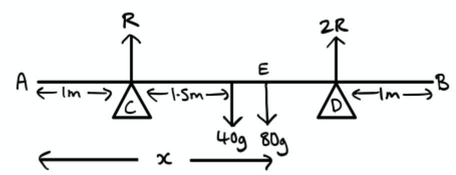
Equilibrium

If a rigid body is in equilibrium, this means that

1. The resultant moment about any point is 0 Nm
2. the resultant force in any direction is 0 m.

The above two facts play a key role in solving problems where rigid bodies are in equilibrium.

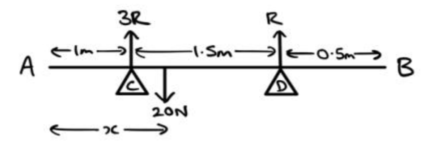
Example 3: A uniform beam AB, of mass 40kg and length 5m, rests horizontally on supports at C and D, where $AC = DB = 1m$. When a man of mass 80kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C. By modelling the beam as a rod and the man as a particle, find the distance AE.

Start by drawing a diagram detailing all the forces acting. The rod is non-uniform and we are not told where the centre of mass is, so we let the distance of the centre of mass from A be x.	
The rod is in equilibrium, so we can resolve in the vertical direction:	$R + 2R = 40g + 80g \therefore R = \frac{120g}{3} = 40g$
The resultant moment is 0 about any point (since the rod is in equilibrium) so we can sum up the moments about A and equate to 0: (here we took the anticlockwise direction to be positive)	Moment of R $= R(1) = R$ anticlockwise Moment of 2R force $= 2R(4) = 8R$ anticlockwise Moment of beam $= 40g(2.5)$ clockwise Moment of man $= 80g(x)$ clockwise $\therefore\quad R + 8R - 40g(2.5) - 80g(x) = 0$
We know $R = 40g$ so we can solve for x:	$\Rightarrow 80gx = 2548 \therefore x = \frac{13}{4}$

Centres of mass

So far, we have only considered uniform rods where the weight acts at the midpoint of the rod. You may need to solve problems involving rods which are non-uniform. This means that the weight no longer acts at the midpoint.

Example 4: A non-uniform rod AB is 3m long and has weight $20N$. It is in a horizontal position resting on supports at points C and D, where $AC = 1m$ and $AD = 2.5m$. The magnitude of the reaction at C is three times the magnitude of the reaction at D. Find the distance of the centre of mass of the rod from A.

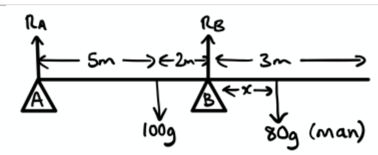
Start by drawing a diagram detailing all the forces acting. The rod is non-uniform and we are not told where the centre of mass is, so we let the distance of the centre of mass from A be x.	
The rod is in equilibrium, so we can resolve in the vertical direction;	$3R + R = 20 \therefore R = \frac{20}{4} = 5N$
The resultant moment is 0 about any point (since the rod is in equilibrium) so we can sum up the moments about A and equate to 0: (here we took the anticlockwise direction to be positive)	Moment of $3R$ force $= 3R(1) = 3R$ anticlockwise Moment of R force $= R(1+1.5) = 2.5R$ anticlockwise Moment of weight $= 20(x)$ clockwise $\therefore\quad 3R + 2.5R - 20x = 0$
Finding x	$\Rightarrow x = \frac{5.5R}{20} = \frac{5.5}{20}(5) = \frac{11}{8}m$

Tilting

Finally, you need to be able to solve problems involving rods on the point of tilting. The following fact is vital for such questions:

- If a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.

Example 5: A uniform plank of mass 100kg and length 10m rests horizontally on two smooth supports, A and B, as shown in the diagram. A man of mass 80kg starts walking from one end of the plank, A, to the other end. Find the distance he can walk past B before the plank starts to tip.

Start by drawing a diagram detailing all the forces acting: Let the distance of the man from B at the point of tilting be x.	
When the man is at a distance x from B, the rod is on the point of tilting about B so the reaction at A is 0. Taking moments about B:	Moment of man's weight $= 80g(x)$ clockwise Moment of plank's weight $= 100g(2)$ anticlockwise.
Using the fact that the resultant moment is equal to 0: (here we took the clockwise direction to be positive)	$80g(x) - 100g(2) = 0$ $\therefore x = \frac{200g}{80g} = \frac{5}{2}m$
Finding x	$\Rightarrow x = 2.5m$